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PRESENTED BY
Dr. Marcus Dalyman in
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LINEAR DRAWING,
SHOWING THE APPLICATION OF
PRACTICAL GEOMETRY
TO TRADE AND MANUFACTURES.

BY
ELLIS A. DAVIDSON,
AUTHOR OF "PROJECTION," "BUILDING CONSTRUCTION," "DRAWING FOR
CARPENTERS AND JOINERS," "DRAWING FOR MACHINISTS,"
"PRACTICAL PERSPECTIVE," ETC., ETC.

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INTRODUCTION.

This work is intended firstly as a text-book for teachers in Schools of Art and Science, Training Colleges, National and other schools, and secondly as a manual for self-instruction for artisans and the public generally.

It is impossible to over estimate the importance of a knowledge of Geometry, forming as it does the basis of all the mechanical and decorative arts; and therefore the first volume of a Technical Series has been devoted to this subject, from which, as from one common highway, branches may diverge, adapted to the various industrial arts.

In order to fit the course of instruction, not only to the young, but to adults whose elementary education may be deficient, the author has deemed it best to assume, in the first instance, utter ignorance on the part of the pupils. Thus it will be seen that in Problem 1, no geometrical terms, such as “describe an arc,” &c., are used, the learner being merely told to place the steel point of his compass on a given spot, whilst tracing part of a circle with the pencil leg. Gradually, however, the proper expressions are introduced and explained; and as the previously uncultivated soil becomes firstly broken up into broad
masses and then refined, the pupil is addressed in more scientific terms, and the higher faculties of the mind are called into action.

The subject is not treated as a mathematical, but as a thoroughly practical one, and therefore no absolute system of reasoning is attempted. Still, it has been thought right to give some simple and familiar explanations of the properties of the various figures, and the principles upon which their constructions are based, as it must be obvious that the more the mind comprehends of the relation of one line and form to another, the more will the eye appreciate beauty and refinement, and the more accurately and intelligently will the hand execute.

In order to guide students who may use this work for self-instruction, the processes in each figure are lettered in the order of the alphabet, the consecutive steps will thus become evident. This plan is assisted by the imaginary or constructive portions being drawn in dots or fine lines, the given figures in medium, and the resultant figures in full black lines.

The figures initiate, or lead on to, each other, and the practical application is given where necessary. Thus, Fig. 13 is the construction of an equilateral triangle; Fig. 15a bisects an angle, and is followed by a problem, by means of which a circle is inscribed in a triangle, a figure which is based on the previous operations. Fig. 16 teaches how to draw a circle through three given points (and hence about a triangle), and the student will then find little, if any, difficulty in working Fig. 17, a Gothic trefoil, in which all these steps are united. This system will be found to pervade the whole book, and hence
the author has been enabled to introduce examples of Mechanical and Architectural drawing, showing the practical application of the previous studies to the daily requirements of the workshop; and in order to prepare the students for this practice, the figures are drawn as largely as the size of the work will allow, so that each one may be worked as a lesson in mechanical drawing.

The “definitions” are not given as separate chapters, but are distributed over the whole work, and will thus become impressed on the minds of the pupils, by being associated with the figures which form the subjects of the lessons.

The author cannot, of course, claim originality as far as the geometrical figures themselves are concerned; several new combinations and adaptations are, however, introduced. The instructions for working are entirely rewritten, and the “steps” are such as have been found most successful in an experience in popular teaching extending over eighteen years.

Besides the explanation of technical terms, brief biographical notes are given of some of the most celebrated geometericians, to whom certain figures are ascribed.

In addition to the illustrations connected with the lessons, six pages of the application of geometrical drawing to iron and wood work, masonry, mechanism, and design are appended. To these and numerous other branches of Technical education, as well as to Isometric and Orthographic Projection, separate volumes will be devoted.

On these grounds, therefore, the author ventures to entertain the hope that he may have contributed one mite towards the scientific education of his countrymen,
in simplifying a study hitherto deemed abstruse, and intelligible to the learned only; but which is so all-important to the artisan, in enabling him to construct the forms required in his trade by rapid and certain means, instead of blindly following the traditional methods existing amongst the men in "the shop," in giving him accuracy of eye and refinement in execution, and above all, enabling him, when once acquainted with the "grammar of form," to originate and invent, and so keep pace with the progress made, not only in foreign countries, but in our own.

ELLIS A. DAVIDSON.
**LINEAR DRAWING**

*(GEOMETRICAL).*

To bisect the line A B (viz., to cut it into two equal parts). (Fig. 1.)
Use the compass having a pencil leg.

Place the steel point in A.
Open the compass until the distance between the points is much more than half the line (say to a).
Keep the steel point in A, and with the pencil point draw a part of a circle, C D.
Place the steel point in B, and with the same length* in the compass draw E F, which will cut through C D in the points G and H.
Draw a line from G to H, which will cut the line A B into two equal parts.

**To erect a Perpendicular at the end of the line A B.** (Fig. 2.)

![Fig. 2.](image)

Place the steel point of the compass in any point above the line, as C.
Extend the pencil point until it reaches B. With the length (called the radius) from C to B, draw a part of a circle cutting the line A B in the point D.
From D draw a line passing through C and cutting the arc in E.
Draw a line through E to B, which will be perpendicular to A B.

* The length with which a circle or part of a circle is struck, is called the radius; it is the length from the centre of a circle to its outer line or circumference.
† Perpendicular means "square" with another line; to say that a line is "perpendicular," does not necessarily mean that it is upright, for the sides
The same, by another method.  (Fig. 3.)
Let it be required to erect a perpendicular* from the point C, which may be at the end or at any other part of a line, A B.

From C, with any radius, draw an arc† D d.
From D, with radius C D, draw an arc cutting the arc D d in E.
From E, with the same radius, describe the arc C G, which will cut arc D d in F.
From F, with the same radius, describe the arc E H, cutting F G in I.
Draw a line through the point I to meet the point C, and this line will be the perpendicular required.

* The teacher is advised to draw perpendiculars to the line A B when placed vertically or obliquely, with the view of showing the pupils that a line may be perpendicular to another though not upright.
† Explain, when two lines are perpendicular to each other, they form a "right angle."
† Arc. Part of a circle.

Or edges of a carpenter's or mason's square are perpendicular to each other in whatever position the square may be placed—
Definitions concerning Parallelograms.

A parallelogram is a four-sided figure whose opposite sides are parallel to each other.

When the four sides are equal, and the four angles are right angles, the figure is called a SQUARE, as A.

When one pair of sides is of a different length to the other, but the sides remain parallel to each other in opposite pairs, the angles being right angles, the figure is called a RECTANGLE, or Parallelogram, as B.

When the four sides are equal and the opposite sides parallel to each other, but the angles not right angles, the figure is called a RHOMBUS, or Lozenge, as C.

In this figure the opposite angles will be equal to each other, as \(a, b\).

When the figure is formed of two pairs of parallel sides, each pair being of a different length, but neither of the angles being right angles, it is called a RHOMBOID, as D.
To construct a Square on the given line A B.
(Fig. 4.)

Erect a perpendicular at A (prob. 2).

Make the perpendicular A C equal in length to A B. This is best done by using A as a centre, and A B as radius, then describing the arc (in this case a quadrant, or quarter of a circle) A C, which will cut off the perpendicular at the required length.

From B, with radius A B, describe an arc; and from C, with same radius, describe another arc, cutting the former one in D.

Draw the lines C D and D B.

Then A B C D will be a square.*

* In a square, all the four sides must be equal; and all the angles must be right angles. If both these conditions be fulfilled, both the diagonals will be equal. Diagonals are lines crossing to opposite angles, as A D and B C.
**LINEAR DRAWING.**

To construct an oblong of a given size (in this case 2½ in. long, and 1½ in. wide).* (Fig. 5.)

![Diagram of a rectangle with labeled points A, B, C, D, and E.]

- Draw the line AB 2½ in. long.
- At B erect the perpendicular BC 1½ in. high.
- From C, with radius AB, describe an arc (c).
- From A, with radius BC, describe an arc (c'), cutting the former arc (c) in D.
- The lines CD and DA will complete the oblong (or rectangle) of the required dimensions.

* The greatest pains should be taken from the first to acquire the power of measuring with perfect accuracy, which is of the utmost importance in geometrical drawing when applied to trade purposes. In drawing, it is best to take the measurements with compasses from the rule. This is more likely to result in exact measurement, than laying down the rule on the paper and marking from it. The proper way to "put down" this measurement is $2\frac{1}{2} \times 1\frac{1}{2}$". One dash ('') over a figure means feet, two dashes (') mean inches—thus $2' \times 1' \frac{1}{2}$" means that the surface is 2 ft. 6 in. long by 1 ft. 8 in. broad.
To draw a line perpendicular to \( A \ B \) from a point lying away from the line, as \( C \). (Fig. 6.)

![Fig. 6.](image)

From \( C \), with a radius rather shorter than from \( C \) to \( B \), draw an arc, cutting \( A \ B \) in the two points \( D \) and \( E \). From \( D \) and \( E \), with any radius, draw arcs cutting each other in \( F \).

Draw \( C \ F \), which will be perpendicular to \( A \ B \).

![Fig. 6a.](image)

This mode would apply if the point \( C \) were under the line, or if \( A \ B \) were placed obliquely, &c. (as in Fig. 6a).
To draw a line parallel* to A B, and at a given distance from it. (Fig. 7)

Mark any two points on the line—viz., C and D.
Set off equal distances, e f and g h, on each side of C and D.
From e f, g h, with any radius, describe arcs cutting each other in I and J.
Draw lines from C and D through I and J.
On these perpendiculars set off from C and D the required distance between the parallels—viz., C K and D L.
Draw the line K L, which will be the required parallel.

To draw a line parallel to A B, and passing through the point C. (Fig. 8)

From C, with any radius (as C D), describe an arc (E), cutting the line A B in the point D.

* Parallel means running in the same direction, and keeping the same distance apart from each other. The metal rails on a railway are parallel to each other, so are the "ruts," or marks on a road made by the wheels of a cart which has passed over it.
LINEAR DRAWING.

