NOTES FOR THE INTRODUCTION OF AN INFORMATION VARIABLE IN MODELS OF DISTRIBUTION OF RETAIL TRADE

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I. THE BASIC MODEL OF RETAIL DISTRIBUTION AND GROWTH

In the basis of all models of retail development and distribution, one simple relation will be present:

Retail sales are the function of the size of the population in trading areas.

More sophisticated models of retail distribution will, in fact, take note of some particular part of the stated relation and from a different standpoint such as:

- What will be the relation of size or number or class among different retail establishments (over time)?
- What is the trading area or what are the probabilities for attracting the consumers from particular areas, etc.?

The basic relation, though fundamental for all other models, by itself is not practically useful as a model that can be applied to the distribution of the retail potential inside the city and its direct hinterland. The reason for this is because the relation holds directly only if the main part of the retail-trade-exchange-system between retail establishments and population is maintained within the level of the unit that is under observation. And the lowest level on which that occurs is the city and its hinterland. No matter how simple and obvious, the relation should be examined and proven empirically.

An analysis of the relation between retail sales and population in different retail trade systems shows that the stated relation holds almost perfectly: the sales are not only a function of the size of the population but take the most simple linear form:
FIG. 1—Frequency distribution of towns according to total sales per person.

FIG. 2—Relationship between sales and population.
\[ y = ax + b \]

where \( a \) appears to be close to 1 and \( b \) close to 0.

As an empirical example, we can take the relations found in Scotish towns in a study by T. B. Fleming (1961). The correlation coefficient between the size of sales and the size of population of the city and its hinterland was 0.98, and the regression line actually was (without constant; that is balancing the actual scale of measures of population and money on the \( x \) and \( y \) axis):

\[ y = 0.824X + 0.721 \]

We could ignore completely the given deviation if we did not know that it represented at least two of many possible influencing variables other than the physical existence of the population:

a) Eventual unequal distribution of purchasing outside of own city retail system in favor of bigger cities.

b) Eventual unequal consumer per capita expenditures between larger and smaller cities in favor of larger cities as a consequence of the difference in the income of the two.

The latter was probably not the case in this example because the majority of cities in Scotland, taken in this analysis, form a conurbation with quite similar conditions in respect to income per capita in the different cities inside it. (see figure 2)

But it might cause remarkable deviation in the national systems where the process of urbanization and general development is still very dynamic and also the difference between cities of different size in different regions with respect to per capita income and expenditures can
be more than double. This is the case in some Yugoslav regions. For example, the difference in per capita income between Slovenia and Kosmet in 1960 was 4:1 but the difference in per capita value of retail establishments in the same regions was even larger - 6:1.

The described model for determining the retail variable as the function of the size of the city and the time variable (which represents in fact the variable of consumer expenditures) is, no doubt, the most universally and the most extensively applied model in actual city planning practice. But there it does not appear as any specific and explicit theoretical and empirical model but usually in the form of GIVEN PER CAPITA STANDARDS FOR RETAIL SPACE REQUIREMENTS

Our previous discussion allows us to make some remarks upon this basic model of retail distribution:

a) It is valid only when starting from the lowest level of formation of the retail exchange system - and that is the city itself. Using it, we can eventually determine or predict what future requirements (on performances) of the total city retail system will be, but we will not be able to determine where in the city and in what relation to particular communities these total requirements or capacities should appear.

b) In the national systems with differences in income between the cities of different size (or up to the level of expected equalization in time) the model must be three-dimensional at least, i.e., the size of the city, the time point (or level of consumer per capita expenditures) must be distinguished in
order to determine the temporary standard for retail requirements (or performances).

c) As the influence of the time variable makes changes in the system and its requirements exponential, the determining of only one point on the exponential curve as the representative of all the relevant points (and that is, in fact, the meaning of a fixed standard of, say, retail space per capital) does not appear as a very meaningful procedure.

d) There is a case in which one value of $y$ could be a reasonably accurate representative of all future values of $y$. This case is when the exponential growth of economic systems as well as consumer expenditures stabilize after a certain point.

This stabilization can be a consequence of limited natural resources (Meadows 1972) or a consequence of the decision of creative human beings to prevent further devaluation of the human world (Marx 1844) (whatever occurs first), and in that case the exponential force of growth would be changed to logistic form.

As the conditions for relevant qualifications of a "logistic" form of retail growth are beyond the abilities of this paper, and probably beyond the practical reach of today's information system, we will stay within the actual problems of growth.

Most often, the basic model of retail growth and distribution which is built on the basis of the relation of size of population to some representative variable of retail systems (sales level, total space, or
number of employees, etc.) can be effectively used in a form of given per capita standard relations (or per family or any other population unit measure) if:

a) the level in question is not lower than the city and its hinterland, and,

b) the standard relation between population variable and retail variable is not expressed as a single value but rather as a matrix that contains different values in rows (for different sizes of cities) and different values in columns (for different future years).

For planning action below the city level of aggregation, additional relations, assumptions, and models are necessary.
II. GENERAL SYSTEM THEORY AND RETAIL GROWTH DISTRIBUTION

Though the city, in general, and its systems of central activities (exchange activities), of which retail is probably the most important one, appear to be an ideal field for initiating and testing the methods and theories of the general system approach, very few practical inputs and references have come from this side up to now. This is easily understandable if the ever-growing complexity of the systems, from static structures to socio-cultural systems, is taken into account. The decreasing availability of theories and models that can practically and effectively operate is related not only to the retail system but to almost all socio-cultural systems.

Naturally, many models and problem solving algorithms in city planning use some elements of General System Theory or Cybernetics (feedback, trial-and-error, etc.). But one of the rare direct applications of the systems approach to the problems of growth and distribution of cities in general, and retail in particular, resulted in (well-known but rarely classified to any particular discipline) The Rank-Size Rule.

The Rank-Size Rule states that if the existing cities in any system of cities (national, continental, or universal) were ordered by their rank (rank by size) on the abscissa and if the values of the ordinate represented the size of the city (population), then the relation between rank and size of the city would always follow the same, simple, and permanent mathematical regularity.

This regularity is found valid for many other systems and their parts and so is found valid for retail systems too.
Fig. 9-2. Metropolitan districts. One hundred largest in the U. S. A. in 1940, ranked in the order of decreasing population size.

Fig. 9-6. Retail stores (including chain stores) in the U. S. A. in 1939, ranked in the order of decreasing number of stores of like kind.
Usually, rank-size rule stays out of classifications and attachment to any particular discipline, but in fact, it is the direct contribution of General System Theory to the urban planning field, because the argument that tries to explain the cause of regularity, given by K. G. Zipf (1949) is the typical system-approach argument.

According to a footnote of K. G. Zipf (1949, pp. 374), the first recognition of regular rectilinear distribution of rank and size of communities in a country came from Felix Auenback in 1913, and was taken over by A. T. Lotka in 1925. It is worth noticing that A. T. Lotka was indicated by L. von Bertalanffy (1968, pp. 11) as formulator of the basic concepts of general system theory. Later extensive studies in the problems of regularity of distribution of city rank, size and frequency of occurrence, were presented by K. G. Zipf (1940, 1942, 1949) and even more extensively by B. Berry (1967 and others) with particular emphasis on retail establishments.

Zipf's formulation of rank-size rule assumes that:

\[ R \cdot P^2 = K \]

where:

- \( R \) = rank of community in terms of population size
- \( P \) = the actual number of inhabitants of the community
- \( 2 \) = the positive exponent parameter
- \( K \) = a constant for the system

The perception of regularity is much more obvious if the graphic presentation is plotted in a doubly logarithmic coordinate system, because that way the hyperbola in the equation attains a linear form in logarithmic space as:

\[ P = \frac{-1}{2^R} \cdot C \]
and more than that, in many rank-size or rank-frequency distributions, the line forms an angle of 45° in respect to the coordinate axes.

Explanations of this regularity by K. G. Zipf have been seen in the general tendency of most of the active human systems and subsystems to arrange their parts according to general harmonic series of which equation 4 is one expression. But the cause of the tendency for harmonic arrangement is not anything but the natural tendency of human systems to achieve the principle of least effort.

In general, the principle of least effort states that: any active human system will tend to arrange its working parts and actions in the way which will require the least possible effort, or the best (most even) distribution of effort overtime or, preferably, both.

It seems very easy and useful to accept the principle of least effort, not only because of its simplicity and quite strong empirical argumentation but also because one or another formulation of least effort (least work, or time, or money cost) has been traditionally the most common criteria for any rational planning evaluation and remain the most common behavioral assumption in the models with behavioral assumptions.

The main property of rank-size distribution is its permanent regularity. That means from one state of the system it is easy to derive the next state by the assumption of change in any of its parts, or by assumption the total sum of changes in the whole system, and to distribute the appropriate quantities to each individual part.

A similar equation of competition between organs in the process of growth is known in biology as the Allometric equation (Berta Lanfy, 1968,
pp. 64) where if:

\[
\frac{dQ_1}{dt} = a_1 Q_1 \\
\frac{dQ_2}{dt} = a_2 Q_2
\]

is one organ, \( Q_2 \) another or total organisms, \( t \) is time (\( a_1 \) and \( a_2 \) are parameters) and allometric growth then holds:

\[
\frac{dQ_1}{dt} : \frac{1}{Q_1} = \frac{dQ_2}{dt} : \frac{1}{Q_2} = \kappa,
\]

or

\[
\frac{dQ_1}{dt} = \kappa \cdot \frac{Q_1}{Q_2} \frac{dQ_2}{dt}
\]

i.e. the relative growth rates of the parts under consideration (\( Q_1 \) and \( Q_2 \) stand in constant proportion during a reproduction cycle for which the allometric equation holds.

By eliminating time in allometric equation solution we can get:

\[
Q_1 = bQ_2^\gamma
\]

with \( \gamma = a_1/a_2 \), \( b = c_1/c_2 \)

which is similar to Zipf's equation (Bertalanff, 1968, pp. 63-64). This emphasises the validity of the principle of least effort in reproduction of living organisms, as well as the chance to rationalize the popular "organic" analogy of the city and its subsystems. It is worth remarking that the well known criterion of "organic architecture" as given by Frank Lloyd Wright contains, in fact, the meaning of the allometric equation.

The problem with application of the rank-size or frequency distribution
rule in planning retail growth and distribution is similar to the problem of application of the basic model of retail distribution and growth discussed earlier. It asks for the complete system to be internalized inside the unit of planning or observation in order to hold.

Very meaningful findings in respect to constant distribution of retail and service establishments were reported by B. L. Berry (1967 and elsewhere) similar to these examples. At the same time, the starting point of Berry's approach was Central Place Theory rather than the Principle of Least Effort or Allometric Growth. This can give the necessary link between General System Theory and some assumptions of the Central Place Theory.

One of the groups of models that are, in fact, using some concepts of general system theory are the "intervening opportunities" models.

Their general assumption, as given by Stouffer (1940, p. 846) states that:

"...the number of persons going a given distance is inversely proportional to the number of intervening opportunities".

Different applications for this basic concept have been found, but one of the best known and earliest applications in the field of retail distribution was made by Britton Harris as a component of a comprehensive model for the Penn-Jersey transportation study (Harris 1964).

The concept of competition is the most characteristic for the "intervening opportunities" and that leads it close to the Alometric growth and distribution, because the same variable is of the most important determinative influence.
It is interesting to note that the new generation of gravity models in retail trade has also included influence of competition (Huff 1966, Lakshmannan and Hansen, 1965) which means in fact that modern gravity models are a combination of gravity and intervening opportunities concepts. An interesting comparison between system theory models and gravity models is given by Ellis and Van Doran (1966), in connection with the problem of choice of recreational facilities in the region; but analogy to the problem of retail sales distribution could also be found.
III. CENTRAL PLACE THEORY - THE URBAN GEOGRAPHY APPROACH TO RETAIL DISTRIBUTION

The central place theory is another complex one that is difficult to classify in any separate discipline. Since the method is geographic, most starting assumptions are economic, and the result describes general behavior of the system, central place theory could be classified into any of those disciplines.