From D, with the same radius, describe the corresponding arc (F), cutting the line A B in the point G.

Measure (with compasses) the length of the arc F, viz., from G to C, and mark off this length on the arc E, viz., from D to H.

Draw a line through C and H, which will be parallel to A B.

To divide the line A B into any number of equal parts (in this case ten). (Fig. 9.)

![Diagram](image)

Draw a line (C D), parallel to A B.

(The line C D may be any length—that is, it may be drawn indefinitely for the present.)

From C set off along this line the number of parts into which the line A B is to be divided—viz., 1 to 10. These parts may be any convenient size, but must be all equal.

Draw C A and 10 B, and produce* both lines until they meet in E.

From each of the points, 1, 2, 3, &c., draw lines to the point E, which passing through A B will divide it into 10 equal parts.

* To “produce” a line means to carry it on further, or to make it longer in the same direction.
Application No. 1 of the foregoing figure. (Fig. 16.)

This problem may also be used for dividing a line proportionally to another—that is, to find divisions on a line, which shall be in the same proportion to it, that certain divisions are to another line either larger or smaller.

Thus, let it be required to cut off from a part which shall have the same proportion to it that the division E D has to the line C D.

Place A B parallel to C D.
Join C A and D B, and produce the lines until they meet in F.

From E draw E F, which passing through A B will cut off G B, which will have the same proportion to A B that E D has to C D.

This process is constantly used in finding the proportions of architectural mouldings, windows, mechanical details, &c., in making reduced or enlarged drawings.
This problem is also most useful in finding a particular point in a line which may be so small as to render accurate division very difficult.

Example: The length from A to B in a spur wheel (Fig. 11), including a tooth and a space, is called the pitch, and the circle on which such distances are set off is called the pitch circle.

Now, although in many drawings the space and tooth are made equal, they are not so in a real spur wheel, the space being a very little larger than the tooth. This small difference is most important, for if the tooth and space were equal, the tooth of a wheel when in gear with another would not clear itself. The difference of one-eleventh is found in practice to be sufficient for all purposes. Thus, if the "pitch" is divided into 11 equal parts, the tooth will be five-elevenths, and the space six-elevenths.

But dividing the space A B (which in many cases is by far smaller than given above) will be found liable to
some inaccuracy; by this problem, however, the required point of division may be found with ease and exactness.

Let A B (Fig. 12) be the length of the pitch, measured from A B in Fig. 11. Draw any line, C D, parallel to A B, and set off on it 11 equal divisions (any length).

Draw C B and D A, and produce the lines to meet in E.

From point 5 draw a line to E, which will divide A B as required, the one part being \( \frac{5}{11} \) and the other \( \frac{6}{11} \).

Set off these lengths on the pitch circle.*

**To construct an Equilateral Triangle on the given line A B. (Fig. 13)**

![Equilateral Triangle Diagram](image)

From A, with radius A B, describe an arc.

From B, with the same radius, describe a corresponding arc, cutting the former one in C.

Lines joining A C and B C will complete the triangle, which will be equilateral—that is, all its sides will be equal.

A triangle having only two of its sides equal, is called an Isosceles Triangle (\( \triangle \)).

![Isosceles Triangle](image)

When all three sides are of unequal length, the figure is called a Scalene Triangle, as B.

* For full instruction concerning the modes of drawing the various forms of teeth of wheels, the student is referred to the volume on Mechanical Drawing.
In a right-angled triangle, one of the angles, as C, is a right angle.*
A right-angled triangle may be either isosceles, as D, or scalene, as E.
The longest side of a right-angled triangle, viz., the side opposite to the right angle, viz., F, is called the Hypotenuse.

To construct a Triangle of given dimensions.
(Fig. 14.)

Let it be required that the sides of the triangle should be 1½", 1", and 1½".
Make A B 1½ in. long.
From B, with a radius of 1½ in., describe an arc.

*When a line, C D, stands perpendicularly on another line, A B, it divides the space into two right angles; if produced beyond C, four right angles will be formed: but if the line C E be drawn, dividing the space unequally, the angle A C E is an obtuse (or wide) angle, being more than a right angle, and the remaining portion, B C E, is an acute (or sharp) angle, being less than a right angle.
From A, with a radius of 1 in., describe an arc cutting the former one in C.
Draw A C and B C, which will complete the triangle of the required dimensions.

**To bisect an Angle, A B C.** (Fig. 15a.)

![Diagram of bisecting an angle](image)

From B, with any radius, describe an arc, cutting the lines B A and B C in D and E.
From D and E, with any radius, describe arcs cutting each other in F.
Draw B F, which will bisect the angle.

**To inscribe a Circle in the Triangle A B C.** (Fig. 15b.)

![Diagram of inscribing a circle in a triangle](image)

Bisect any two of the angles (by Fig. 15a).
Produce the bisecting lines until they meet in D.
From D, with the radius D E, which is a perpendicular from D on A B, a circle may be described which will touch all three sides of the triangle. This is called the inscribed circle.

To draw a Circle through three points, however they may be placed (provided they are not in an absolutely straight line). (Fig. 16.)

Let A B and C be the three given points. Join A B and B C. Bisect A B and B C, and produce the bisecting lines until they cut each other in the point D.

Then D will be equally distant from each of the three points. Therefore, from D with radius D A, D B, or D C, a circle may be drawn which will pass through the three given points.

It will be evident that if A and C were joined, the figure would be a triangle; and thus this problem serves also for describing a circle which shall touch the three angles of a triangle. This is called the circumscribing circle.

The Gothic Trefoil.* (Fig. 17.)

This figure will serve as an application of the construction of the equilateral triangle and the bisecting of

* Trefoil. A figure much used in Gothic architecture. It is formed of three leaves, or lobes (hence its name), meeting at a centre, as in the three-leaved clover. It is sometimes enclosed in a circle, as in window tracery, but not always, as in many wall piercings.
angles. It is here introduced with the view of showing students the importance of absolute accuracy in the early problems, as well as in the subsequent operations.

Construct an equilateral triangle, $a v c$.

Bisect the angles, and produce the bisecting lines, $d e f$.

Observe, that in an equilateral triangle, the lines which bisect the angles will, if produced, bisect the sides opposite to the angles as well, and thus the points $g, h, i$ are obtained.

From $a, b$ and $c$, with radius $a g$, equal to half the side of the triangle, describe the arcs $j k l$, and the others, which it will be plain are concentric* with them.

* Concentric. Drawn from the same centre.
LINEAR DRAWING.

The arcs \( m \) and \( n \), and those corresponding to them, are also drawn from the same centres.

The outer circles and the arcs \( j \), \( g \), \&c., are drawn from the centre of the triangle \( o \).

To construct on the given line \( D \quad E \) an Angle similar to the angle \( A \quad B \quad C \). (Fig. 18.)

From \( B \), with any radius, describe an arc cutting the sides of the angle in \( c \) \( d \).
From \( E \), with the same radius, describe an arc cutting \( E \quad D \) in \( F \).
Measure the length from point \( c \) to \( d \).
Mark off the same on the arc from \( F \)—viz., to point \( G \).
Draw a line from \( E \) through \( G \).
The angle \( F \quad E \quad G \) will be equal to \( A \quad B \quad C \).

On the given line \( A \quad B \), to construct a Triangle similar* to \( C \quad D \quad E \). (Figs. 19 and 20.)

At \( A \) construct an angle similar to the angle \( H \quad C \quad G \)—viz., \( J \quad A \quad I \).

* When a figure is said to be similar to another, it means that it is of the same shape.
When it is said to be equal, it means that it is of the same area—that is, it contains precisely the same space.
A figure may be equal to another without being similar in shape, as in
At B construct an angle similar to the angle K D L—viz., M B N.
Produce the lines A J and B M until they meet in O; which will complete the triangle required.

Definitions concerning Four-sided Figures which are not parallelograms.

A figure having four sides, which are neither equal nor parallel to each other, is called a TRAPEZIUM, as A.

But any two of its adjacent (or adjoining) sides may be equal to each other, so long as they are not parallel to the opposite sides, as in the figure B.

If any two of the sides are parallel to each other, the figure is called a TRAPEZOID, as C.

To construct a Trapezium similar and equal to another (C D E F). (Figs. 24 and 25.)

Draw A B equal to C D.
At A construct an angle similar to that at C.

Fig. 205, where the square is equal to the rectangle; and a figure may be similar without being equal, as in Figs. 26 and 27.
"Similar and equal," means being of both the same shape and size as another figure, as in Figs. 24 and 25.
Make A G equal to C E.
At B construct an angle similar to that at D.
Make B H equal to D F.
Join H G, and the trapezium on A B will be similar and equal to C D E F.

It is advisable that the students should be repeatedly exercised in constructing figures similar and equal to each other; and as the correct result of the higher figures depends on the refinement of their construction, the most intense accuracy should, as the pupils advance, be insisted upon.

To construct, on the given diagonal A B, a trapezium similar to another (C E D F). (Figs. 26 and 27.)

Fig. 26.

Fig. 27.

Draw the diagonal C D in the given trapezium. From C and D, with any radius, draw arcs cutting the diagonal C D in G and H; C E and C F in I and J, and D E and D F in L and K.

From A and B, with the same radius, describe arcs cutting diagonal A B in M and N.

From the point M, cut off on the arc the length G J, viz., to O, and also the length G I, viz., to P.

From the point N, cut off on the arc the length H K, viz., to Q, and also the length H L, viz., to R.

Draw B Q and B R.

Also A O and A P.

Produce these lines until they meet in S and T.

A T B S will be the trapezium required.

This result would be the same whatever might be the length of the diagonal or the relative sizes of the figures, as an angle is not altered by the length of the lines of which it may be formed.
To construct a Trapezium from the following given dimensions (Fig. 28)—

![Diagram of a trapezium]

Fig. 28.

Sides C A and C B are to be adjacent to each other, forming an angle similar to A C B.

- C A is to be $1\frac{1}{2}$ inches long.
- C B
- A D
- B D

Before commencing to work out any questions, the student is recommended to think over the given conditions, and to consider most carefully which would be the best starting-point.