Regardless, it is geographer Walter Christaller who made the first attempt to generalize the observed pattern of distribution of central places in South Germany (Christaller, [1933] 1966). Through several theoretical assumptions, he tried to obtain the general laws that would make it possible to determine the number, size, and distribution of central places.

As the chief function, according to Christaller's assumptions, the city (town) is to be the center of its region. The main spatial relations between different places will be derived from the ability of central places to supply the surrounding region with central goods and services. All towns are not central places but can be specialized dispersed places that produce some special products for the markets that are not at all connected directly with their region. As we can see from this brief summary, it is the retail sector, in fact, that has the dominant role in central place theory.

The different central goods are services, naturally, have different upper ranges and different lower thresholds. But the tendency for concentrated agglomeration of establishments leads to the formation of a
discrete hierarchial order and spatial network of centers in which the lowest-rank centers will serve their basic hexagonal areas. The center of highest order will service its own basic areas with the central goods and services of the lowest rank, but it will also service several immediately attached basic hexagons with the central goods and services of the highest order, etc.

The combinations of cities at different hierarchical rank in the described hexagonal network can be different, depending on whether central goods or services have influenced the historical development of the region. So for retail and trade in general, the $K = 3$ network relation is most likely to occur (for every central place of higher rank in specialization, there are three central places of lower rank so that the hierarchy follows a geometric progression 1, 3, 9, 27 ... etc.). If the transportation principle was dominant, the $K = 4$ arrangement will occur; or if the administrative influence in arrangement was dominant, then $K = 7$, etc. The illustration of the $K = 3$ network was taken from Johnson's (1970) review in order to illustrate the meaning more clearly.

Criticism and revision of the central place theory was coming from many directions; but the most unrealistic seemed to be rigid hierarchical order produced by Christaller's generalization.

Paying more attention to the actual economic requirements of different central goods and services for the size of trading areas, Loch ([1943] 1954) came to the conclusion that the spatial arrangement of retail establishments does not follow uniform hierarchical order where the highest-order control goods have a hexagonal area that totally contains several trading
areas of lower centers. Instead, every single line of central goods and services has the hexagonal area of specific size, and if their overlapping occurs in a specific town center, the content of that center will probably be a different combination from one to another.

The control point which coordinates the different networks in the region is the metropolitan center, as well as the present tendency in the system to form the centers with as much overlapping retail and service lines as possible. The complexity of the system that is founded on these assumptions is far greater than that of Christaller's and, therefore, seems to fit better to actual contemporary conditions.

Naturally, both generalizations have their strong and weak points, but it seems that they are dealing with the different basic conditions in the space. For the areas with low density of population and small purchase power, the forces of centralization will, probably, be stronger than competition for somewhat over-extended market areas for some lines of goods whose requirements for trading areas are between the particular discrete magnitudes of centers of lower and higher rank. That would result in Christaller's pattern rather than in Loch's. For dense areas with high purchase power, Loch's pattern is more likely to result than Christaller's.

We already mentioned the work of B. Berry (1967) in connection with the rank-size rule approach but it is worth mentioning that he found in Iowa the romgoid pattern of central place distribution which leads to the conclusion that transportation networks in Iowa influenced the spatial arrangements as if the space itself has properties of Ribons along the
In this system there is a constant ratio between the number of trade centres at various levels of specialization and between the areas of their respective zones of influence.

A SIMPLE LÖSCH NETWORK

Hexagonal service areas are again assumed, with distinctive-sized zones of influence for different types of services. Here, however, there is no constant ratio between the various sizes of hexagonal service areas, so that they do not "nest" in the manner characteristic of Christaller's system.
communication lines where the central place competition and arrangement is going on, while only villages could use the continuous plane and space in between the main roads.

A promising method in central place analyses seems to be the application of topology, as presented by Nedvedkov (1968).

It is worth noting that almost all contemporary urban planning organizations of new cities or reorganization of old ones, in respect to central activities, tend to have the hierarchical structure which is more similar to Christaller's than to Lösch's interpretation of the system. However, in intensive and dense areas - such as the city - the main natural reason for that can be questioned.

Out of that fact, during the last century, the countries that have been forced to use city planning theories in practice have all reached a high level of urbanization. What must inevitably in that case, be put in doubt is the basic assumption that the chief function of the city is to serve its hinterland and region. Under this distribution, the opposite is more likely to be true: region and hinterland are serving the city. The consequence of this inversion is the tendency for cities to agglomerate, conurbate, metropolize, etc. without limits of even hierarchical distribution over space, in the places that are favorable for the cities themselves (like coastal lines or major transportation corridors, etc.) In other words, the majority of today's large cities are not central places any more according to Christaller's definition; but central place theory covers the cases of distribution of towns in low-urbanized areas.

Mathematical formulations of hierarchy like the one of Martin Bechmann
(1958), can not perform satisfactorily inside the city structure for the reason discussed above.

A serious attempt to solve this problem was made several times by Brian Berry, where one of the most interesting results was the model developed on the general assumptions of central place theory for the city of Chicago and Northeastern Illinois planning commission (Berry 1965) using a set of simple linear connections among sixteen chosen variables.

One of the outcomes of the model was the presented monograph, the utility and applicability of which, under all natural limits, are obvious.
1. **FUNCTION** Number of different Standard Industrial Classification four-digit types of business found in the center.
2. **ESTABLISHMENTS** Number of distinct retail establishments (stores) in the center.
3. **TOTAL CENTER AREA** Total site area of a planned center; total area within defined land value boundaries for an unplanned center.
4. **SHOPPING CENTER AREA** Total area minus parking for a planned center; total area minus streets and alleys but including off-street parking for unplanned centers.
5. **GROUND FLOOR AREA** Ground floor area of business establishments in center.
6. **POPULATION OF TRADE AREA** Total population residing within the intensive trade area of the center.
7. **AREA OF TRADE AREA** Intensive trade area in square miles.
8. **MEDIAN INCOME** Median income level of the market area as reported in 1960 census.
9. **SOCIAL CLASS** Factor score for the market area on the first dimension of a factor analysis of census-reported socio-economic data for the market area.10
10. **FAMILY CLASS** Factor scores on a second dimension of social and economic structure.11
11. **TOTAL COMPETITION** Total number of establishments located in the market area, excluding the center itself.

12. **PLANNED COMPETITION** That part of 11 located in planned centers.
13. **UNPLANNED COMPETITION** Likewise, in unplanned centers.
14. **RIBBON COMPETITION** Likewise, ribbon retail development.
15. **DISCOUNT COMPETITION** Likewise, discount shopping centers.
16. **POPULATION DENSITY** \( d = \frac{6}{7} \)

Let the 16 variables listed above be represented by the following letters: \( F, E, T, S, G, P, A, Y, C_6, C_7, I, J, K, L, d \). Then for unplanned centers:

\[
\begin{align*}
(7) & & E &= 1.154 + 0.016F & r^2 &= 0.96 \\
(8) & & T &= 3.946 + 0.855E & r^2 &= 0.91 \\
(9) & & S &= 3.619 + 0.922E & r^2 &= 0.88 \\
(10) & & G &= 3.374 + 0.968E & r^2 &= 0.87 \\
\end{align*}
\]

and for planned centers:

\[
\begin{align*}
(11) & & E &= 0.800 + 0.028F & r^2 &= 0.82 \\
(12) & & T &= 4.385 + 1.022E & r^2 &= 0.79 \\
(13) & & S &= 4.287 + 0.836E & r^2 &= 0.67 \\
(14) & & G &= 4.025 + 0.823E & r^2 &= 0.69 \\
\end{align*}
\]

Apparently the fits were closer when the individual

---

**Figure 1: Centers Serving Higher Income Communities**

Nomograph illustrating the interdependency of variables descriptive of unplanned business centers serving the higher income communities of the City of Chicago. To use, place a straight edge horizontally across the page and read the expected sales (1958 dollars), population of market area, number of different kinds of business provided, number of establishments, and ground floor area for any desired size of center.

<table>
<thead>
<tr>
<th>Sales ($1,000,000)</th>
<th>Population Served (000s)</th>
<th>No. of Business Types</th>
<th>No. of Establishments</th>
<th>Ground Floor Area (000s of Sq. Ft.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log. 10 - 7,000</td>
<td>Log. 4,000</td>
<td></td>
<td>1</td>
<td>Log. 20</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>50</td>
<td>100</td>
<td>4,000</td>
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<tr>
<td>30</td>
<td>60</td>
<td>80</td>
<td>100</td>
<td>5,000</td>
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<td>40</td>
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<td>6,000</td>
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<td>5,400</td>
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<td>90</td>
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<td>50</td>
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<td>5,800</td>
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<td>100 - 8,000</td>
<td>600</td>
<td>50</td>
<td>200</td>
<td>5,800</td>
</tr>
</tbody>
</table>
IV. MICROANALYSIS

Microanalysis of the retail establishment and its present or future location is the most commonly used technique in dealing with the retail structure of the city. Needless to say, it is the most accurate method too. Naturally, the different parts of microanalysis technique could not, by definition, be generalized enough to serve as means for designing and planning retail systems on the level of the whole city or region. Microanalytic methods in retail locations are designed to discover particular conditions for business operations of individual retail establishments in already existing conditions.

We can not elaborate much on the different principles of interest, but can mention that the general procedure includes the following simple steps:

a. Separation of a few possible locational alternatives.
b. Estimation of present and future trade areas of the retail outlet and possible sales.
d. Estimation of additional sales that can be expected at the location (by taking over the shoppers from competitive or complementary shops, interception of potential shoppers from other areas, sales to working daytime population, etc.), as well as increase of sales through the use of knowledge of behavioral and preferential patterns of consumers.
e. Income-cost analysis for alternative locations and a
decision about the optimal one.

Though the marketing approach and micro-economic analysis of locations for retail establishments were the methods developed to insure or increase the sales of the particular firm in expansion, they formed a very fertile basis for theoretical developments in two general areas:

- investigating the measure and meaning of retail trade areas and the influence of particular retail establishments;
- investigating patterns of consumer behavior and preferences.

These were of extreme importance for developing models and methods for retail distribution with a systematic approach on the higher level of the city or region.

One of the best classical works of this approach containing methodological, theoretical, and practical application information from this domain is probably Nelson's "The Selection of Retail Locations". His most important contribution to the theory might be the analysis of the compatibility of aggregated retail firms with each other and centers that they form (Nelson, 1958).
V. GRAVITY MODELS

A method which is equally able to be applied to the complex conditions of numerous retail establishments and their agglomerations inside the city or for the whole region appears in the form of a different family of models of gravitational interactions between retail centers and population.

The basic assumption of this family of models states that the number of interactions between some population distributed in a particular urban or inter-urban space and some center of activity in the same space will tend to be proportional to the size of the center of activity (measured in some adequate units) and inversely proportional to the distance between the population and the center. The actual form of mathematical expression of this relation differs from model to model.

The first notion of this regularity was probably defined by E. G. Ravenstein (1885) as a process of dispersion and absorption of migrants where "... migrants enumerated in a certain center of absorption will consequently grow less with the distance and proportionately to the native population which furnishes them" and: ... "the process of dispersion is the inverse of that of the absorption and exhibits similar features".

It is interesting to note that since that time all the way up to Huff's conceptualization of the probabilistic model in the early sixties the concept of dispersion of population was not used at all.

The reason for this was the traditional marketing conception of retail trade areas as closed and defined areas with the dominant influence of one
center or retail establishment. The real need for introducing concepts of probability and dispersion of population in gravity models occurred, indeed, only with intensive interest of city planners in problems of retail distribution because individual firms can approximate estimations of their own sales well enough from the concept of closed area, but city planning has had to see the particular elements working in the city system - the detail and the complex at the same time.

The real use of the gravity model methods started intensively from an empirical approval of gravitation-like regularity in patronizing alternative cities by a population whose residences were between them after hypothesis and research by W. J. Reilly (1929).