Now, in the figure here required, the first fixed condition is, that the sides C A and C B are to make an angle similar to the given angle A B C.—Therefore at any point, construct this angle (a C b) and produce the lines until C A is $1\frac{1}{2}$ inches, and C F $1\frac{1}{2}$ inches long, viz., to A and B.

From A, with 1 inch radius, describe an arc.

From B, with $1\frac{1}{4}$ inch radius, describe another arc cutting the former in D.

Draw A D and B D, which will complete the figure from the given dimensions.
To construct a Square on a given diagonal A B.
(Fig. 29.)

Bisect the diagonal A B in the point C.
From C, with radius C A, describe a circle cutting the
bisecting line in D and E.
Draw A D, D B, B E, E A, which will complete the
square on the given diagonal A B.

To construct a Parallelogram when the
diagonal A B and the length of one pair of
sides C are given. (Fig. 30.)

Bisect A B in the point O.
From O, with radius O A, describe a circle.
From A and B set off the length of the line C on the
circle—viz., A D and B E. Join these points, and the
required figure will be completed.
To describe a Square about a Circle. (Fig. 31.)

Draw two diameters* A B and C D at right angles to each other.
From A and C, with radius equal to the radius of the circle (O A), describe arcs cutting each other in E.
From C and B with same radius, describe arcs cutting each other in F. From A and D with same radius, describe arcs cutting each other in G.
From D and B describe arcs cutting each other in H.
Draw E F, F H, H G, and G E, which will complete the square about the circle.

To describe a Circle about the Square A B C D. (Fig. 32.)

Draw the diagonals A D and B C.
From their intersection† (O), with radius O A (O B,

* Diameter. A line drawn across a circle, and passing through the centre, it is thus equal to two of the radii (plural of radius), and is the longest straight line that can be drawn in a circle.
† Intersect. To cut through.
O C, or O D), describe the circle touching the four angles of the square.

**To find the Centre of a Circle.** (Fig. 33.)

![Diagram of circle and its center]

Draw a chord $\ast$ as A B, and bisect it by a line cutting the circle in C and D.

Bisect C D by the line E F.

The intersection O is the centre of the circle.

**To inscribe a Square in a Circle.** (Fig. 34.)

![Diagram of square inscribed in a circle]

Find the centre of the circle, and draw two diameters at right angles to each other.

$\ast$ *Chord.* A line cutting off any part of a circle. The parts into which the circle is thus divided are called *segments.* A part of a circle contained between two radii, as D O E, in Fig. 34, is called a *sector.*
From their extremities draw lines A B C D, which will form the square in the circle.

To construct a Gothic Quatrefoil.* (Fig. 35)

Construct a square on the diagonal A B (see Fig. 29).

Bisect the sides by the lines E G, F H, cutting the lines A C, C B, B D, and D A, at i, j, k, l.

From A, C, B, and D, with radius A i—that is, half the side of the square—draw the arcs i, m, n, o, and those concentric with them.

The outer circles are drawn from the centre O.

* Quatrefoil. A figure based on four leaves or lobes. See remarks on the Trefoil, Fig. 17.
To inscribe a Square in any Triangle, A B C.
(Fig. 36.)

From C drop a perpendicular, C D.
From C draw a line parallel to A B—viz., C E.
From C, with radius C D, describe a quadrant cutting C E in F.
Draw F A, cutting C B in G.
From G draw G H parallel to A B.
And from G and H draw lines G I and H J parallel to C D, which will complete the square in the triangle.

To inscribe a Square in a given Trapezium, A B C D.  (Fig. 37.)

Draw the diagonals A C and B D.
Draw D E at right angles and equal to D B.
Draw E A, cutting C D in F.

C 2
Draw $FG$ parallel to $AC$.
Draw $GH$ and $FI$ parallel to $DB$.
Join $HI$, which will complete the square in the trapezium.

To inscribe a Circle in a given Trapezium, $ABC$, $D$, of which the adjacent sides are equal. (Fig. 38.)

![Diagram](image)

Draw the diagonal $AB$, which will bisect the angles $C \parallel D$, and $C \parallel D$.
Bisect the angle $AD$.  
Produce the bisecting line until it cuts $AB$ in $O$.
Then $O$ is the centre from which a circle may be described, touching all four sides of the trapezium.

To trisect* a Right Angle, $ABC$. (Fig. 39.)

![Diagram](image)

From $B$, with any radius, describe the quadrant $DE$.

*Trisect. To cut into three equal parts.
From D, with the radius D B, describe an arc cutting E D in F.
From E, with the same radius, describe an arc cutting E D in G.
Draw lines B F and B G, which will trisect the right angle.

The Measurement of Angles. (Fig. 40.)

Angles are estimated according to the position which the two lines of which they are formed occupy as radii of a circle.
The circle being divided into 360 equal parts, called "degrees," it will be evident that the lines A, O, C, contain 90 degrees (written 90°) or a right angle.
Similarly B O C is a right angle.
Now, if these right angles be trisected (as per last problem), each of the divisions will contain 30°, thus:

A O E is an angle of 30°
A O F " 60°
A O C " 90°
A O G " 120°
A O H " 150°
A O B is in reality not any angle at all, being a perfectly straight line; but the slightest divergence from it would cause it to become an angle; as 179°, &c.

Each of these angles being again divided into three parts will give tens, which may again be divided into units; and thus angles may be constructed or measured with the greatest accuracy.

**Example No. 1 of the foregoing.** (Fig. 41.)

To find the angle contained by the lines A B C.

Erect a perpendicular at B.

Draw the quadrant D E, and trisect it.

Divide the arc G E into three equal parts by points H and I. (70° and 80°.)

Bisect the arc H I, and it will be seen that the line B C falls precisely on the bisecting point.

A B C is therefore an angle of 75°.

Had the line B C not fallen exactly in the bisecting point, further subdivision would have been necessary.
Example No. 3. (Fig. 42.)

To construct at a given point B an angle of a required number of degrees, say 100°.
At B erect a perpendicular, B C.
Trisect the right angle, carrying on the arc beyond the perpendicular, C.
Divide any one of the three divisions into three equal parts representing tens.
Set off one of these tens beyond C, viz., to D.
Draw B D.
Then A B D will be an angle of 100°.

To construct a Triangle, when the length of the base and the angles at the base are given.
(Fig. 43.)

Let it be required that the base should be 2' 5 (2 deci-
mal 5, or 2 and 5 tenths, which is 2 1/2 inches long, that the angle at A should be 50°, and that at B 45°.

Draw the base 2 1/2 inches long.

At A erect a perpendicular; draw a quadrant and trisect it in E D.

Divide the middle portion, D E, into three equal parts, and the second division from E will be 50°.

Draw a line from A through point 50 and produce it.

At B erect a perpendicular, and bisect the right angle thus formed (as 45° is one-half of 90°).

Produce the bisecting line until it meets the line of the opposite angle in F.

Then A B F will be the required triangle.

Note.—All the three angles of a triangle are always equal to two right angles, that is 180°, and therefore, as one of the above angles is 50°, and the other 45°—total 95°—the vertical angle, that is, that opposite the base, will be 85°.

The Protractor. (Fig. 44.)

For measuring and constructing angles, there is, in most cases of mathematical instruments, a brass semicircle called a Protractor. This has a short line marked at C, and two rows of figures round the rim—the one reading from right to left, and the other the reverse way.

In order to measure an angle by means of the protractor, place the edge A B on the straight line which is to form one of the sides of the angle, with the point C exactly against the point of the angle to be measured. Then the line C D will be seen to correspond with the point 60°, and B C D is therefore an angle of 60°; or, reading from the left side, A C D is an angle of 120°.

In constructing an angle, place C against the point at which it is desired to construct an angle; mark a point on your paper exactly against the figure corresponding to the number of degrees required; remove the protractor, and draw a line through the point thus obtained, to C, which will give the desired angle.

Protractors are sometimes made of wood or ivory, and of a rectangular form, as E F. These are used in a manner precisely similar to the semicircular instruments, but are not generally thought as useful or exact in practice.
To construct an Isosceles Triangle on a given base, and having a given vertical angle (say 30°). (Fig. 45.)

Before commencing to work this figure, it is desirable that attention should be called to the principle upon which the construction is based.

It has been shown (page 30) that all the angles of a triangle, of whatever shape it may be, will always be equal to two right angles (viz., 180°).

Every straight line then is equal to the bases of two right angles; for a perpendicular drawn at any point will at once form two right angles, equal to 180°, upon it.
Now let it be supposed that £180 are to be divided between three persons—that one of them is to receive £30, and the remainder to be equally divided by the other two.

It will be seen at once that, when the first condition has been fulfilled, and £30 deducted from £180, the remainder will be £150, or £75 for each of the remaining claimants.

It is on a similar principle that this operation is based; and this mode of procedure is rendered necessary because we cannot commence by constructing the vertical angle—for, as the base A B is fixed, we should not know where to commence the vertical angle, so that the sides

might not cut through A B (Fig. 1), or pass beyond it (Fig. 2), and thus we are compelled to construct the angles at the base firstly, and of such a number of degrees, that they should meet in the required angle.

Now it has been shown that 180° stand on every line.

Produce A B (Fig. 45), and at A construct an angle of 30°—viz., C A D.

So that out of the whole sum of 180° we have set aside 30°, the fixed number.

Bisect the remaining angle D A B in E.

Draw A E.

At B construct an angle A B F, similar to the angle B A E.

Produce lines A E and B F, which will meet in G, and will form the required angle of 30°.
To construct an Isosceles Triangle when the vertical angle is given in lines and not in degrees. (Fig. 46.)

Let A B be the given base, and C the given vertical angle.
Produce A B towards D.
At A construct the angle D A E, similar to the angle C (the given vertical angle).
Bisect the angle E A B in F.
At B construct an angle similar to the angle F A B—viz., angle G B A.
Produce A F and B G until they meet in H.
Then the vertical angle at H will be similar to the given angle C.

Within the given Square, A B C D, to inscribe the largest Equilateral Triangle it will contain. (Fig. 47.)
Trisect the right angle D A B.
Bisect the angles E A F and G A H by the lines A I and A J.
Join I J. Then—
A I J is the largest equilateral triangle that can be contained in the square A B C D.
The principle on which this construction is based, is, that as the angle of the square is 90°, and that of the
The equilateral triangle is 60°; there is an overplus of 30°. If, then, the two outer angles (E A F and G A H) which are each 30°, are bisected, and half of each added to the angle F A G (30°) an angle of 60° is obtained centrally placed, leaving 15° on each side. It will be seen that the sides of the equilateral triangle are larger than those of the containing square.

To construct an Equilateral Triangle of the given altitude (or height) A B. (Fig. 48.)

At A and B draw lines C D and E F at right angles to A B.
LINEAR DRAWING.

From A, with any radius, describe the semicircle G H.
From G and H, with radius A G, cut the semicircle in I and J.
From A draw lines through I and J, cutting E F in K and L.
A K L will be the equilateral triangle of the required altitude.

To draw a Tangent* to a circle at a given point, C. (Fig. 49.)

![Diagram](image_url)

(1) Draw a radius from the centre O to the point C.
At C construct a right angle, O C D.
Then D C is the required tangent.

* A tangent is a straight line which touches a circle at one point, but does not cut off any portion of the circumference. A tangent is always at right angles to the radius drawn from the point at which it touches.
Or (2) let E be the given point.
Draw radius O E and produce until E F equals E O.
Bisect F O by the line G H, which will be the tangent required.

**To construct an Equilateral Triangle about a given circle.** (Fig. 50.)

![Diagram](image)

From any point in the circle, as A, with a radius equal to the radius of the circle, describe an arc cutting the periphery* in B and C.

From B and C, with radius B C, cut the periphery in D.
(It will be seen that if B C, B D, and D C are joined, an equilateral triangle will be inscribed in the circle).

From B and C, with radius B C, describe arcs cutting each other in E.

From B and D, with the same radius, describe arcs cutting each other in F.

From D and C, with same radius, describe arcs cutting each other in G.

* *Periphery.* The circumference, or boundary line, of a circle, ellipse, or any other regular curvilinear figure.
Join $F G$, $F E$, and $G E$, which will complete the triangle about the circle.

It will be seen that by this problem an equilateral triangle may be constructed about another, the whole consisting of four equilateral triangles.

**WITHIN a given circle, to inscribe a triangle similar to a given triangle, $A B C$. (Fig. 51.)**

---

**Fig. 51.**

Draw a tangent to the circle.

From the tangent point $D$, with any radius, describe a semicircle cutting the tangent in $E$ and $F$.

At $A$ and $B$, with radius $D E$, describe arcs cutting the sides of the triangle in $I J$ and $G H$.

From the point $F$, mark on the semicircle the length of the arc $G H$—viz., $F K$.

From $E$, mark on the semicircle the length of the arc $I J$—viz., $E L$. 
From D, draw a line through L cutting the circle in M. From D, draw a line through K cutting the circle in N. Draw M N, which will complete the triangle in the circle.

(The various steps have here been given in detail in order to adapt the process for self-instruction, or for junior pupils; otherwise it will be seen that the most brief mode of explaining the figure would have been: "Construct on one side of D an angle similar to A B C, and on the other an angle similar to B A C.)

**ABOUT a given Circle, to construct a Triangle similar to A B C, (Fig. 52.)**

Produce the base of the original triangle A B C towards F and L. D
LINEAR DRAWING.

Draw any radius in the circle, as O D.

At O draw a line which shall make with the line D O an angle similar to that which B C makes with F B—viz., the angle D O G.

In a similar manner construct at O the angle D O H, similar to the angle I A C.

There will thus be three radii in the circle—viz., O D, O G, and O H.

Draw tangents (viz., lines at right angles) to each of these radii, and these tangents meeting in J, K, and L, will form a triangle about the circle similar to A B C.

Definitions concerning Polygons.

All figures having more than four sides are called Polygons, and are distinguished by names denoting the number of their sides and angles—thus:

A Polygon of 5 sides is called a Pentagon.

" 6 " " Hexagon.

" 7 " " Heptagon.

" 8 " " Octagon.

" 9 " " Nonagon.

" 10 " " Decagon.

" 11 " " Undecagon.

" 12 " " Duodecagon.

When all the sides of a polygon are equal, and all its angles equal, it is called “regular.”

When they are not equal, the polygon is said to be irregular.

By drawing lines from the angles of a regular polygon to the centre, the figure may be divided into as many triangles as the polygon has sides. In the regular hexagon these triangles will be equilateral, but in all other regular polygons they will be isosceles.
To construct a regular Polygon—in this case a pentagon—on the given line A B. (Fig. 53.)

Polygons may be constructed by either general methods—that is, by rules, which by simple variation as to division of parts will apply equally to all polygons—or by special rules which apply to particular figures only.

The above mode of constructing a pentagon on a given line is a general one.*

- Produce A B on each side.
- From A, with radius A B, describe a semicircle cutting A B produced in C.
- Divide the semicircle into 5 equal parts.
- From A, draw A D to the second division.
- From B, with radius B A, describe a semicircle cutting A B produced in E.
- From E, mark on this semicircle the length of the arc C D—viz., to F.
- From D and F, with radius A B, describe arcs cutting each other in G.
- Draw D G and F G, which will complete the pentagon on A B.

*That is, any other polygon may be thus constructed. To construct a heptagon by this method, divide the semicircle into 7 equal parts; for an octagon, into 8, and so on; but it must be remembered that, whatever may be the number of parts, the line A D must always be drawn to the second division.
To construct a regular Pentagon on the given line A B, by a special method. (Fig. 54.)

From A and B, with radius A B, describe arcs cutting each other in C and D.

Draw a line through C and D perpendicular to A B.

From C, with radius C A (equal to A B), describe an arc cutting the perpendicular in E, and also cutting the two previously drawn circles in F and G.

Draw lines from F and G, passing through the point E, and cutting the two circles in H and I.

Draw A I and B H.

From H and I, with radius equal to the side of the pentagon (viz., A B, A I, or B H), describe arcs cutting each other in J.

Draw H J and I J, which will complete the pentagon.
To inscribe a regular Polygon—in this case a pentagon—in a given circle. (Fig. 55.)

Draw the diameter $AB$, and divide it into as many equal parts as the polygon is to have sides (in this case five).

From $A$ and $B$, with radius $A B$, describe arcs cutting each other in $C$.

From $C$, draw a line passing through the second division and cutting the circle in $D$.

Draw $DB$, which will be one side of the polygon.

Set off the length $DB$ around the circle—viz., $EFG$. Join these points, and thus complete the figure.

Any polygon may be thus formed, by dividing the diameter into the number of parts corresponding with the sides of the required polygon; but the line $CD$ must, in every case, be drawn through the second division.
To inscribe a regular Pentagon in a circle, by a special method. (Fig. 56.)

Draw the diameter A B.
At O erect a perpendicular, O C.
Bisect O A in the point D.
From D, with radius D C, describe an arc cutting A B in E.
From C, with radius C E, describe an arc cutting the circle in F.
Draw C F, which will be one side of the pentagon.
Set off the length C F around the circle—viz., G H I.
Draw lines F G, G H, H I, and I C, which will complete the figure.
Application of the foregoing principle in the construction of Gothic tracery. (Fig. 57.)

Draw a circle, divide it into five equal parts, and draw the radii, A B C D E.
Bisect one of the radii, and set off the half on each of them—viz., F G H I J.
Join these points, and a regular pentagon will be formed.
Bisect the sides of this pentagon, by the lines K L M N P.
Draw a small circle in the centre, and another, Q, concentric with it.
From Q to the sides of the pentagon draw lines parallel
to O K, O L, &c., at a small distance on each side of
them—viz., R S, T U, &c.
Produce the sides of the pentagon indefinitely from
F G H I J, and with radius H U, describe circles cutting
the produced sides of the pentagon in V W and the
corresponding points.
Draw V C, W C, and similar lines from the other
circles, and the remaining lines will be parallel to, and
concentric with, those already drawn.

About the given Pentagon A B C D E, to de-
scribe a pentagon whose sides shall be parallel to
it, and equal to the line F G. (Fig. 58.)

Find the centre of the pentagon by bisecting two adja-
cent angles.
Draw the five radii, and produce them indefinitely.
Produce one of the sides, as B C, until it is equal to
F G—viz., B H.
From H draw a line parallel to O B, cutting the radius O C in I.
From O, with radius O I, describe a circle cutting the produced radii in K L M N.
Draw K L, L M, M N, N I, I K, which will complete the pentagon of the required size, described about the given pentagon.

To construct a regular Hexagon on the given line A B. (Fig. 59.)

From A and B describe arcs cutting each other in O.
From O, with radius O A, or O B, describe a circle.
The radius with which a circle is struck will divide it into six equal parts, therefore set off the length O A,
which is equal to A B, around the circle—viz., A C E F D.
Join A C, C E, E F, F D, and D B, and a regular hexagon will be formed.

To inscribe a regular Hexagon in a Circle.
Find the centre of the circle (figure 33), set off the radius around it, and join the points.
**Example 1 of inscribing a Hexagon in a Circle.**

-To draw a simple Fly-wheel. (Fig. 60.)

![Diagram](image.png)

- Fig. 60.

Draw the circles $A$ and $B$, representing the outer and inner edge of the rim.

Divide the circle $B$ into six equal parts, and draw the radius $CDEFGH$.

Next draw the circles $I$ and $J$, representing the end of the shaft and the boss, or central part of the wheel; the small parallelogram at the side of the inner circle represents the “key,” by which the wheel is held on the shaft.

On the edge of the boss set off equal distances, $KL$.

Draw the circle $M$, and on it, on each side of the radii, set off distances rather less than $K$ and $L$—viz., $N$ and $O$.

Draw the sides of the arms, $KN$ and $LO, &c.$

With any convenient radius describe the small arcs connecting the arms with the rim at $N$ and $O$.

The length $PQ$ set off from $P$ and $Q$ on the radius,
will give the point R, which is the centre for striking the arc, caused by the elliptical arm meeting (called penetrating) the elliptical rim.