Reilly's hypothesis was that two cities attract retail trade from an intermediate populated space approximately in direct proportion to the population of the two cities and in inverse proportion to the square of the distance between each of the two cities to the intermediate point; or stated symbolically:

\[
\frac{Ba}{BB} = \frac{Pa}{PB} \frac{Db}{Da ^ 2}
\]

where

- \(Ba\) = the proportion of trade from intermediate space units attached by city A
- \(Bb\) = the proportion of the trade attracted by city B
- \(Pa\) = the population of city A
- \(Pb\) = the population of city B
- \(Da\) = the distance from city A to intermediate unit
- \(Db\) = the distance from city B to the same unit

Reilly used population size of the city as a measure of the attracting
forces. This could be questioned as an "adequate measure of retail activity" only if his investigation would not operate between two completely formed and separate retail systems represented by the cities in which those centers were situated. If we recall the observation of a 0.98 correlation between the population of the city and retail sales in Scottish cities reported by T. B. Fleming (1954), the substitution made by Reilly appears as perfectly valid and very rational, considering the generally easier availability of population rather than business data.

After an analysis of 255 cases of linking cities and towns of various sizes in Texas, Reilly concluded that the exponent of the population was the first power and the exponent of distances was "nearer to the second power than to any other power ..." (after SCOT 1970, p. 169).

"... Reilly subjected his hypothesis to extensive empirical examinations and was so impressed with its predictive ability that he called it "the law of retail gravitation" (D. L. Huff, 1962, p. 7). "... Despite continued attacks and modifications this 'law' continued to have a long life and success and, up to the present, is the dominant tool for intercity retail trading area determination." (D. H. Revzan, 1971).

As it happens in science, fresh early concepts are often better and more "representative" than many later "improvements." This was partially the case with Reilly's law concept.

- It treated the intermediate city as open to influences of both competing central cities (which is close to the concept of dispersion and probability than to the accepted concept of definite trading areas of the retail center.
- The method of model building was the statistical estimation of model parameters (exponents of population variable and exponents of distance variables), after which he concluded that population has power 1 and distance power 2. This is the same procedure that would be used today (Reilly 1937).

The general gravity model form appears later applied to some other social influences by

T. Q. Steward (1941, see Zipf, p. 557),
to retail again by
P. D. Converse (1943 - p. 557),
to traffic by
G. K. Zipf (1946) etc.

The next major transformation was made by Curtis Publishing Company in 1947 during the process of mapping trading areas in the U.S.A. (Strohkarck, Phelps, 1948).

For this purpose, the relevant information was a boundary line between trading areas, rather than proportion of trade in intermediate points between cities. The solution was found in reformulating Reilly's equation, and the point where distribution of population between two centers becomes 50:50, was called the breaking point and was also the element of boundary between trading areas. The transformation from Reilly's formula by Curtis Publishing Company was presented by D. L. Huff (1962, pp. 8-9).

After Curtis Publishing Company revised the Reilly formula to a form
1. Reilly's formula can also be expressed as follows:

\[
\frac{D_a}{D_b} = \frac{\frac{P_a}{D_a^2}}{\frac{P_b}{D_b^2}}
\]

(i)

2. The breaking point between city A and B would be where \( B_a = B_b \)
or where:

\[
\frac{\frac{P_a}{D_a^2}}{\frac{P_b}{D_b^2}} = \frac{\frac{P_a}{D_a^2}}{\frac{P_b}{D_b^2}}
\]

(ii)

3. By multiplying both sides of equation (ii) by the denominator, we obtain:

\[
\frac{P_a}{D_a^2} = \frac{P_b}{D_b^2}
\]

(iii)

4. Transposing in equation (iii) brings about:

\[
\frac{P_a}{P_b} = \frac{D_a^2}{D_b^2}
\]

(iv)

5. If the exponents in equation (iv) are transposed, we derive:

\[
\frac{D_a}{D_b} = \sqrt{\frac{P_a}{P_b}}
\]

(v)

6. Since \( D_a + D_b = D_{ab} \) and therefore \( D_a = D_{ab} - D_b \); Substituting this expression for \( D_a \) in (v) we get:

\[
\frac{D_{ab} - D_b}{D_b} = \sqrt{\frac{P_a}{P_b}}
\]

(vi)

7. If like terms are cancelled in equation (vi), we obtain:

\[
\frac{D_{ab}}{D_b} - 1 = \sqrt{\frac{P_a}{P_b}}
\]

(vii)

8. By adding + 1 to both sides of equation (vii), the resulting equation is:

\[
\frac{D_{ab}}{D_b} = 1 + \sqrt{\frac{P_a}{P_b}}
\]

(viii)

9. Inverting equation (viii) brings about:

\[
\frac{D_b}{D_{ab}} = \frac{1}{1 + \sqrt{\frac{P_a}{P_b}}}
\]

(ix)

10. Finally, by transposing \( D_{ab} \) in equation (ix), we obtain the breaking point formula:

\[
B_b = \frac{D_{ab}}{1 + \sqrt{\frac{P_a}{P_b}}}
\]

where \( B_b = \) the breaking point between city A and city B in miles from B;

\( D_{ab} = \) the distance separating city A from city B;

\( P_b = \) the population of city B; and,

\( P_a = \) the population of city A.
which was able to produce the trading area boundaries between centers
without connection to any particular unit of population between the centers,
the use of gravity models became extensive in delineating trading areas for
different retail lines or services. For example, Rand McNally applied it
in 1952 (Revsan, 1970, p. 89) as primarily retail oriented; the American
Medical Association has applied a gravity model for establishing boundaries
of 757 medical service areas (Dickinson 1954).

In 1955, J. D. Carroll presented a method for defining urban trade
areas (Carroll 1955) introducing a somewhat different output of the model -
the magnitude of influence of a city on its surrounding hinterland, i.e.,
the attracting power of the city. So appeared the form:

\[ UJ_a = (K) Pa \left( \frac{f}{D} \right) \]

where

- \( UJ_a \) = the attractive power of the city A;
- \( Pa \) = population mass of city A;
- \( D \) = variable measure of distance;
- \( f \) = unknown rate at which the incremental distances modify the urban influence; and,
- \( k \) = a constant used to provide balance to the equation.

Carroll formed also the composite equation for the simultaneous delineating
influence between several cities and tested it with data pertaining to
five Michigan cities.

In 1959, Duncan Luce (1959) referred to the problem of individual
choice behavior (see Huff, 1962, p. 15) in the form that will soon be
improved by D. L. Huff.

The present form of gravity model for determination of intra-urban
retail trade areas, was formulated by David L. Huff after criticism of the main weak points of existing formulations (Huff 1962).

The central thesis of Huff's study was that consumer spatial behavior is best described as a probabilistic phenomenon, i.e., the output of the model should be the probability \( P \) of a given alternative \( j \) being chosen from among all alternatives of subset \( Jo \) (subset of really available alternative shopping centers for some population among all shopping centers in the system), and it is proportional to \( Uj \) (positive "payoff" function \( Uj \) indicates the specific utility of each shopping center to a particular population).

The main improvement of Huff's method was final reformulation of the gravity model so that it can act inside the level of the city retail system and investigate and determine internal relationships among different shopping centers in the city with respect to the population and their choice-decisions.

At the same time, Huff provided partial explanation for gravity-like regularity in the attraction of centers for a population, stating that the influence of perceived utility of a particular center brings out the tendency of shoppers to patronize the center proportionately to that perceived utility, which results in an adequate probability result in the aggregated population.

The high applicability of Huff's basic formulation was soon proved in the study of Potentials for Retail Growth in the Baltimore Region where
Terms and Definitions

The following basic elements are incorporated in the model of consumer spatial behavior:

1. A set of alternative shopping center choices which is represented as set \( J \);
2. A subset of alternative shopping center choices which is represented as \( J_0 \). The subset \( J_0 \) of alternatives represents available alternatives which are in accord with a consumer's tastes and preferences. Any given alternative within the subset \( J_0 \) is represented as \( j \) (where \( j = 1, \ldots, n \)) and,
3. A positive "payoff" function \( u_i \) is associated with each alternative shopping center indicating its "utility" to a consumer.

Basic Propositions

Given the preceding elements, the following propositions are set forth:

1. The probability \( P \) of a given alternative \( j \) being chosen from among all alternatives in the subset \( J_0 \) is proportional to \( u_j \). That is,

\[
P_j = \frac{u_j}{\sum_{j=1}^{n} u_j}
\]

such that, \( \sum_{j=1}^{n} P_j = 1 \); and, \( 0 < P_j < 1 \).

2. The ratio between the probabilities of a consumer's choosing any one of two particular shopping centers does not depend on the existence of other centers. This ratio is called the ratio of utilities of the two centers to a consumer. Therefore,

\[
\frac{P_i}{P_j} = \frac{u_i}{u_j}
\]

3. The properties of the pair \((P_i, P_j)\) that determine the utility in \((u_i, u_j)\) are: (1) the "size" \( S_i \) of a given shopping center; and (2) the distance \( T_{ij} \) in time units, from a consumer's travel base \( i \) to \( j \).

4. The utility \( u_{ij} \) of a shopping center is directly proportional to the ratio \( S_i / T_{ij} \lambda \) where \( \lambda \) is a constant. That is,

\[
P_{ij} = \frac{u_{ij}}{\sum_{j=1}^{n} u_{ij}} = \frac{\frac{S_i}{T_{ij}^{\lambda}}}{\sum_{j=1}^{n} \frac{S_j}{T_{ij}^{\lambda}}}
\]

where \( P_{ij} \) = the probability of a consumer at a given point of origin \( i \) traveling to a given shopping center \( j \);

\( S_i \) = the size of a shopping center \( j \);

\( T_{ij} \) = the travel time involved in getting from a consumer's travel base \( i \) to shopping center \( j \); and,

\( \lambda \) = a parameter which is to be estimated empirically to reflect the effect of travel time on various kinds of shopping trips.

5. The expected number of consumers at a given place \( i \) and shopping at center \( j \) is proportional to the number of consumers at \( i \) and to the probability that a consumer at \( i \) will select \( j \) for shopping. Therefore,

\[
E_{ij} = P_{ij} \cdot C_i = \frac{S_i}{\sum_{j=1}^{n} \frac{S_j}{T_{ij}^{\lambda}}} \cdot C_i
\]

where \( E(C_{ij}) \) = expected number of consumers at \( i \) that are likely to travel to shopping center \( j \); and,

\( C_i \) = the number of consumers at \( i \).
Lakshmannan and Hansen (1965) developed and applied the gravity model formulation, basically identical with Huff's, which has shown great practical use value in generating relevant information for evaluating the planning alternatives.

The Lakshmannan-Hansen procedure is a form of great practical value, simple enough, modest in information requirements, leaving the space for direct human decision in initial location of the retail center, as well as for evaluation of alternatives.

The inability of the classical gravity model to allocate retail growth (or distribute retail facilities) among different locations inside the model was partially solved by introducing the concept of "intervening opportunities" by B. Harris (1964) in Penn Jersey Transportation Study, as well as it was solve in the retail component of the Model of Metropolis by I. S. Lowry (Lowry 1964).