**Example 2 of the application of the Hexagon in Mechanical drawing.** (Fig. 61.)

![Diagram of a nut and bolt with hexagon projection](image)

In the above drawing of a nut and bolt, the plan, that is, the appearance it would have if your eye were directly over it, and you looked down upon it, is to be drawn firstly.

The two largest circles being described, the inner one is to be divided into six equal parts, and a hexagon inscribed in it.

Perpendiculars drawn from each of the angles of the hexagon will give the projection of the widths of the sides of the nut.
The rest of the figure may be copied without further instructions, and the whole subject of projection, of elevations, &c., from given plans, &c., will be fully treated of in the subsequent volume.

To construct a regular Heptagon on the given line A B. (Fig. 62.)

Erect a perpendicular at B, and draw the quadrant A C. Divide the quadrant into seven equal parts. Continue the arc A C beyond C, and set off on it from C, three of the divisions—viz., to D.
Draw B D.

Bisect A B and B D, and from the intersection O of the bisecting lines, with radius O A or O B, describe a circle.

From A and B set off the length A B around the circle —viz., E F G H.

Draw D E, E F, F G, G H, and H A, which will complete the heptagon.

To construct a pentagon on this principle, divide the quadrant into five parts, and set off one beyond C.

For a hexagon, divide the quadrant into six, and set off two beyond C.

For an octagon, divide into eight, and set off four beyond E, &c.

To inscribe a regular Heptagon in a given circle. (Fig. 63.)

From any point, as A, with the radius of the circle, describe arcs cutting the circumference in B and C.
Draw the line B C and the radius A O, which will bisect B C in D.

From A set off the length D B around the circle, join the points A E F G H I J, and the heptagon will be completed.

It will be seen that B C is one side of the equilateral triangle, which could be inscribed in the circle, and thus as D B is half of B C, half of the side of the inscribed equilateral triangle gives the side of a regular heptagon, which can be inscribed in the same circle.

**To construct a regular Octagon on the given line A B.** (Fig. 64.)

![Fig. 64.](image)

Produce A B on each side.

Erect perpendiculars at A and B.

From A and B, with radius A B, describe the quadrants C D and E F.
Bisect these quadrants, then A G and B H will be two more sides of the octagon.

At H and G draw perpendiculars, G I and H K, equal to A B.

Draw G H and I K.

Make the perpendiculars A and B equal to G H or I K—viz., A L and B M.

Draw I L, L M, and M K, which will complete the octagon.

To inscribe an Octagon in the square A B C D.
(Fig. 65.)

Draw diagonals, A D and C B, intersecting each other in O.

From A B C and D, with radius equal to A O, describe
quadrants cutting the sides of the square in E F G H I J K L.

Join these points, and an octagon will be inscribed in the square.

**To inscribe an Octagon in a given Circle.**

(Fig. 66.)

![Octagon Diagram](image)

Draw the diameter A B, and bisect it by C D.

Bisect the quadrants A C, C B, A D, and B D, in the points E F G H.

Draw lines connecting all the eight points, which will complete the required octagon.

As all other polygons may be constructed on the prin-
To inscribe an Equilateral Triangle in a regular Pentagon, A B C E D. (Fig. 67.)

From E, with any radius, describe a semicircle, F G.

From F and G, with the same radius, describe arcs cutting the semicircle in H and I.

(The radius with which a semicircle is struck, divides it into three equal parts).

From E draw a line through H and I, cutting the sides of the pentagon in J K.

Draw J K, which will complete the equilateral triangle in the pentagon.

E
To inscribe a Square in a Regular Pentagon.  
A B D E C.  (Fig. 68.)

Draw C D, and C F at right angles, and equal to it.
Draw F E, cutting the side of the pentagon in G.
Draw G H parallel to C F.
And H I parallel to C D.
Draw I J parallel to H G.
And G J parallel to H I.

Then G H I J will be the required square contained in the pentagon.
To inscribe an Equilateral Triangle in a regular Hexagon, A B C D E F, so that its sides shall be parallel to three sides of the hexagon. (Fig. 69.)

Bisect the alternate sides, as E D, F A, and C B of the hexagon in the points G H and I. Join these points, and the lines will form an equilateral triangle.

To inscribe in a Regular Hexagon the largest Equilateral Triangle it will contain. (Fig. 70.)

Draw lines joining the three alternate angles of the hexagon, as A B C, which will form the required triangle.
LINEAR DRAWING.

Within the Equilateral Triangle, A B C, to inscribe six equal Circles. (Fig. 71.)

![Diagram of equilateral triangle with inscribed circles]

Draw the lines B D, A F, and C E, bisecting the sides and angles of the triangle, and intersecting each other in O.

Bisect the angle O A E, and the point (G) where the bisecting line cuts C E, will be the centre of one of the three isosceles triangles, into which the equilateral triangle has been divided.

Through G draw H I, parallel to A B, and from H and I draw H J and I J, cutting B D and A F in K and L.

From G H I J K and L, with radius G E, draw the six circles.
To inscribe three equal Circles in a Circle.
(Fig. 72.)

At any point, as A, draw a tangent, and A G at right angles to it.

From A, with radius O A, cut the circle in B and C.

From B and C draw lines through O, cutting the circle in D and E, and the tangent in the point F (and in another not given here, not being required).

Bisect the angle at F, and produce the bisecting line until it cuts A G in H.

From O, with radius O H, cut the lines D C and E B in I and J.

From H I and J, with radius H A, draw the three required circles, each of which should touch the other two, and the outer circle.
To inscribe in an Equilateral Triangle, A B C, the three largest circles it will contain. (Fig. 73.)

Draw A G, B F, and C E, bisecting the angles and sides of the triangle, and intersecting in O.

Bisect the right angle A E O.

Produce the bisecting line until it cuts A G in H.

Draw H I parallel to A B, H J parallel to A C, and I J parallel to B C.

From H I and J, with radius H K, draw the three circles, each of which should touch the other two, and two sides of the triangle.
To inscribe four equal Circles in a Circle, each touching two others and the containing Circle. (Fig. 74)

Draw the diameters A B and C D at right angles to each other.

From A B C D, with radius of the circle, describe arcs cutting each other in E F G H.

Join these points, and a square will be described about the circle.

Draw the diagonals E H and G F

Bisect the angle C F O, and produce the bisecting line until it cuts C D in I.

From O, with radius O I, describe a circle cutting the lines A B and C D in J K and L.

From these centres, with radius I C, describe the four required circles.
To inscribe seven equal Circles in a Circle.
(Fig. 75)

Around the circumference of the circle set off the radius, thus dividing it into six equal parts, A B C D E F, and draw the radii.

Divide one of the radii, as O A, into three equal parts—viz., O G, G H, H A.

From O, with radius O G, describe the central circle.
From O, with radius O H, describe a circle which, cutting the radii, will give the points I J K L M.
From these points, with radius O G, describe the six circles, each of which will touch the central circle, two others, and the containing circle.

Similarly, a circle O G being given, to draw six equal circles to touch it and each other, divide the given circle
into six equal parts. Draw radii and produce them. From G set off GH, equal to GO. From O, with radius OH, describe a circle which, cutting the produced radii, will give the centres J K L M of the six circles.

**Within a Circle to inscribe any number of equal Circles, each touching two others and the containing Circle.** (Fig. 76.)

![Diagram](image.png)

Divide the circle into equal sectors, corresponding to the required number of circles—viz., A B C D, &c., and bisect the sectors by the lines E F G, &c.

Produce any two of the radii, as A and B, and draw the tangent HI parallel to AB.

Bisect one of the angles at the base of the isosceles
triangle thus formed, and produce the bisecting line until it cuts O F in J.
From O, with radius O J, describe a circle cutting each of the lines which bisect the sectors in K L M N, &c.
From these points, with radius J F, describe the required circles.
By drawing P Q parallel to A B, and bisecting the angle at the base of the triangle, the centre for another circle may be found; and by continuing the process as before, another series of circles may be drawn.

**Application of the division of a Circle in drawing a Rack and Trundle.** (Fig. 77.)

![Diagram of a rack and trundle wheel]

The circle A, on which the centres of the circles representing sections of the bars (or teeth of the trundle) are placed, is called the *pitch circle*; and the line on which are the points of contact between the teeth of the rack and those of the wheel, is called the *pitch line*.
The pitch circle must be divided into the given number
of teeth, and spaces B C D, and the same lengths E F G must be set off on the pitch line H I.

The rest of the construction will be readily understood on reference to the figure.

Numerous studies in this branch of the subject will be given in the special “Manual of Mechanical Drawing for Engineers.”

The above (Fig. 78) is an example of the division of circles in drawing the Plan and Elevation of a Column,
and is introduced here in order to impress on students the necessity of acquiring the utmost accuracy in division of spaces.

The circle forming the boundary of the plan is to be divided into a number of parts, corresponding to the required number of flutes—viz., 1, 2, 3, &c.; half the width of the filets is then to be set off on each side of these divisions, as $a$, $b$, &c., and semicircles drawn from the centres of the remaining spaces.

The elevation of the column is projected by drawing perpendiculars from the various points in the plan.

For full instruction in "projection" of elevations from plans, &c., see the second portion of this work, "Solid Geometry and Projection," and the various orders of columns will be found in the volume devoted to Architectural Drawing for Masons.

**To divide a Circle into any number of equal parts, having the same area.** (Fig. 79.)

![Diagram](attachment:image.png)

Divide the diameter $AB$ into the required number of equal parts, $AC$, $CD$, $DE$, $EF$, $FB$.
From points \( a \), midway between \( A \) and \( F \), describe semicircles, \( F \) and \( A \). From point \( E \), describe the semicircle \( C \) and \( B \). From \( D \), describe the semicircle \( F \). From \( b \) and \( d \), midway between \( C \) and \( E \), draw the semicircles \( E \) and \( D \). From \( C \) and \( F \), draw the semicircles \( D \) and \( E \), which will complete the figure.

**To divide a Circle into a given number of Concentric Circles, having the same area.**

(Fig. 8o.)

Draw a radius \( A \), and on it describe a semicircle. Divide the radius \( A \) into the number of equal parts corresponding with the number of circles required. From the points of division, \( 1 \), \( 2 \) and \( 3 \), raise perpendiculars cutting the semicircle in \( 1 \), \( 2 \) and \( 3 \).
Through these points (from B as a centre) draw circles; then the belts C D E and the circle F will have the same area.