Retail establishments are divided into m groups, each of which has a characteristic production function; the elements of this production which enter directly into the model are: minimum efficient size of establishment,* number of clients required to support one employee, and number of square feet of space per employee. Since local consumer demand provides the market for establishments of this sector, we may treat employment in each line of retail trade as roughly a function of the number of households in the region:

$$E^k = a^k N$$  

*Actually the minimum number of employees per tract; these employees may represent more than one establishment of the same type.
The distribution of this retail employment among the square-mile tracts depends on the strength of the market at each location. Assuming that shopping trips originate either from homes or from workplaces, the market potential of any given location can be defined as a weighted index of the numbers of households in the surrounding areas, and the number of persons employed nearby.

\[
E^k_j = b_k \sum_{i=1}^{n} \left( \frac{c^k N_i}{T^k_{ij}} \right) + d^k E_j \tag{2}
\]

This equation could easily be made more general; however, we have assumed that none but short-range pedestrian trips originate from workplaces, so that the only relevant origins are those in Tract \(j\). Those originating from home are often longer vehicular trips, but the likelihood of a shopping trip from \(i\) to \(j\) diminishes with intervening distance. (The variable \(T^k_{ij}\) is a positive function of this distance, fitted from an analysis of home-based vehicular shopping trips.) The coefficients \(c^k\) and \(d^k\) measure the relative importance of homes and workplaces as origins for a particular type of shopping. Finally, \(b^k\) is a scale factor which adjusts the retail employment in each tract to the regional total determined in equation 1.

\[
E^k = \sum_{j=1}^{n} E^k_j \tag{3}
\]

In this way we determine the amount of employment in any tract for each line of retail trade. The sum of these employment figures, plus the quantity of basic employment allocated to the tract is total employment
for that tract.

\[
E_j = E_j^B + \sum_{k=1}^{m} p_k^j
\]  

(4)

Finally, with the aid of exogenously-determined employment-density coefficients \((e)^k\) for each line of trade, we can determine the amount of land in each tract which will be occupied by retail establishments.

\[
A^R_j = \sum_{k=1}^{m} d_k^j e_k^j
\]  

(5)

Lowry 1964, pp. 10, 11)

As the size and form of the equations in gravity models became too complex and cumbersome to handle if they are to be used frequently, they are often written in changed form with negative exponents instead of the functional form. The way of transferring from one form into another is given with all steps in the following extract from the review of models in planning by C. Lee (1973). (See pages 34 and 35)
Experience with the early formulations of the gravity model showed that they tended to over-predict the volume of short trips in an area. This experience led to a re-statement of the gravity model, which can be generally described in the following terms: the amount of interaction between two or more zones is directly proportional to the size (or attractive power) of the zones, and is inversely proportional to the distance between the zones and the relative attraction of competing zones. In symbolic terms this means that a balancing factor is introduced as the denominator of the gravity model to represent the competing attraction of other zones:

\[
T_y = \frac{G \frac{P_i P_j}{d_{ij}^b}}{G \frac{P_i}{d_{i1}^b} + G \frac{P_i}{d_{i2}^b} + G \frac{P_i}{d_{i3}^b} + \ldots + G \frac{P_i}{d_{im}^b}}
\]  

(8)

This equation, which is useful to show the way in which the structure of the model has developed, is cumbersome if it is to be used frequently. Fortunately it can be substantially simplified by the use of mathematical conventions. For example, the expression

\[
G \frac{P_i P_j}{d_{ij}^b} \text{ means } G \times P_i \times P_j \times \frac{1}{d_{ij}^b}
\]

However, as we showed in Chapter 3,

\[
\frac{1}{d_{ij}^b} \text{ can be written as } d_{ij}^{-b},
\]

so that the expression

\[
G \frac{P_i P_j}{d_{ij}^b}
\]

can be rewritten as \(GP_i P_j d_{ij}^{-b}\).

Equation (8) can therefore be written as:

\[
T_y = \frac{GP_i P_j d_{ij}^{-b}}{GP_1 d_{i1}^{-b} + GP_2 d_{i2}^{-b} + GP_3 d_{i3}^{-b} + \ldots + GP_n d_{in}^{-b}}
\]  

(9)

This can be further reduced by the use of the summation sign (see Chapter 3), where

\[
G \sum_{j=1}^{n} P_j d_{ij}^{-b}
\]

means the sum of the individual terms \(GP_1 d_{i1}^{-b}\) to \(GP_n d_{in}^{-b}\). Equation (9) therefore becomes:

\[
T_y = \frac{GP_1 P_j d_{ij}^{-b}}{G \sum_{j=1}^{n} P_j d_{ij}^{-b}}
\]  

(10)

In equation (10), the constant term \(G\) is common to both parts of the fraction and can therefore be cancelled out. Equation (10) then becomes:

\[
T_y = \frac{P_i P_j d_{ij}^{-b}}{\sum_{j=1}^{n} P_j d_{ij}^{-b}}
\]  

(11)
A useful way of interpreting equation (11)—and one which we will use to
develop our practical examples at the end of the chapter—is to regard the
term
\[
\frac{P_i d_{ij}^{-b}}{\sum_{j=1}^{n} P_j d_{ij}^{-b}}
\]
as representing the probability of interaction between any zone \(i\) and zone \(j\),
based on the attraction of \(j\) compared with the attraction of all other zones.
The actual amount of interaction is obtained by multiplying the probability
of interaction by the total activity in zone \(i\), \(P_i\).

It is possible to simplify equation (11) further by rewriting the denomi-

\[\frac{1}{d_{ij}^b}\]

so the expression
\[
\frac{1}{\sum_{j=1}^{n} P_j d_{ij}^{-b}}
\]
can be written as
\[\left(\sum_{j=1}^{n} P_j d_{ij}^{-b}\right)^{-1}\].

If we call this term \(A_i\), we can write the gravity model formula much more
simply as:
\[
T_{ij} = P_i A_i P_j d_{ij}^{-b}.
\]  \(\text{(12)}\)

In this equation, the probability of interaction between \(i\) and \(j\) is represented
by \(A_i P_j d_{ij}^{-b}\). The equation is usually written as:
\[
T_{ij} = O_i A_i D_j d_{ij}^{-b}
\]  \(\text{(13)}\)

where \(T_{ij}\) = the number of trips between zones \(i\) and \(j\),
\(O_i\) = the number of trips originating in zone \(i\),
\(D_j\) = measure of attraction of zone \(j\).

( Lee 1973 pp.62,63,64 )
VI. MODEL WITH ASSUMPTION OF MINIMIZING TRANSPORTATION COSTS IN SHOPPING

One of the models of consumer shopping behavior that has clear theoretical assumptions from the field of economy is the model proposed by R. W. Bacon (1971).

Under the assumption that the shopper, tending to minimize his transportation costs in shopping, is not only using the nearest retail outlet but is also using available economies of scale and joint economies offered by more distant but larger shopping centers, Bacon developed an algorithm which is able to distribute the trips from the neighborhood units to the shopping centers, if quality and frequency of demand as well as the offer level of particular centers are known.

The Bacon model is obviously very attractive from the point of view of particular optimization (transportation costs), but if minimizing transportation costs would not be empirically proven as by far the most important behavior variable for shoppers (which is not necessarily so) - the model would lose a lot of its theoretical force. On the following 2 pages is the example of the algorithm of Bacon's model, with a remark by Bacon that mathematical solution of it is not yet possible.
An Example of the Transport Cost Minimisation Algorithm

The shopper under consideration buys 10 goods at the following frequencies per period of time:

\[
\begin{array}{ccccccccccc}
A & B & C & D & E & F & G & H & I & J \\
20 & 15 & 15 & 12 & 10 & 8 & 5 & 3 & 2 & 1 \\
\end{array}
\]

There are eight shopping centres available to the consumer. These are arranged below in ascending order of transport cost from the home.

Using the notation ‘X’ denotes possession of a given type of shop and ‘-’ non-possession, the centres contain the following shops:

\[
\begin{array}{ccccccccccc}
A & B & C & D & E & F & G & H & I & J \\
C(1) & X & - & - & - & - & - & - & - & - \\
C(2) & - & X & X & - & - & - & - & X & - \\
C(3) & X & - & - & - & - & X & - & X & - \\
C(6) & X & - & X & X & - & - & X & - & - \\
C(7) & X & X & - & - & X & X & X & - & - \\
C(8) & X & X & - & - & X & X & X & - & X \\
\end{array}
\]

Step 1: Examination for ‘dominated’ centres

A preliminary step in the analysis examines the centres to see whether any centre is ‘dominated’ by nearer centres. If this is the case, that centre can be ignored in the subsequent analysis.

Now C(2) contains all the types of shop contained by C(5) and hence the latter would not be used in the minimum cost solution and can be discarded.

It can be seen that C(1) and C(2) together contain all the shops contained by C(7). If \((t_1 + t_2) < t_7\), then it would always be cheaper to make a trip to each of these nearer centres rather than a trip to C(7). It is assumed that in fact C(7) can be discarded from consideration.

Step 2: The basic solution

Every good is allocated to the nearest centre which sells it. The tableau now reads

\[
\begin{array}{ccccccccccc}
C(1) & 20 & - & - & - & - & - & - & - & - \\
C(2) & 15 & 15 & - & - & - & 3 & - & - & - \\
C(3) & 0 & - & - & - & - & 8 & - & - & 2 & - \\
C(4) & - & - & - & 12 & - & 5 & - & - & - & - \\
C(6) & 0 & 0 & 0 & 10 & - & - & 0 & - & - & - \\
C(8) & 0 & 0 & - & - & 0 & 0 & 0 & 0 & 1 & - \\
\end{array}
\]

The initial cost = \(T = 20t_1 + 15t_2 + 8t_3 + 12t_4 + 10t_6 + t_8\)

Step 3: External economies

First all external economies generated by C(6) are realised

\[
\begin{array}{ccccccccccc}
C(2) & - & 15 & 15 & - & - & - & 3 & - & - \\
C(3) & 0 & - & - & - & - & 7 & - & - & 2 & - \\
C(4) & - & - & - & 12 & - & 5 & - & - & - & - \\
C(6) & 0 & 0 & 0 & 9 & - & - & 0 & - & - & - \\
C(8) & 1 & 0 & - & - & 1 & 0 & 0 & 0 & 1 & - \\
\end{array}
\]

Next those generated by C(6) alone:

\[
\begin{array}{ccccccccccc}
C(1) & 10 & - & - & - & - & - & - & - & - \\
C(2) & - & 15 & 15 & - & - & - & 3 & - & - \\
C(3) & 0 & - & - & - & - & 7 & - & - & 2 & - \\
C(4) & - & - & - & 5 & - & 5 & - & - & - & - \\
C(6) & 9 & 0 & 7 & 9 & - & - & 0 & - & - & - \\
C(8) & 1 & 0 & - & - & 1 & 1 & 0 & 0 & 1 & - \\
\end{array}
\]

There is one ‘tied’ external economy for C(2)—good B is reallocated to C(8) and simultaneously good C to C(6).

\[
\begin{array}{ccccccccccc}
C(1) & 10 & - & - & - & - & - & - & - & - \\
C(2) & - & 14 & 14 & - & - & - & 3 & - & - \\
\end{array}
\]
C(3) 0 - - - 7 - - 2 - C(4) - - - - - - - 5 - - - C(6) 9 - 1 7 9 - - 0 - C(8) 1 1 - - 1 1 0 0 - 1

There are no possible external economies at C(4) since all the shops are used the same number of times.

Next come the external economies generated by C(3).

C(1) 3 - - - - - - - - C(2) - 14 14 - - - - 3 - - C(3) 7 - - - 7 - - 2 - C(4) - - - - - 5 - - - C(6) 9 - 1 7 9 - - 0 - C(8) 1 1 - - 1 1 0 0 - 1

There are no more external economies to be realised.

Step 4: Joint economies

The first centre to be analysed is C(3). In fact it does not sell all the maximum frequency goods purchased at C(1) and C(2) so there is no possibility of a joint economy.

C(4) does not sell any of the maximum frequency goods at C(1), C(2) or C(3) and the analysis turns to C(6).

Although C(6) sells the maximum frequency goods at C(1) it does not sell all of those at C(2), C(3) or C(4). Also C(4) and C(6) combined do not sell all of the maximum frequency goods of C(1), C(2) and C(3) so there is no possibility of a 'joint' economy.

In a similar fashion there are no joint economies to C(8) alone. However C(6) and C(8) combined do sell the maximum frequency goods at C(2), C(3) and C(4). Five 'joint' economy trips can be made. This limit is reached when no more shopping is done at C(4). For the first two of these trips the frequency of shopping at C(6) need not be increased since D can generate two external economies at C(6). For these occasions the cost criterion is

\[ t_8 \geq t_4 + t_3 + t_2. \]

It is assumed that the joint economy is cost saving.

Also it is assumed that

\[ t_6 + t_8 < t_4 + t_3 + t_2 \]

so that the next three possible trips actually are cost saving.

The position after all five economies have been realised is:

C(1) 3 - - - - - - - - C(2) - 9 9 - - - - 3 - - C(3) 2 - - - - 2 - - 2 - C(4) - - - - 0 - - 0 - - C(6) 12 - 6 12 9 - - 0 - - C(8) 3 6 - - 1 6 5 0 - 1

The number of trips to C(8) cannot be further reduced because good I is not sold at any higher order centre.

However, these 'joint' economies have generated further 'external' economies. Good A can be moved from C(1) to C(3) without increasing the number of trips to the latter.

C(1) 0 - - - - - - - - C(2) - 9 9 - - - - 3 - - C(3) 2 - - - - 2 - - 2 - C(4) - - - - 0 - - 0 - - C(6) 12 - 6 12 9 - - 0 - - C(8) 6 6 - - 1 6 5 0 - 1

Step 5: Reversal moves

At C(8) good E was moved from C(6) before the joint trips increased the frequency of shopping at C(6). Now E can be moved back to C(6) without altering the transport costs.

No other reversal moves are possible and the final tableau is

C(1) 0 - - - - - - - - C(2) - 9 9 - - - - 3 - - C(3) 2 - - - - 2 - - 2 - C(4) - - - - 0 - - 0 - - C(6) 12 - 6 12 10 - - 0 - - C(8) 6 6 - - 0 6 5 0 - 1

The final transport cost is:

\[ T = 9t_2 + 2t_3 + 12t_5 + 6t_8. \]

(Bacon 1971 pp. 62, 63)
PART 2

INFORMATION AND SHOPPING PROCESS
A common element of many shopping models is the assumption of perfect knowledge by shoppers about the retail system in the city, what commodities it offers, where, what are the prices, distances, etc. After that, shopping behavior is a problem of calculation of optimal solution in terms of total distance that must be traveled. The behavior of the shopper can be inconsistent for each individual - but will become consistent and regular in terms of large numbers.

The first assumption, that shopping process is most of all a distance-minimizing process, is not necessarily the most obvious pattern. It is very probable that this would be the case under the abstract assumption of "other things being equal", but, in practice, other things are never equal.

Furthermore, even if they are, shoppers must know first that it is so, but in reality, perfect knowledge of shopping conditions in their own city is not granted. It is not probable at all for the cities that are bigger than a few neighborhoods. It is possible that at a high enough level of aggregation the actions of a large enough population will appear enough alike to assume a behavioral pattern, but that would only prove the statistical - not behavioral - regularity. (Isard 1960, p. 513)

Therefore, we think that the first steps in approximating behavioral models of shopping should introduce the assumption of different, imperfect information about the shopping system in different segments of population, which would result, also, in different outcomes in final shopping distribution.

According to that, our hypothesis would be that shopping is an
information obtaining process more than anything else, or at least as much as anything else. If that is so, then models of shopping behavior should introduce the aspect of information, too.
I. DECISION-MAKING SCHEME FOR SHOPPING FOR THE PAIR OF SHOES

In order to support the statement about the importance of the information nature of the shopping process, let us analyze the scheme of communication and decision-making in one abstract example of shopping for a pair of shoes:

The shopper has the intention to buy the specific pair of shoes that will satisfy his criteria of a good fit, in respect to style, model, price range and size. This process is composed of successive choices between available alternatives, and checking the proper fit of the chosen one afterwards - on different levels of agglomeration of shoe - sales facilities in the retail system of the city.

Progressive changes of levels of agglomeration we shall call steps in shopping, and each decision which reduces the number of alternatives left we shall denote by one bit of information. The reason for that is to get the impression necessary to accomplish one simple shopping case.
II. DECISION-MAKING STEPS IN SHOPPING FOR THE PAIR OF SHOES

Step 1 - Choice of Cities

Should one go to city A, B, C, or D? For decision needed: 1st bit of information

     either A or B, but not C or D. 2nd bit,

Does the city A have good shopping centers?

If yes, go to A and to step 2.

If no, make another selection of the cities. 3rd bit,

Step 2 - Choice of Shopping Centers in the City A

Should one go to shopping center:

     a, b, c, or d? 4th bit,

     either a or b, but not c or d. 5th bit,

     a - yes    b - no

Does the shopping center a have good shoe shops?

If yes, go to a and step 3.

If no, make another selection of shopping centers

(and if no one shopping center has good shoe stores, come back to step one and make another
selection of city). 6th bit,

Step 3 - Choice of Shoe Store in the Shopping Center a

Should one go to the shop I, II, III, or IV?

Either I or II, but not III or IV. 7th bit,

     I - yes    II - no 8th bit,

Does the store I have the sort of shoes that one

is looking for?
If yes, go to store I and step 4.
If no, make another selection of store (or even shopping center if needed).

9th bit,

Step 4 - Choice of Model of Shoes That Fits One's Demand

Should one take the model:

\[ \text{W, X, Y, or Z?} \]

Either W or X, but not Y or Z.

W - yes. X - no.

Does one really like the model W, in all aspects of design and materials used?

If yes, ask for model W and go to step 5.
If no, make another selection of model (or go to another store if necessary).

10th bit,

11th bit,

12th bit,

Step 5 - Choice of the Price Ranges of the Models

Should one pay given prices for the shoes:

\[ \text{W, X, Y, and Z?} \]

Either W or X, but not Y or Z.

W - yes. X - no.

Is the chosen model W in the desired price range?

If yes, ask for it and go to step 6.
If no, make another selection of price range (or model if necessary, or store, or shopping center...).

13th bit,

14th bit,

15th bit,
Step 6 - Choice of the Sizes of the Chosen Model

Should one ask for size:

9, 10, 11, or 12?  
Either 9 or 10, but not 11 or 12.

Does the chosen size of chosen model fit ones feet well enough?

If yes, ask for these shoes and go to step 7.
If no, make another selection.

Step 7 - Final check of steps 4, 5 and 6 does not require any essentially new information but only combination of already given in order to establish the proof for right decision. If final check confirms the decision, the final technical phase of exchange can be started.

Would the store accept credit card:

E, F, G, or H?  
Either E or F, but not G or H.  
E - yes.  F - no.

Is the card E all right?

If yes, pay and accomplish the shopping.
If no, offer another card. If no card is acceptable, . . . etc. . . . etc.

Some information might further concern the question of delivery (one or two bits) but at some steps the choice, maybe, would not be four
or three but only two real alternatives (size, for example). What would reduce the needed information amount? Anyway, we have gotten the answer to the question of what was the order of magnitude of the measure of information that was necessary to accomplish one simple shopping task.

In the case of a shopping procedure like this, 21 bits of information were necessary to obtain the successful exchange (purchase). In cases of special high requirements, or contradictory requirements of the individual shopper, success in the search will not happen inevitably in a direct line, and failures in particular stores (no size, or too high prices, or no model at all like that desired) are likely to happen. That would increase the amount of information needed in advance - or would turn the shopping process into an extensive and systematic time-and effort-consuming search, from shop to shop, from shopping center to shopping center, which is, in general, very unpleasant - even for the most enthusiastic shoppers.
DECISION-MAKING AND INFORMATION OBTAINING EFFORT

The bit as a measure of information is a simple one, defined as the amount of information necessary to make a decision between two alternatives. The total number of bits that one decision-making process will require to reduce the total number of alternatives to one (or the desired number of alternatives) will be expressed as an exponential function, i.e.

\[ A = B^i(\text{bit}) \]

\[ -i(\text{bit}) = \log_B A \]

where \( A \) = total number of alternatives

\( B \) = logical basis of decision-making system

(in our case, the cause system is binary: yes-no)

\( i(\text{bit}) \) = number of required bits of information to reduce number of alternatives to desired level

When we buy a detergent in a supermarket, the process of shopping, though similar to the example of the buying the shoes in regard to total amount of information necessary to accomplish the communication and exchange process, has also one essential difference.

The ratio of Routhinised and previously-made and solved decisions for the purchase of detergent, compared to new added decisions and information in that particular shopping trip, is significantly different. While 20, out of 21 theoretically necessary bits, (and 7 out of 7 steps would be known and decided in advance for the purchase of detergent in a familiar neighborhood supermarket) almost 20 bits out of 21 bits of information will be new information for new decisions, necessary to accomplish
successful shopping for the shoes.

The bit as a measure of information usually is not as familiar to people as a measure of distance or value in everyday life. Still, we can assume that it makes a lot of difference in behavior, if the communication process and decision-making requires 7, rather than 21 new bits of information. We can get a rough notion about differences between actual values of 1, 7, 14, 20 and 21 bits of information if we use an equation from the definition of this measure, and count out the theoretical number of alternatives from which is derived the decision that asked for 1, 7, 14, 20, and 21 bits of information which were some characteristic values in the previous discussion. (Attneav 1959)

\[
\begin{align*}
\log_2 2 &= 1 \\
\log_2 128 &= 7 \\
\log_2 16,384 &= 14 \\
\log_2 1,048,574 &= 20 \\
\log_2 2,097,152 &= 21
\end{align*}
\]

While the actual universum of alternatives involved in shopping action was only two for a detergent, it was over one million for shoes.

There is no reason in the world to think that the decision-making process for reducing more than one million alternatives to one single solution is effortless. On the contrary, it is a significant mental effort, and a hypothesis which assumes that the consumer tends to minimize this effort too (and not only the effort of traveling) would not seem senseless.
IV. VALUE OF UNIT OF INFORMATION IN TERMS OF DISTANCE OR TIME OR MONEY COSTS

In previous discussion we tried to establish some sense for the general connection between information and the shopping process. Numerous particular connections can be derived from this general basis.

One of them is the connection of information with trip-distance-saving. As most of the shopping models are trip-effort-saving oriented, this particular connection appears as an important one. In the case of Bacon's type of model (optimisation of shopping trip-effort over time using available external and joint economies of more distant but larger centers) the connection is obvious:

- no one external or joint economy can be utilized unless proper knowledge of their availability exists
- (Bacon himself did not introduce differences in information), and perfect information of the shopper was an implicit assumption.

Similar are probabilistic gravity models, though their information-level-implicit-assumption is a more elastic one. Gravity models in fact assume in the general experience of the shopper that the probability of finding a desired product is proportional to the number of alternatives offered at the same place, i.e., proportional to the mass (size, number, volume of sales, etc.) of the retail facility.

No doubt it is possible to measure the distance-saving value of particular information in any particular shopping action and particular shopping environment, in many ways.
Let us examine again one of the possible simple abstract schemes of relation between information and distance, supposing again a "full search" situation (Bucklin 1963).

Shopper has his residence: 10 miles from city A
16 miles from city B

Cities A and B are equal in other respects - each has:
- 4 shopping centers located on 2, 4, 6 and 8 miles respectively from the highway entrance to the city A as well as B.
- Each shopping center has the shop that sells the product that shopper wants to buy, but only can meet all shopper's criteria of a proper fit - (say, the stores in shopping centers $b_2$ and $a_2$).

Our task is to see what is the average difference in average travel that results from input of partial information in the shopper's decision-making process concerning the questions: which shopping center to visit first in which city, etc. assuming the desire to accomplish the purchase successfully with minimal distance traveled.

**Step I**

No knowledge exists at all about particular structure of shopping centers, distances and where the desired commodities are likely to be found. A full search will therefore be a random visit to shopping centers
in cities A or B without any order, as long as the product is not found.

R-Q2-R is minimum traveled distance possibility: \(2 \times (10+6) = 32\) miles

R-b1-b3-b4-b2R is maximum traveled distance possibility: \(44+16+8+4 = 72\) miles

probable average: \(\frac{72+32}{2} = 52\) miles

Probable distance traveled without any particular information would average 52 miles.

**Step II**

Information:

City B is further away from R but in other respects cities are equal

This information limits search only to city A, eliminates city B: 1 Bit

R-a2-R minimum traveled distance possibility: \(2 \times (10+6) = 32\) miles

R-a2-a3-a4-a2-R maximum travelled distance possibility \(32+16+8+4 = 60\) miles

possible average: \(\frac{60+32}{2} = 46\) miles

Distance between average with 0 and with 1 bit of information

\[d_1 = 52-46 = 6\] miles, i.e., one bit of information was worth 6 miles traveling.

**Step III**

Information:

The closest place where the desired product can be
bought is in shopping center $a_2$!