**The Cone and its Sections.** (Fig. 81.)

![Diagram of a cone and its sections](image)

A Cone is a solid, the base of which is a circle, but which tapers to a point from the base upward.

If a cone be cut horizontally—that is, parallel to the base—all such sections will be circles; but if it be cut obliquely across, as at A B, the section or cutting is called an Ellipse—as A B and C D.

The line drawn from C to D is called the long diameter (or transverse axis); that across from E is called the short diameter (or conjugate axis).

Both ends of an ellipse are equal, and in this its form differs from that of the Oval (see Fig. 95) which is egg-like (from *ova*, an egg), that is, broader at one end than at the other.
A perfect Ellipse may be drawn by means of a piece of string and pins—a method which is of great service to masons, joiners, gardeners, &c. (Fig. 82)

Fig. 82.

Place the given diameters A B and C D at right angles to each other at their centres E.

From D, with radius E A, cut the long diameter in F F.

These two points, F F, are called the *foci* of the ellipse.

Place a pin in each of these, and another in D. Pass a string round the three pins, and tie it securely, thus forming a triangle of string, F F D.

Take out the pin at D, and substitute a pencil, which may be drawn along, moving within the loop, and the point will thus trace a perfect ellipse.

* *Foci*—plural of focus*
To draw an Ellipse—the diameters A B and C D being given. (Fig. 83.)

Place the diameters A B and C D at right angles to each other, intersecting in E.

Find the foci as in the last figure.

Between E and F, mark off any number of points, as 1 2 3 4 5. (It is advisable that these points should be nearer together as they approach F.)

From F F, with radius 1 B, describe the arcs G G G G.

From F F, with radius 1 A, describe arcs H H H H.

The arcs H H H H will intersect the arcs G G G G in I I I I, and these will be four points in the curve.

Proceed to strike arcs from F F, firstly with 2 B, and then with 2 A; and these intersecting will give four more points.

When arcs have been struck with the lengths from all the points to A and B, the curve of the ellipse must be traced by hand through the intersections.

As there are many of the higher curves in geometry, and numerous forms in mechanical and architectural drawing which are better drawn by hand than by instruments, the student is urged to practice Free-hand Drawing at the same period that he studies the scientific portions of mechanical art.
To draw the curve of an Ellipse, by another method. (Fig. 84.)

Place the two diameters, A B and C D, at right angles to each other, at their centres E.

From E, with radius E C, describe a circle; and also from E, with radius E A, describe another circle.

Divide either of the circles into any number of equal parts, as 1, 2, &c.

From the centre draw radii through these points, and cutting the other circle in 3 and 4. From 3 and 4 draw perpendiculars, and from 1 and 2 draw horizontals, cutting the perpendiculars in F and G. The curve must then be traced from C, through F and G to A; and this will give one quarter of the ellipse. Proceed in the same manner to obtain the points H I J K L M, through which the remaining portion of the curve is to be drawn.
To construct a Semi-Elliptical arch, of which A B is the span, and C D the height. (Fig. 85.)

Divide C A and C B into any number of equal parts.
Divide A E and B F into a corresponding number of equal parts.
Number the parts as in the figure.
Produce D C, and make C G equal to C D.
From D, draw lines to the points 1 2 3 4 5, in the lines E A and F B.
From G, draw lines through the points 1, 2, &c., in the line A B, and produce these lines until they cut those of corresponding numbers drawn from D to the points in the lines E A and F B.

Thus—G 1 will cut D 1 in a.
G 2 " D 2 in b.
G 3 " D 3 in c.
G 4 " D 4 in d.
G 5 " D 5 in e.

The curve is to be drawn through these intersections.
Strictly speaking, no portion of an ellipse is a part of
a circle, and the curve cannot therefore be drawn with compasses so as to be mathematically correct; but there are many ways in which figures nearly approximating to ellipses may be drawn by arcs of circles; and as these are very useful for general practical purposes, the following three methods are given.

**To construct an Elliptical figure by means of arcs of circles.** (Fig. 86.)

![Diagram](image)

Fig. 86.

Place the two given diameters, A B, C D, at right angles to each other, at their centres E.

From A, set off A F equal to C D.

Divide F B into three equal parts.

Set off two of these parts on each side of E—viz., G G.

From G G, with radius G G, describe arcs cutting each other in H H.

From H H, draw lines through G G, and produce them.

From H H, with radius H C or H D, describe
arcs cutting the lines H G produced, in the points I J K L.

From G G, with radius G A or G B, describe arcs meeting those drawn from H H in I J K L, which will complete the figure.

To describe an Elliptical figure, when one diameter, A B, is given. (Fig. 87.)

![Diagram of an Elliptical figure]

Divide A B into four equal parts.

From C and D, with radius C A or D B, describe circles touching each other in E.

From C and D, with radius C D, describe arcs cutting each other in F and G.

Draw lines G C, G D, F C, and F D, and produce them until they cut the circles in H I J and K.
From F and G, with radius F K or G I, draw arcs uniting H with I and J with K, which will complete the figure.

To construct an Elliptical figure by means of two squares, A B C D, B D E F. (Fig. 88.)

Draw diagonals in each of the squares, intersecting each other in G and H.

From B, with radius B C, describe the arc C E.

From D, with same radius, describe the arc A F.

From G, with radius G C, describe the arc C A.

From H, with the same radius, describe the arc E F, which will complete the figure.
An Ellipse being given, to find the axes* and foci. (Fig. 89.)

Draw two parallel chords, A B and C D.
Bisect each of these in E and F.
Draw E F, touching the ellipse in G and H. This line divides the ellipse obliquely into two equal parts.
Bisect G H in I, which will be the centre of the ellipse.
From I, with any radius, draw a circle cutting the ellipse in J K L M.
Join these four points, and a rectangle will be formed in the ellipse.
Lines N O and P Q, bisecting the sides of the rectangle, will be the diameters (or axes) of the ellipse.
To find the Foci.—With radius equal to half the long diameter—viz., P I, describe an arc from N cutting P Q in R and S, which points will be the foci.

* A axes—plural of axis.
To draw a line perpendicular to the curve of an Ellipse at a given point A. (Fig. 90.)

Find the axes and foci as in last figure. From the foci F1 F2, draw lines through A and produce them. Bisect the angle A B C, and the bisecting line A D will be perpendicular to the curve of the ellipse.

The divisions of the voussoirs* in an elliptical arch are found by repeating this process at each point of division.

To draw a Tangent to the curve of an Ellipse at a given point E. (Fig. 90.)
Draw F1 E and produce it.

* Vousoir. The stones of which an arch is formed, the middle one of which is the "key-stone."
Draw F2 E, meeting the former line in E.

Bisect the angle F2 E G by the line H I, which will be the required tangent.

If a Cone be cut so that the section A B C is parallel to the slant of the cone, E D, the section is called a Parabola, as in Fig. 91.

If the cone be cut so that the section F G H is upright,

that is, parallel to the axis of the cone, the section is called an Hyperbola, as in Fig. 92.

The mode of projecting these sections directly from given cones are shown in the volume on Solid Geometry and Projection.
To construct a Parabola, the base A B and abscissa C D being given. (Fig. 93.)

Draw E F through D parallel to A B, and A E and B F parallel to C D.

Divide C A and C B into any number of equal parts—viz., 1 2 3 4 5.

Divide A E and B F into the same number of equal parts.

From D, draw lines to the points in E A and F B.

From points 1 2 3 4 5 in A C and C B, draw perpendiculars, to meet the lines drawn from D to the points in E A and F B.

The curve is to be drawn through the points where the perpendicular 1 meets D 1, where perpendicular 2 meets D 2, &c. &c.
To draw an Hyperbola, having given the diameter A B, the abscissa and double ordinate C E. (Fig. 94.)

At D and E erect perpendiculars.

Through B draw a line parallel to D E, meeting the perpendiculars in F and G.

Divide D C and C E into any number of equal parts, 1, 2, 3.

Divide F D and E G into the same number of equal parts.

From B, draw lines to the points in F D and E G.

From A, draw lines to the points in D E.

Draw the curve through the points where the lines correspondingly numbered intersect each other.
To construct an Oval (or egg-shaped figure), the width A B being given. (Fig. 95.)

Fig. 95.

Bisect A B by the line C D, cutting A B in E, and from E, with radius E A, draw a circle cutting C D in F.
From A and B, draw lines through F, and produce them indefinitely.
From A and B, with radius A B, draw arcs cutting the last two lines in G and H.
From F, with radius F G, describe the arc G H, to meet the arcs A G and B H, which will complete the oval.

To construct a Spiral* of one revolution. (Fig. 96.)
Describe a circle, using the widest limit of the spiral as a radius, as A XII.

* The spiral is a curve, which makes one or more revolutions round a fixed point, but does not return to itself.
LINEAR DRAWING.

Divide the circle into any number of equal parts, as 1 to XII, and draw radii.
Divide one of these radii, as A XII, into a corresponding number of equal parts, as 1 to 12.

Fig. 96.

From the centre, with radius A 1, describe an arc cutting the radius 1 in B.
From the centre, continue to describe arcs from points 2, 3, &c., cutting the corresponding radii 11, 111, &c., in the points C D E F G H I J K L.
From XII trace a curve passing through all these points, which will be an Archimedes* spiral of one revolution.

* Archimedes, the most celebrated of ancient mathematicians, was born at Syracuse, B.C. 287. He cultivated particularly the branches of science relating to the areas of curves and sections of curved surfaces. He proved that the area of a circle is equal to half the rectangle contained by its circumference and radius, and showed how to approximate, as near as may be required, to the quadrature of a circle. The spiral was invented by Conon, but its properties having been demonstrated by Archimedes, it is in honour of him called by his name.
To describe a Spiral of any number of revolutions—in this case three. (Fig. 97.)

Divide the circle into any number of equal parts, as I to XII, and draw radii.

Divide one of the radii, as A XII, into a number of equal parts, corresponding with the required number of revolutions—viz., A'B', B' C', C XII.