This information leads straight to the center $a_2$
reducing eight original shopping center information units to one. The value of information therefore is:

$$\log_2 8 = 3 \text{ bits}$$

$R_{a_2-R}$ is the probable minimum and average travel equals 32 miles.

Difference between averages with 0 and 3 bits of information is:

$$d_3 = \frac{52-32}{3} = \frac{20}{3} = 6.66 \text{ miles},$$

which can be taken as a representative value for one bit of information in terms of distance-saving in this particular example.

In other words, if our shopper knew that he could buy exactly what he was looking for in the shopping center $a_2$, he would probably save 20 miles in traveling which is about as much as the minimal necessary travel itself is.

If we would analyze the value of information also in the further steps in shopping (choice and decision-making inside shopping centers, inside shops, inside price ranges, models, etc.), what would be possible in terms of time-saving if not in the distance-saving terms any more (traveled distance can be expressed in time units. Both can be expressed in money terms if desired), even more remarkable the relevance of the information variable for shopping facility choice might be deduced.

The demonstrated tools for measuring the importance of information in the shopping process are:
- value of a particular bit of information in terms of decision-making effort (as a range of numbers of new alternatives that should be reduced to a single one in the particular shopping action);
- value of one bit of information in terms of savings in traveled distance;
- value of one bit of information in terms of time-saving, as well as money saving if desired. These have been very primitive but hopefully illustrative enough. Probably it would be easy enough to develop much more sophisticated relations, for use in the development of the hypothetical information-oriented shopping behavior model.
V. FAMILIARITY OF THE ENVIRONMENT AND SHOPPING BEHAVIOR

There are many ways to reduce effort in the shopping decision-making process.

One is the natural process of standardization in industrialized production which permanently reduces the relevant universum of alternatives, though the appearance of a universum can simulate diversification (as in the automobile industry, for example, where total number of models appears as innumerable, but the number of significantly different technical and functional alternatives is probably smaller than at the beginning of the century). The process of standardization has important informative consequences. Knowing one product from the class, all other products of the same class are known in general, too.

Another process that is connected with the previous one is the reduction of the level of the shopper's criteria for a good fit (or greater elasticity of these criteria at least). Lowering down the shopper's criteria and the standardization of products are both conditions for successful and strong decentralization of the retail system. Suburbanization of shopping and speciality goods in large cities are symptoms which show that both conditions actually exist.

In his study of the relationship between social stratification and use of the inner city of Paris, B. Lamy found that shopping trips to downtown areas were determined the most significantly by three factors:

- membership in the upper-class,
- social relationships in Paris, and
- place of work in Paris.
The significance was great for any one factor and cumulative if two or all three stimulative factors existed.

... "When one of these factors is positive, then the number of families who did the greater part of their shopping in Paris is twice that of families who had some of these characteristics; when two factors are positive it is four times greater, it is five times as much, finally, when the three factors become positive." (Bernard Lamy, 1967, p. 358)

The influence of the social class factor we can easily explain as the relation between standardization offer of suburban retail compared to the highly sophisticated offer in central area. That matches better the highly selective demand of Paris' upper class shopper, i.e. that is the influence of the relevant size of the universum of offered alternatives in shopping. The other two factors - working place in Paris and social relations in Paris can give us additional arguments in favor of appreciating the information variable in shopping.

The main way of reducing the actual amount of information and decision-making steps necessary in each individual shopping act is to memorize relevant shopping information in advance and to make routine as many decision-making steps as possible. This practice does not change the absolute amount of information required, but spreads the process of obtaining information over a longer period of time. Practically, it is a merging of the process of gathering shopping information with other independent activities, like journey-to-work, or to social connections, or other shopping.

In respect to information, the basic difference between places that
one visits frequently and regularly, and the ones that one visits rarely is in the degree of familiarity, i.e. the amount of relevant information that one possesses, concerning the particular environment.

Further on, it is very probable that the general familiarity of some space to a potential shopper is highly correlative with the amount of particular shopping information, which leads to the next hypothesis:

**Shoppers will tend to shop in the most familiar shopping environment (other things being equal).**

The rationale for this hypothetical tendency could be the difference in memorized stock of previous information which makes the shopping and decision-making easier for every given level of demanded success in shopping. The partial illustration of the effect of familiarity on shopping facility choice can be explained through the analysis of the tendency of shoppers to have or not have a usual place for shopping.

The following tables from research of shopping patterns in Oakland County, Michigan (W. M. Ladd, P. D. Cousens, D. A. Pampu, 1966, p. 60) are very illustrative for this purpose. [See p. 57]

All shopping outlets in the list are more often patronized as someone's "usual place", even the furniture store. At the same time, aggregated distances do not show significant differences which would lead to the conclusion that proximity of the "usual" store was the crucial factor in its "familiarity".

It is worth mentioning that both patterns (repetition rate caused by familiarity and distance) are different for non-shopping actions, i.e. movie and restaurant visits.
### USE OF ESTABLISHMENTS, BY INCOME

(Percentage Distribution)

<table>
<thead>
<tr>
<th>Establishments</th>
<th>Total County</th>
<th>Less than $6,000</th>
<th>$6,000-$9,999</th>
<th>$10,000-$14,999</th>
<th>$15,000 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GROCERY STORE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>85</td>
<td>83</td>
<td>89</td>
<td>85</td>
<td>80</td>
</tr>
<tr>
<td>Have gone</td>
<td>14</td>
<td>14</td>
<td>11</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>Have not gone</td>
<td>1</td>
<td>3</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>DEPARTMENT STORE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>68</td>
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<td>29</td>
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<td>24</td>
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<tr>
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<td>12</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>DRUG STORE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>85</td>
<td>77</td>
<td>89</td>
<td>87</td>
<td>84</td>
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<tr>
<td>Have gone</td>
<td>12</td>
<td>16</td>
<td>9</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>Have not gone</td>
<td>3</td>
<td>7</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>RESTAURANT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>35</td>
<td>25</td>
<td>38</td>
<td>34</td>
<td>39</td>
</tr>
<tr>
<td>Have gone</td>
<td>44</td>
<td>31</td>
<td>39</td>
<td>52</td>
<td>55</td>
</tr>
<tr>
<td>Have not gone</td>
<td>21</td>
<td>44</td>
<td>23</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>MOVIE HOUSE OR THEATER</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>21</td>
<td>11</td>
<td>25</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td>Have gone</td>
<td>25</td>
<td>17</td>
<td>20</td>
<td>30</td>
<td>37</td>
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<tr>
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<td>54</td>
<td>72</td>
<td>55</td>
<td>47</td>
<td>41</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>LUMBER AND HARDWARE STORE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>60</td>
<td>36</td>
<td>62</td>
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</tr>
<tr>
<td>Have gone</td>
<td>14</td>
<td>10</td>
<td>14</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>Have not gone</td>
<td>26</td>
<td>54</td>
<td>24</td>
<td>11</td>
<td>17</td>
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<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>FURNITURE STORE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Usual place</td>
<td>27</td>
<td>14</td>
<td>31</td>
<td>29</td>
<td>28</td>
</tr>
<tr>
<td>Have gone</td>
<td>18</td>
<td>42</td>
<td>13</td>
<td>19</td>
<td>28</td>
</tr>
<tr>
<td>Have not gone</td>
<td>55</td>
<td>44</td>
<td>56</td>
<td>52</td>
<td>44</td>
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<td>Total</td>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

* Less than .5 per cent.

### MEDIAN DISTANCE IN MILES, BY WHETHER HAVE A USUAL PLACE TO SHOP

<table>
<thead>
<tr>
<th>Establishments</th>
<th>Have A Usual Place</th>
<th>Have No Usual Place</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total County</td>
<td>A</td>
</tr>
<tr>
<td><strong>GROCERY STORE</strong></td>
<td>1.6</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>DEPARTMENT STORE</strong></td>
<td>4.7</td>
<td>3.4</td>
</tr>
<tr>
<td><strong>DRUG STORE</strong></td>
<td>1.4</td>
<td>0.8</td>
</tr>
<tr>
<td><strong>RESTAURANT</strong></td>
<td>3.7</td>
<td>2.9</td>
</tr>
<tr>
<td><strong>MOVIE HOUSE OR THEATER</strong></td>
<td>3.8</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>LUMBER AND HARDWARE STORE</strong></td>
<td>1.7</td>
<td>1.3</td>
</tr>
<tr>
<td><strong>FURNITURE STORE</strong></td>
<td>4.9</td>
<td>2.7</td>
</tr>
</tbody>
</table>
On the basis of previous discussion it appears that the starting hypothesis that the consumer's shopping process is deeply influenced by the information structure of the shopping process itself and the information structure of the retail system in the city in relation to the individual consumer could eventually be proved by special research.

It also appears that models based on minimizing consumer's effort in shopping and that all the vast majority should take serious care concerning information-gathering and decision-making processes (inevitably undertaken by the consumer) as the processes that represent substantial effort. This effort should be the subject of minimizing in models of consumer's shopping behavior, too.
VI. INFORMATION VALUE OF THE LOCATION

In previous parts of this paper we have discussed only the influence of information on the shopper's side of the shopping process. Naturally, a connection between information and shopping process exists even more explicitly on the side of the offer, where optimization of shopping process usually means the promotion of sales for a particular retail establishment.

The subject of advertising as an information process is at this moment out of our scope, but the question of some general relations between location and information process cannot be neglected as they are not neglected in the practice of location behavior of retail establishments.

In explanations of the agglomeration forces that are forming urban centers, different functional economies are always mentioned (as, for example, external economies of scale, joint economies, distance minimizing in transportation, face to face contact, etc.). From our particular point of view, we can think that economy of information and information process (as a part of general communication and exchange processes that are taking place in the urban centers) are underestimated as agglomeration forces in forming the urban centers.

In the part of the paper that was dealing with the decision-making structure of a shopping action, we have established the connection between information and different steps in shopping but we did not mention that sort of information these were, or from where they were coming.

One of our many possible classifications of reception of the information for shopping decisions can be the following one:

1. Personal experience from previously shopping for the
same kind of product.

2. Personal reception of relevant information through mass media.

3. Indirect information from other people's personal experience or reception of information through mass media.

The first type of reception would, theoretically, lead to the exception of a highly repetitive pattern of shopping.

The second type of reception would lead to expectation of direct proportionality of shopping frequency to information activity of the retail facility (volume of advertising for example).

The third type of reception of information would finally lead to expectation of some similarity of shopping patterns inside social groups.

Almost all retail outlets are counting on, and are trying to promote all three types of information flow towards potential customers because the total stock of information in a shopper's memory is never single-source originated but rather composed of many levels of all three types.

In the present organization of the life, production, and consumption, it seems that personally experienced information - provided by mass media - appears as the most important section of the three (at least observing the tremendous intensity of advertising in all available mass media).

Development of telecommunications, press and advertising on one side, telephone and mail shopping on the other, predicting probable development of two way cable television connections between retail store and consumer's home that would finally enable the consumer to avoid shopping trip and
search in exterior traditionally have led city planners to the conclusion that a general disappearance of the central structure in a city is the most probable process (Alpaas, 1966) of further development.

We can not estimate at this moment any probability that this or an opposite process will occur. We shall rather assume that the trends towards developing a clearly organized central structure, or towards more and more decentralized and amorphous forms of spatial organization of the city, are functions of the current proportion of the influence of spatially sensitive media or spatially insensitive ones upon the information obtaining process.

Press, radio, television, telephone, etc. are almost insensitive in relation to space variables. But there are mass media that are very sensitive to spatial variables such as distance, direction, length, height, etc.

One of the major mass media of that kind in the city is public space itself.

It is loaded with many communication and information processes that are developing at the same time. The basic form of connection between public space and citizen is the visual contact of a man who is moving in that space with visual messages that are organized in public-communication space.

A great deal of competition for favorable location does not mean anything but competition for favorable position in public-communication space.

By favorable position in communication space, we assume that the position of a communication outlet of the retail facility (windows, or
identification or advertising signs, or the building as a symbol of the activity that happens in it...) is such as to be able to deliver a maximal amount of information with maximal frequency, to the largest number of potential shoppers, with least effort to both sides.

In practical terms, that which is public-communication-space has, naturally, many spatial forms. If we start from the community room in some neighborhood unit as an extreme of the scale, and go to theaters and stadiums, etc. on the other end of the scale, as the most important and the most public communication space in the city, we shall find, probably, the main shopping street.

If we look more closely at the pattern of location of unplanned business centers (for example, in the Chicago region, as presented in the figure from B. Berry, 1967 [See page 65]), though the streets and roads are not drawn at all, it is easy to conclude that all major regional centers as well as smaller regional centers are located directly at several major streets and roads that are taking the majority of everyday commuting in the region.

If only the price of products that are being sold in the shopping centers would matter in attracting the consumer, centers would tend to locate in the space between major traffic lines where land is cheaper, and not directly by the roads.

If price of land (therefore product) and accessibility were the only two variables that really matter, the centers would tend to come in proximity of the major traffic arteries, but not directly on them. (As driving a few hundred yards more is practically irrelevant). But, as
there exists the third very relevant variable, the information variable, centers centers are directly next to the road, because they must be visible from the road. Centers themselves must repeat the information about their existence to the passing commuters, as frequently as possible, in order to establish their own place in the memory of the potential shopper, which will, at some latter occasion, play an important role in the decision-making process of the consumer.

This fact makes all locations which are relatively invisible from the road inferior and uncompetitive compared to the locations that have better visibility. In the sense of information-transferring ability, the location which can establish adequate visual contact with more potential shoppers has higher informative value than some other less visible location.

To measure directly informative value of the location in terms of BITS, as a unit of specific measure of information, at this moment is a too complicated, uncertain, and extensive job, where too many arbitrary assumptions should be made. Therefore, quantitative accuracy would be lost and the whole enterprise would hardly have paid off.

The main problem is how to distinguish information from noise in visual communication between visitor (commuter or walker, shopper ...) and signs in public communication space, where all messages emitted in the environment are not necessarily information, i.e. do not reduce some relevant set of alternatives to some particular individual, especially do not necessarily concern any shopping action.

Theoretically and empirically this is not an unsolvable problem, but at this moment we can not go deeper into it.
Instead of that let us use the effect of the connection of level of retail sales and differences in land values of locations of similar qualities in other respects with differences in number of commuters and pedestrians at the same time and location.

One of the earliest studies that established a clear connection between volumes of traffic and retail sales was that of Paver and McClintock (Paver, McClintock 1935).

Their level of aggregation of problem, though it gives clearly the connection between existence of retail outlets and potential shoppers in the network of public communication space, still does not allow the conclusion that it is just the information variable that matters and not also accessibility or some other reasons.

In order to deduce the connection between potential sales and the information value of location, we can look to the following figure on page 65 taken from study of retail system of Chicago (B. Berry 1967).

According to these two graphs, land value of location is almost completely proportional to the number of pedestrians at the same place. The peak values are at the corner locations while the lots in between drop in value 2, 3 or more times. As the difference in accessibility (40-50 yards or about 30 seconds) is negligible for pedestrians, and for the driver of a passenger car the corner of a busy street is the most inaccessible place of all, it would not be premature to conclude that difference in information value of the corner location rather than the between-the-corners one, is in fact the variable that matters most.

The fact that corner location offers and gets at least twice as many
All business centers: Chicago region.

Land values and pedestrian counts along Ashland Avenue.
communicative contacts, and twice as much potentially relevant shopping information (by the simple fact that at least twice as many pedestrians and drivers pass intersection points rather than other points in between) gives us the possibility of expressing the information value of the location either in terms of land value (which is easy and reliable in existing situations but hard and unreliable where the problem is prediction), or in terms of number of passengers and pedestrians who can have a visual contact with different pieces of information offered by the retail outlet.
VII. REMARKS UPON STRUCTURE OF INDIVIDUAL SPACE IN THE CITY

In the previous discussion about information aspects of shopping, we concluded that all places and locations in the city do not have equal informative value. The measure of their informative value at some point will always be some kind of function of the number of people who are in the communicative distance of one unit of time, i.e., the number of possible communication contacts which would provide that emitted information would be received with maximum frequency.

If we look upon this relation from the point of view of an individual who is acting in the space of the city, the same relation would appear in the following form:

- The places with the highest potential of informative value for each individual, are the places where the individual comes most frequently in his normal daily, weekly, monthly, etc. activities.

The more frequent are visits to one place by one individual, the more familiar are the form and content of the activities that are taking place in it, and the more likely is it that this place will be chosen for undertaking some activities from its content rather than at any other place with similar content but less familiarity to the particular individual.

If we take a look at the patterns of life, movement and frequency of use of a city space by its individual residents, we shall easily conclude that there do not exist any two same patterns and two identical maps of use of city space and city facilities by its inhabitants. We can distinguish, according to the frequency of usage, between the groups of
places within the classes of similar usages and of frequency of usage.

That way the whole city from the standpoint of an individual would be transformed first into two basic sets of spaces.

- The space in the city that one uses; and
- The space in the city that one does not use at all.

Further distinction in the individual set of space that is used by the individual can diverge into many possible classifications (by function, by the reason for being there, by status as destination or transit space, etc.) but we shall be interested only in the general familiarity of different sets of space in the city to different individuals, and we shall suppose that familiarity is a function of chances for communication contact between the individual and particular activities that are taking place.

Frequency of potential communication contact is first of all a function of frequency of being in the place.

In that sense, we might, inside the set of relevant space for an individual in the city, note the sets of space of primary importance (very frequent everyday use which means at the same time and the most familiar space), of secondary importance, tertiary, etc.

Obviously, the residence itself would generate the highest frequency of use both by the number of arrivals and by the time spent.

Place of work would be the next by both criteria. Place of work and residence are connected by the transport-space link. The frequency of use of that space is also very high but the time spent in any point of it is very limited. This limitation poses restrictions on the form and content of information that can be exchanged, but information value of
Two individual residents of the same neighbourhood, in the same city, have different sets of space even if some points have to be common for both.
that space exists and it is not low in any case.

If, for example, the means of transportation that one uses to commute from home to job is underground, the link between home and job would have zero information value and the space between home and job as an element of the familiar set of space would not exist.

Most of the contemporary planning proposals for reconstruction of cities or building new ones start from the assumption of hierarchical organization of territorial urban units with their respective urban centers.

If we interpret these concepts in terms of organizing individual sets of space in the city, then we must conclude that the basis of reorganization of the city in the multilevel, hierarchical type of organization is the assumption that:

- all individual sets of urban space usage for the residents of the same territorial unit are identical or should be identical if they are not. That way residents of one neighborhood unit should all shop in their neighborhood center, or in "their" district center together with residents of other neighborhoods from "their" district, etc.

The main question here is: How realistic is this assumption that all individual sets of spaces in neighborhoods are the same or can be the same? Instead of taking the position that this is, in general, a far too simplified notion of urban functioning, which would take a few pages of discussion and maybe lead us to another opposite oversimplification, let us rather state that:
Introverted hierarchical organization of central activities and urban units in general will show a goodness of fit proportional to a degree of similarity of individual sets of urban space for the individuals in the same residential unit.

The higher is the number of individuals with identical or similar sets of space (which means inevitably: similar occupations and working places, similar recreational, social, and cultural habits, the higher is the amount of usage of the same central or, particularly, retail facilities that can be expected, and the higher is the probability of successful hierarchical organization. In the case of lack of similarity in individual sets of space in the residential unit the probability for a good fit of hierarchical organization can not be very high.
VIII. PUBLIC COMMUNICATION SPACE AND URBAN CENTERS

If all individual sets of space in city space were completely different, the city would not have any structure, and public-communication-space, as mass media, would not exist.

But that is not the case. Individual sets of space are different and are never identical but they all are overlapping at some points and lines; they all have some common elements, i.e., some places in the city are an element of all individual sets of space of all urban residents (like the very central place in the shopping and business district); some are the element of individual sets of space of a very large segment of population (like airport or railway station or stadium); and some places are the element of only a few individual sets of space (like an alley behind a few houses or delivery space for the big department store inside the block, even in CBD).

The rank and informative potential of public communication space depends, first of all, upon how many people have this particular place as an element of their individual sets of urban space.

Second, it depends upon the rank of this space inside each individual set of space (primary, secondary, etc.)

Further, in relation to some particular content of information it appears as important who are the individuals of whose sets of space this place is an element. Obviously for information about a new model of motor boat, a small road which is leading to the regional marina is as good as, or even better, than any street in CBD. Segmentation of potential users of information is of great importance.
Finally, the fourth relevant property of public-communication space refers to its spatial and functional characteristics which are promoting it to be a highly effective communicative and informative space, or make it less able to exchange respective kinds of information.

While a pedestrian shopping street is able to provide the information worth many bits for shopping purposes (window-shopping done in a pedestrian shopping street could bring about 14, out of theoretically-needed 21 bits of information in our shopping example of purchasing a pair of shoes) - the four-lane, divided highway, with green belts on both sides, would be able, at best, to provide the exchange of a small fraction of one bit of information by means of advertising panels, or about 1 or 2 bits of information concerning existence and shape of the shopping centers that are visible and accessible from the highway.

All four properties of public communication space are influencing each other and compensating the unfavorable differences or accumulating them. For example, the process of transformation of a shopping street into a vehicle-free pedestrian shopping zone, would exclude some segments of population (car passengers and drivers) and, therefore, would lower its ability for information transmitting by this criteria. But the ability to transfer a greater amount of information of a different, more qualitative, level would increase, and the overall result would be improvement and promotion in informative value of the space, rather than the opposite (HUD International 1972, pp. 49-50).

On the basis of this we can draw some general conclusions that concern the question of growth and development of urban structure.
Each city, at any point in time, has a practically defined structure of its elements, and one of the most important elements of the city structure is its public communication space. Public communication space is the highly sophisticated network of particular places and lines where communication contact between some information content and man can occur.

The information value of each point of public communication space is different in respect to some qualitative and quantitative aspects of the four mentioned criteria: How many individual sets of space the particular point is an element of, what the rank of this point in individual sets is, who the people whose sets of space the given point is an element of are, and what the functional informative performance of this point is.

Publicly oriented activities tend to occupy the most favorable positions in the public communication space, and the most favorable positions are the locations with the greatest information value.

As the information value of the location has a direct influence on the successful or unsuccessful financial development of business, and as there exists competitive demand for locations with particular information value, as well as a competitive supply of such locations, information value has also its market value in many terms. From this sequence of assumptions, obviously we can explain the concept of urban rent differences (Alonso 1960), taking note of differences in information position of the locations, more precisely than if we introduce only functional economies. We can further provide more realistic and more flexible initiating reasons for derivation of the structure of the city in models of urban structure of, for example, Mill's type, with the explanatory assumption (for an otherwise well working model) that:
"...Two commodities are produced in the urban area under consideration. Commodity one is export goods. The number of units to be exported, X, is exogenous, and all units must be shipped to a predetermined point. ...Which is the city center." (Hills 1972, p. 85) This insufficiently resembles what common sense says the function of a city center in fact is.

In our opinion, the main attribute of the main city center which distinguishes it from all other places in the city is the fact that it is an element of individual sets of space in the city of all city residents as well as the element of restricted sets of space of all external visitors to the city.

Therefore, the information value of the focus of public communication space in the city is so high, that it would probably be the sufficient reason for high agglomeration of public activities which depend on communication with the public and information about their existence and content of activity to the public.

Different activities have different types of reasons for using the information potential of the location. Shops tend to promote their sales and enlarge the number of potential customers by securing the place for their stock in the information gathering and decision-making process of the shopper. To locate the shop at the location that is an element of all individual sets of space in the city is obviously very favorable in respect to this process.

Business offices tend to secure or enlarge their clients by informing
them of the existence of a particular office in the context of the central business district which can mean also that the scope of their business is of regional or broader importance rather than only local.

Finally, symbolic reasons can be often the motive for governmental institutions to look for their location in the space that is an element of a-l individual sets of space in the urban area under consideration, because that will symbolize their importance and domain, as well.