Divide each of these into the same number of equal parts as there are radii—viz., 1 to 12.

It will be evident that the figure consists of three separate spirals—one from XII to C', another from C' to B', and another from B' to A'.

Commence, as in the former spiral of one revolution, drawing arcs from 1, 2, 3, &c., to the correspondingly numbered radii, thus obtaining the points marked with the largest capitals; and the first revolution having been
brought up to \( C' \), proceed in the same manner to draw
arcs from the points \( 1, 2, 3, \ldots \), contained between \( B \) and
\( C \), cutting the corresponding radii in the points marked
with the italic capitals, and draw the curve through these
points, thus reaching \( B' \).

Proceed in the same manner to draw arcs from the points
between \( B' \) and \( A' \), thus obtaining the points marked with
the smallest capitals, and the spiral may then be brought
up to the centre.

To describe a Spiral adapted for the volute of
an Ionic column, by means of quadrants. (Fig. 98)

![Diagram](image)

Divide the given height into eight equal parts.
From 3 and 4, draw lines at right angles to \( A \) \( B \).
LINEAR DRAWING.

Between these two lines describe a circle (the eye of the volute), the centre being at a distance from A B equal to four of the divisions.

Inscribe a square in this circle.

Bisect the sides of this square, join the bisecting points, and thus a smaller square will be inscribed in it.

Divide each of the semi-diagonals into three equal parts, join these points, and two more squares will be formed within the former one.

The quadrants are drawn in rotation from the angles of each square commencing at 1 with radius 1 C.

The next is drawn from 2 with radius 2 D.
The next 3 3 E.
The next 4 4 F.

The process is then continued from the inner squares.

The Involute. (Fig. 99.)

If a perfectly flexible line is supposed to be wound round any curve, so as to coincide with it, and, kept stretched as it is gradually unwound, the end of, or any point in the line will describe or trace another curve, called the *involute* of the curve — being in reality the opening out, or *unrolling*, of the periphery of the first curved surface.

Thus, if a circular piece of wood were fastened on a board, and a string equal to the circumference, fastened by one end to it and rolled round it, a pencil placed in a loop in the end of the string would, as the string is gradually unrolled, trace the involute.

The circle (or other original curve) is called the *evolute*.

To construct the Involute of the circle A. (Fig. 99.)

Divide the circle into any number of equal parts (1 to 12), and draw radii.

The relation of *Involutes* and *Evolutes* was first discovered by Huygens, a native of Holland (born 1629).
Draw lines (tangents) at right angles to these radii. On the tangent to radius No. 1, set off a space equal to one of the parts into which the circle is divided. And on each of the tangents set off the number of parts corresponding to the number of the radius.

Tangent No. 12 will then be the circumference of the circle unrolled, and the curve drawn through the extremities of the other tangents will be the Involute.

**The Cycloid.** (Fig. 106.)

If a mark were to be made with chalk on the iron tire of a wheel at the exact spot where it touches the ground,
the white mark, as the wheel rolls along a level road, would be observed to move in a peculiar form, which is called the "Cycloid" curve. Whilst the centre of the nave of the wheel (1), although moving onward, would travel in a horizontal line, that is, it would keep exactly the same distance from the ground, however far the wheel might roll.

When the wheel is at B, its centre is at I, and the point A is at A 1.

When the wheel has moved to C, the centre will be at II, and the point A will be at A 2.

When the wheel has moved on to D, the centre will be at III, and the point A will be at A 3.

When the wheel has moved on to E, the centre will be at IV, and the point A will be directly over it, viz., at A 4.

When the wheel has moved on to F, the centre will be at V, and the point A will be at A 5.

When the wheel has moved to G, the centre will be at VI, and the point A will be at A 6.

When the wheel has moved on to H, the centre will be at VII, and the point A will be at A 7.

It will thus be clearly seen that the wheel in moving from A 1 to A 7 has passed completely through one revolution, and therefore that the length of the line A 1, A 7 is equal to the circumference of the circle laid out on a straight line.

The straight line on which the wheel rolls is called the Director.

The wheel is called the Generating Circle, and the point A is called the Generator.
To describe the Cycloid. (Fig. 101.)

Draw the Director A B, the Generating Circle C, and a line through the centre, called the line of centres, D E, parallel to A B.

Draw the diameter, VI 6, and divide each half of the circle into any number of equal parts—viz., 1a, 2a, &c.
On each side of point VI, set off the lengths, \(V\alpha\), \(IV\alpha\), \(III\alpha\), \&c., and \(V\beta\), \(IV\beta\), \(III\beta\), \&c., equal in size and number to the divisions in the circle.

From \(O\) \(I\alpha\), \(II\alpha\), \(III\alpha\), \&c., erect perpendiculars cutting the line \(D\ E\) in \(I\), \(II\), \(III\), \&c.
From each of these points describe circles equal to the generating circles.

From \(V\alpha\), set off on the circle of which \(V\) is the centre the length of the line VI \(5\alpha\)—viz., \(5\beta\).

Mark off the same length on the corresponding circle, from \(V\beta\).

From \(IV\alpha\), set off on the circle drawn from centre \(IV\) the length of the line VI \(4\alpha\), and do the same on the corresponding circle from \(IV\beta\).

Proceed thus, setting off the lengths of the lines VI \(3\), \(2\), and \(1\), on the circles resting on the points numbered correspondingly (in Roman figures), and through the points marked on the various circles—viz. \(1\beta\), \(2\beta\), \(3\beta\), \&c., draw the curve.

When a circle, instead of rolling along a straight line, rolls around the edge of another circle, any point in it will describe the curve known as the Epicycloid (Fig. 102).

To describe the Epicycloid.

Draw the directing circle \(A\ B\ C\), and the generating circle \(D\).

From \(A\), with radius \(A\ D\), describe the circle of centres \(E\ F\).
The Epicycloid and Hypocycloid (Fig. 102)
Divide the generating circle into any number of equal parts, 1a, 2a, &c., and set off these lengths from VI on the directing circle C B—viz., the points marked I, II, III, &c., in the larger Roman figures.

From A draw lines through I, II, III, &c., cutting the circles of centres in i, ii, iii, &c. (smaller Roman figures.)

From each of these points, as centres, describe circles similar to the generating circle.

From points v, iv, iii, ii, i, set off on the circles resting on them the lengths VI 5a, 4a, &c.; and through the points thus obtained—viz., 1b, 2b, 3b, &c.—the Epicycloid is to be drawn.

If the generating circle rolls inside instead of outside the directing circle, the curve traced is the Hypocycloid (Fig. 102.)

It is constructed in precisely the same manner as the Epicycloid, excepting that the lengths, VI 5a, &c., are set off from V, IV, III, &c., inside instead of outside the directing circle; and the points 5c, 4c, 3c, &c., are thus obtained.

Not. If the diameter of the generating circle were equal to the radius of the directing circle—that is, if VI 6 extended to A—a point in the generating circle, instead of generating a curve, would trace a straight line.

The Involute, Cycloid, Epicycloid, and Hypocycloid curves are much used in drawing the exact curves forming the teeth of wheels working in racks or in gear with each other; and these will therefore be more fully worked out in the volume of this series devoted to Mechanical and Engineering Drawing.
To trace a Cycloid by mechanical means.
(Fig. 103)

Fasten a rail of wood, or any straight edge to the board.
Take a circular piece of wood, and cut a small notch at any point in the edge (A), and fix a small knob or button in the centre (B).
The point of a pencil held in the notch whilst rolling the disc along the straight edge by means of the knob, will describe the Cycloid.*

In order to prevent the disc slipping as it rolls along, it is advisable to glue a narrow strip of sand-paper round the edge of the disc and on that of the rail.

To find a mean proportional between two given lines at A and B. (Fig. 104)

Join the lines A and B, and at their junction C, erect a perpendicular of indefinite height.
Bisect the entire line which has been formed, by uniting the two lines A and B, and from the bisecting point D, with radius D A, describe a semicircle, which will cut the perpendicular C in E.

* The Cycloid was invented by Galileo, an eminent mathematician and natural philosopher. He was born in Pisa in 1564, and died in 1642.
Then C E will be a mean proportional between A and B, the application of which will be found in the following figures:

To construct a Square, equal in area to the rectangle A B C D. (Fig. 105)

Find a mean proportional between the long and the
short side of the rectangle, and this mean proportional will be the required length of the side of the square.

The process shown in the last figure is here repeated.

Produce C D, until D E equals D B.
From D, with radius D B, describe a quadrant cutting C D produced, in E.
Erect a perpendicular at D.
Bisect C E in F.
From F, with radius F C, describe a semicircle cutting the perpendicular D in G.
D G will be the length of the side of the required square.
From D set off D H.
From H and G with radius D H, describe arcs cutting each other in I.
Draw H I and G I, which will complete the square.

**To construct a Rectangle, equal in area to the Triangle A B C.** (Fig. 106.)

From C, drop a perpendicular, C D.
Erect perpendiculars at A and B.
Bisect C D, and the bisecting line, cutting the perpendiculars in E and F, will complete the rectangle equal in area to the triangle.
A square, A G H I, equal in area to the rectangle, constructed as per last figure, will, of course, be equal to the triangle.

To illustrate this figure, make a triangle, A B C, of cardboard.

Cut through the line J K, and also through C L.

Rotate the triangle K L C on the point K, until K C touches K B. Then L will correspond with F.

Rotate the triangle J L C on J, until J C touches J A. Then L will correspond with E, and the parallelogram will be shown to be composed of exactly the same amount of material, or to contain precisely the same space as the triangle.

To construct a Triangle, a Rectangle, and a Square equal respectively to a Circle. (Fig. 107.)

Draw a radius, C A, and a tangent to it, D E, equal to the circumference of the circle (viz., divide the circle into any number of equal parts, and set them off on each side of A on D E.)

Draw D C, C E, and the triangle D C E will be equal to the circle.

Erect perpendiculars at D and E.
Bisect \( C A \) by a line cutting the perpendiculars in \( F \) and \( G \).

The rectangle \( D E F G \) is equal to the triangle, and thus equal to the circle.