We do not mean to say that the reasons of accessibility, or hard and soft functional economies (by hard, we assume shortening necessary technological and transportation links for assembling the products or shipping it; by soft functional economies we assume the possibilities for quick and easy gathering of two or more or many individuals that must communicate and participate in some information processing or in some decision-making process are of little importance. What we mean to say is that place in the system of public communication space is one of the main locational questions for any public activity, where information value of the location is often the main criterion for the feasibility of some particular space for some particular a particular activity.

In the case of trying to explain further growth and development of the central business district in the large contemporary city, despite tremendous traffic congestion, introduction of the information variable will be necessary. Functional economies and today a rather unrealistic "best accessibility" assumption for the CBD alone can not explain the new taller office building in the CBD is raising the land values in CBD instead to lower them down when it produces new level of traffic congestion
and parking problems.

But the fact is that any new activity at some point increases the information value of the surrounding locations too.

The consequences of adopting an information concept of city structure if it could be proved as a relevant one, would be important in the sphere of urban design than in the sphere of quantitative programming of growth, but even there the differences in methods and outcomes can be remarkable ones.
PART 3

RETAIL DISTRIBUTION MODELS AND INFORMATION VARIABLE
In Part II of this paper we discussed some aspects of influence of the information variable on shopping behavior and decisions. The conclusion was that the influence of previously obtained information is substantial. Logical consequence of a conclusion like this, taking into account that the information variable does not figure in any single retail distribution model, would be to build a model which directly involves the information variable as one determinant of consumer decision.

The problem with this is very simple and practical one: the lack of knowledge and adequately structured information that would allow a successful approach based on the information concept is too great, at this moment, and chances for success are insufficient without research in this particular direction.

The explorations in the structure of individual space in the city, and the influence of special characteristics of particular sets of urban space on behavior of an individual have been, up to now, oriented almost exclusively towards questions of perception of visual and symbolic characteristics of urban public space (Lynch 1960, Appleyard 1970), or have been general conceptualization of the methodology for structuring interconnections between urban variables on a basis which gives excellent openings to the relevant questions (Alexander 1964), but not yet the necessary answers.

If we are not able at this moment to present the consequent model with a built-in information basis in the direct way, we can try to see if we can add some representative analogue of the information variable to the existing retail distribution models in order to reach positive corrections to their outcomes as if information variable was included.
That way we can have the most general equation which would state that:

\[ P_{ij} = I_{ij} \cdot A_{ij} \]

Where

(P) probability that shopper will patronize the retail facility j;

(I) amount of information connected to the retail facility j

(or familiarity with the place in general); and,

(A) attraction of the retail outlet to particular population

unit i (as estimated by any chosen model of retail distribution that does not include the information variable).

The concept of the overall model is not deterministic in defining constant trading areas. Probabilistic type of models would be easily adapted to fit into scheme.

This type of model is a specific case of a class of models of interaction, and therefore has two parties, subject and object (residential unit and shopping center). The corrective influence of the information variables can be attached to one, or to the other, or to both, depending on the form and content of the information variable.

At this moment we can suppose two main ways to deal with this problem:

(a) To use the property of differences in informative values of locations of shopping centers under consideration. In that case, the information value of the location is a new variable added to the model but without other serious complications.

(b) To use the effect of segregation of the spatial sets of residents in a particular urban space, in which case the residential unit would have
to be disaggregated into classes of residents with similar spatial sets in the given urban space. This task, by itself, is not a simple one at all. That was probably one of the main reasons why the information variable was not introduced in retail distribution models before, because it is questionable whether the results would pay off the additional effort.
I. INTRODUCTION INFORMATION VARIABLE WITHOUT DISAGGREGATION OF RESIDENTIAL UNIT

If we suppose that all residents of one unit have the same opportunities for getting information, or if we are not in a position to know enough about the differences in their spatial sets, we shall obviously deal with very abstract assumptions and analysis.

In the equation: \( P_{ij} = I_{ij} \cdot A_{ij} \)

we can suppose any value of \( A_{ij} \) as exogenous, as a result of some more or less sophisticated model of shopping behavior. But as the result will depend on the \( I_{ij} \), which, obviously can not be very particular without the segregation, we can start with \( A_{ij} \), also, from the most general level.

The principle of least effort is, no doubt, a good model guide. So the function of attraction of the center for an individual can be stated in terms of maximizing the probability for successful shopping under conditions of minimizing frictions which cause additional effort. This assumption leads to the well known form:

\[
A_{ij} = \frac{M_j}{d_{ij}}
\]

(Attraction of A of center j to individual i is proportional to the mass of the center and inversely proportional to the distance between i and j).

Speaking of i as an individual, or a group of similar individuals, we are dealing with human subjects, for whom both variables M (mass) and d (distance) do not have any objective value that is identical to the real measures of mass and distance, but rather only the empiric perception of
value and influence of each of these variables for shopping goods, etc.). The best way to introduce these influences of the perception parameters would be in the form of exponents over mass and distance (respectively, let them be $a$ and $b$). In that case, the attraction part of the model would have the form

$$A_{ij} = \frac{M_j^a}{d_{ij}^b}$$

It is worth mentioning again that this is, in fact, absolutely the same form that constitutes Reilly's formula of retail gravitation. This form was set before the actual empirical test, after which the conclusion that the exponent of $M$ was the first power and exponent over $d$ was nearer to the second power than to any other power (according to Scott 1970) was formulated in the form of Reilly's law of retail gravitation.

Does this mean that we just discovered something like a "behavioristic and perceptual explanation" for gravity-models' regularity? We would not rely too much upon that, but this can be one of the useful forms of conceptualization of the model because it can lead to the procedure of calibration of parameters on the basis of individual behavior rather than as the statistical outcome of behavior of aggregated groups like residential unit, etc., and that would hopefully lead to some eventual generalization of parameters for some defined number of classes of goods and families. The problem of shifting parameters through time, of course, is untouched.

As the whole idea in this part of the paper was to introduce the information variable but without segregation of the population unit to segments and classes of different information backgrounds, two options
are open.

One is, as said before, to introduce the new variable-information value of the location. Let us call it I, so the previous model would get a form

\[ A_{ij} = \frac{I_i \cdot M_j}{d_{ij}^b} \]

If we want to go another way (without introducing the variable of information value of the location), and try to define the information part of the model \((I_{ij})\) from the form \(P_{ij} = I_{ij} \cdot A_{ij}\) without segregation of population, than \(I_{ij}\) has to be some outcome of the concept of random probability of gaining information at a distance. Again, many existing concepts are available but almost all of them again are a kind of gravity formula with mass and distance as main variables.

As it will hardly take a lot of time to produce a new derivation of some kind of random-interaction-on-a-distance-formula, let us take this time and chance.

If the residence unit under consideration (I) is in a circular space with unrestricted possibility of moving radially in all directions, the probability of the interaction contact between (i) and (j) is proportional to the part of the perimeter that the mass \((M)\) of (j) is covering and the size of the perimeter itself.

As perimeter is a function of a radius \((d)\) (which is distance in the model), then

\[ I_{ij} = \frac{M_j}{2\pi d_{ij}} \]
if the space between (i) and (j) is empty. But as it is not the case, as any point inside the circle with radius d can be the one to be contacted, then the opportunities are not restricted to the perimeter of the circle but to its surface. That way $I_{ij}$ becomes:

$$I_{ij} = \frac{M_j}{\bar{u}d_{ij}^2}$$

In the same manner, some other type of space can be imagined. For example, in the case of the rectangular space, similar to the grid of the street blocks, where motion can be done only in a combination of two fixed directions, the outcome would be:

$$I_{ij} = \frac{M_j}{2d_{ij}^2}$$

or if we generalize the coefficients ($\bar{u}$) and (1/2) into (C):

$$I_{ij} = \frac{C M_j}{d_{ij}^2}$$

As we had at the beginning, the formulation of probability of visiting the center ($P_{ij}$) as a function of level of information ($I_{ij}$) and consumer's judgment of attraction of a center ($A_{ij}$) in respect to his home, then the equation:

$$P_{ij} = I_{ij} \cdot A_{ij}$$

(5) can be written as:

$$P_{ij} = \frac{M_j^{\alpha}}{d_{ij}^2} \cdot \frac{M_j^{\beta}}{d_{ij}^b}$$

(6) which is the same as

$$P_{ij} = \frac{M_j^{1+a}}{d_{ij}^{2+b}}$$

(7) if $M_j$ from the information part and $M_j$ from the attrac-
tion part are the same variables. But if they are not, the first can be \((M^*)\) the total size of all activities in the center and the second the particular one), then the formula would have to become:

\[
P_{ij} = \frac{M_j^a}{d_{ij}^{2+b}}
\]

as the condition that:

\[
P_{ij} = 1
\]

(9) is necessary, and it brings the whole outcome into relative terms, we have been able earlier to neglect the constant \(C\).

The conclusion of this analysis can be the following:

There are many different ways to arrive at the same basic form of the gravity model for the very simple reason: its form is so simple that the only particularity that can be argued or that can refer to better or worse fit, or should ask for some proof or explanation is the value of the exponential parameters \((a)\) and \((b)\).

If the value of parameters \((a)\) and \((b)\) is calibrated in a traditional way, from aggregated number of actual visits in the whole residential unit, without giving any particular content and meaning to the parameters, then
the equations (6) and (7) do not open any new possibility or difference in respect to the well-known forms of gravity models in retail distribution.

The difference could be found in equation (6) where the decision-making part, attraction, is separated from the information part and where parameters (a) and (b) have the meaning of subjective perception of respective mass and distance in connection to a particular shopping goal. This offers different procedures in their calibration (with the more cognitive elements) Practically, that means that equation in surveys of the type:

"How important is the mass of the retail facility when you search for...?" and

"How important is the distance of the retail facility when you search for...?"

should be introduced into questionnaires rather than:

"Where did you purchase your ... the last time?" - which is the most common form.

Answers on the last question can not provide any generalization other than statistical regularity for one particular case. It also can not give any explanation because the question itself did not ask for any explanation. Buy, by separating the different hypothetical influences in the process of decision-making and investigating them separately, some casual relationship can hopefully be established and some generalization of the value of the model parameters as well.
II. RETAIL DISTRIBUTION MODEL WITH DISAGGREGATED POPULATION ACCORDING TO DIFFERENCES IN SPATIAL SETS

The question of individual spatial sets and their connection with the information variable in shopping is discussed earlier, and, at the most reduced level, the hypothesis was that:

The more familiar one environment is to the individual, the more likely it is that he will choose it for undertaking some activity from its content (if the content of activities itself is comparable to the alternative ones).

Familiarity means possession of the relevant information about the environment and its content. One of the major sources of information about environment is frequent presence and visual contact with it.

That means that if we were able to learn about major patterns of daily, weekly, and monthly movements of respective groups of the population, we would be able to suppose that this pattern of movement represents also the analogue to the pattern of familiarity with space in the city to the population under consideration.

The numerous models and methods for describing the pattern of movement of an urban population already were developed and have reached a remarkable level of sophistication and accuracy in the field of transportation studies.

In that sense, the first step towards introduction of the information variable in shopping behavior models would probably lead through joining the most appropriate traffic model that is available for a particular city, to the most appropriate retail distribution model that is chosen for use, whether it be gravity model, intervening-opportunities model, or Bacon-type model.
In any case, the general form would be the same as in the previous part:

$$P_{ij} = I_{ij} \cdot A_{ij}$$

$P_{ij}$ is the probability that a particular fraction of population from (i) with similar spatial sets in the city will patronize shopping center (j);

$I_{ij}$ is the outcome from the traffic model which states the intensity of presence of population from (i) in point (j), and represents the exogenous variable in the model, which introduces differences in information for different centers and different population groups.

$A_{ij}$ is the decision-making part where relevant variables of the shopping center (like mass and friction or competition) are connected, and represents in fact the abstract case of probable individual decision if perfect or uniform information exists.

No doubt, the presented discussion is not any practical and final result which can be applied without problems. In this stage of development it is nothing but a hypothesis of one but of a few possible directions where further research for improvement of models of retail distribution can be directed. The hope was that enough causal relations were built in to permit eventually some predictive performance of the model.
REFERENCES