Construct the square \( D H I J \), as in preceding figure, and it will be equal to the rectangle, and thus equal to the triangle, and therefore equal to the circle.*

To construct an Equilateral Triangle, equal in area to a given Triangle, \( A B C \). (Fig. 108.)

On either of the sides of the given triangle, as \( A C \), construct an equilateral triangle, \( A C D \).

Produce the side \( D A \) indefinitely.

From \( B \), draw a line parallel to \( CA \), cutting \( DA \) produced in \( E \).

At \( A \), draw a line perpendicular to \( DE \).

Bisect \( DE \) in \( F \).

From \( F \), with radius \( FD \), describe a semicircle cutting the perpendicular drawn from \( A \) in \( G \).

\( AG \) is then a mean proportional between \( AD \) and \( AE \).

On \( AG \), construct an equilateral triangle, which will be equal in area to the triangle \( A B C \).

* Theoretically, no right line, or right-lined figure, can be exactly equal to the circumference or area of a circle; but the difference in practice is too small to be of any consequence.
By this figure an equilateral triangle, equal to the triangle D C E in the last figure, may be found, and such equilateral triangle will thus be equal in area to the Circle.

To construct a Triangle equal in area to any regular Polygon—in this case a pentagon. (Fig. 109.)

Fig. 109.

It has already been shown (page 40) that every polygon is equal to as many triangles as the figure has sides.

Now it will be clear, that if the length of the side of the pentagon be set off on a straight line, viz., A B C D E F, and triangles equal to C G D be constructed on them, such five triangles, will together be equal to the pentagon, for if the pentagon had been cut into five triangles, these could be placed on a straight line, and could again be put together so as to re-form the pentagon.

But it has been seen, that triangles need not necessarily be of the same shape to be of equal area, and by Euclid* (Book I., prop. xxxvii.), "Triangles upon the same base, and between the same parallels, are equal."

If, therefore, lines be drawn from A B C D E and F to G, five triangles will be formed, having the same base, and between the same parallels A F and H I. They will therefore be equal to each other, and thus, when added together, will be equal to the same area as the five original triangles, and, consequently, the Triangle A G F will be equal to the pentagon.

* Euclid, one of the most eminent geometricians of antiquity, was born about 320 B.C. His birthplace is by some said to have been Alexandria. He studied geometry in Athens, then went from Greece to Alexandria, where he settled during the time of the first Ptolemy.
Further, an equilateral triangle, constructed as in Fig. 108, equal to A G F, will also be equal to the pentagon.

To construct a Triangle that shall be equal in area to a given Rectilineal figure, as A B C D E. (Fig. 110.)

Draw B D, thus cutting off the angle C.
Draw C F parallel to D B.
Draw D F.
The original irregular pentagon has now been reduced to the four-sided figure A F D E.
Draw D A and E G parallel to it.
Join D G, and the triangle G D F will be equal in area to A B C D E.

By problem 106, the triangle may be converted into a rectangle, and into a square; and by problem 108, into an equilateral triangle, each of which will be equal to the original figure.
To construct a Triangle equal in area to a given Rectilineal figure (continued). (Fig. 111.)

Let \(\text{A B C D E F}\) be the given figure, and let it be required that one angle of the triangle shall coincide with \(\text{A}\).

From \(\text{A}\), draw lines to the other angles of the polygon.
Produce \(\text{D E}\).
Draw \(\text{F G}\) parallel to \(\text{A E}\).
Produce \(\text{C D}\).
From \(\text{G}\), draw a line parallel to \(\text{A D}\), cutting \(\text{C D}\) produced in \(\text{H}\).
Produce \(\text{B C}\).
From \(\text{H}\), draw a line parallel to \(\text{A C}\), cutting \(\text{B C}\) produced in \(\text{I}\).

Draw \(\text{A I}\), then the triangle \(\text{B A I}\) is equal in area to the original polygon, and the side \(\text{A B}\), and the angle \(\text{B}\), are common to both.

As by this figure, and those referred to in the last, irregular forms may be rectified and converted into equilateral triangles, rectangles, and squares, they are of great importance to land-surveyors, architects, &c.
To construct a Square equal in area to a given quadrilateral figure, A B C D. (Fig. 112.)

Draw the diagonal A C, and bisect it by a perpendicular.

From B and D, draw lines parallel to A C, and cutting the perpendicular in E and F.

Draw a line bisecting E F in O, and from A draw a line parallel to E F, and cutting this bisecting line in G.

Find a mean proportional between O E and O G—viz., O H.

Set off the length O H (the semi-diagonal) from O on E F and O G—viz., I J K.

Join H I J K, and the square will be equal to the quadrilateral figure A B C D.
To construct a Square which shall be equal in area to two other squares added together.
(Fig. 113.)

Place the two squares so that a side of the one, as A B, shall be at right angles to one side of the other, as B E.

Draw the line A E.

Now, according to Euclid (Book I., prop. xlvii.), "In any right-angled triangle, the square which is described upon the side subtending the right angle, is equal to the squares described upon the sides which contain the right angle." And it will be seen that A B E is a right-angled triangle, and that the squares A B C D and B E F G

This proposition is said to have been discovered by Pythagoras, a disciple of Thales, who, after travelling in India and Egypt in pursuit of knowledge, settled in Tarentum, in Italy, where he founded the celebrated Pythagorean school, 550 years B.C.
are described upon the sides of it which contain the right angle; and therefore the square A E H I, which is described on (the hypothenuse) A E, which subtends the right angle, is equal to the sum of the two other squares.

To construct a Square equal in area to any number of squares added together. (Fig. 114.)

This is done by merely carrying on the process shown in the last figure.
Let it be required to construct a square, which shall be equal to the areas of the three squares of which A, B and C are the respective sides.
Place B at right angles to A, then the hypothenuse D would be the side of the square equal in area to the squares constructed on A and B.
Place C at right angles to D, draw E F, and construct a square upon it; then E F H G is equal to the squares constructed on C and D, and therefore equal to the squares constructed on all three lines.
Any number of squares may be thus added together.
To divide a given Triangle, A B C, into two equal parts by a line parallel to one of its sides.
(Fig. 115.)

Bisect one of the sides, as C B, in the point D, and erect the perpendicular D E equal to D C.
From C, with radius C E, describe an arc cutting C D in F.
From F, draw F G parallel to A B, which will divide the triangle into two parts of equal area.

To divide a Triangle into two equal parts by a line perpendicular to one side.
From C, draw C H perpendicular to A B.
Bisect A B in I.
Find a mean proportional between B H and B I—viz., B J.
From B, set off B K equal to B J, and the perpendicular K L will divide the triangle as required.
To divide the space contained between the lines A B and C D, into equal parts, by means of lines parallel to A B. (Fig. 116.)

Draw the line E F perpendicular to A B, and set off on it equal lengths corresponding to the number of spaces into which A B C D is to be divided—viz., 1 to 8. These spaces may be any size, but must be equal.

From E, with radius E 8, describe an arc cutting C D in G.

Draw E G.

From E, with radius E 8, E 7, E 6, &c., describe arcs cutting E G in H I J K L M N.

Draw lines parallel to A B through these points, and the space will be divided as required.
To draw a Circle of a given radius, which shall touch another given circle and a straight line. (Fig. 117.)

Let A be the given Circle, B C the straight line, and D E the radius of the required circle.

The question here is, to find a point which shall be the centre of a circle of a given radius, which shall touch the given circle and straight line.

From O, the centre of the given circle, draw a radius and produce it.

From the periphery of the circle, and on this radius, set off F G, equal to D E.

From O, with radius O G, describe an arc.

At any point, as H, in B C, draw a perpendicular, H I, equal to D E.

From I draw a line parallel to B C, cutting the arc drawn from O in J.
LINEAR DRAWING.

From J, with the required radius, describe a circle, which (if the work has been accurately done) will touch the given circle and straight line.

To draw a Circle of a given radius D E, which shall touch both lines of an angle, A B C. (Fig. 113.)

![Fig. 113.]

Bisect the angle by the line B F.
On either of the lines of the angle erect a perpendicular equal to the given radius D E—viz., d e.
From e, draw a line parallel to B C, cutting the bisecting line in O.
From O, with the given radius, draw the circle, which will touch both the lines of the angle.
To draw a Circle which shall touch both lines of an angle, and shall pass through a given point P. (Fig. 119.)

Let A B C be the given angle, and P the given point, through which the required circle is to pass.

Bisect the angle A B C by the line B D.

From any point in B D, as E, draw a circle, touching both lines forming the angle.

From B, draw a line through P, cutting this circle in F. Join F to E, the centre of the circle.

From P, draw a line parallel to F E, cutting B D in G.

From G, draw a line perpendicular to A B (by Fig. 6) —viz., G H.

Then, with radius G H, which will be found to be equal to G P, describe a circle which will touch both lines forming the angle.
To draw a series of Circles to touch each other, and two lines not parallel. (Fig. 120.)

Produce A B and C D until they meet in E.

Bisect the angle A E C by E D'.
Draw the first circle at pleasure, and from its centre, X, draw a radius, X F, at right angles to E A.

From G, draw G H perpendicular to E D.
From H, with radius H G, describe an arc cutting E A in I.
Draw a line at I, perpendicular to A E cutting E D in J.
From J, with radius J I, describe the next circle, cutting E D in K.
From K, draw a line perpendicular to E D, cutting E A in L.
From L, with radius L K, describe an arc cutting E A in M.
From M, draw a line perpendicular to E A, cutting E D in N.
From N, with radius N M, describe a circle touching K, and cutting E D in O.
From O, draw O P at right angles to E A.
From P, with radius P O, describe an arc cutting E A in Q.
From Q draw a line perpendicular to E A, cutting E D in R.
From R, with radius R Q, describe the next circle.
Any number of circles may be thus described.
To draw a Circle of a given radius, to touch two given circles, 1 and 2. (Fig. 121.)

Draw any radius in each circle Ct D, and C2 D, and produce them.

On these radii, beyond the circles, add to each the radius of the required circle—viz., D E 1 and D E 2.

From Ct, with radius Ct E 1, and from C2 with radius C2 E 2, describe arcs cutting each other in F.

From F, with radius E D, describe the required circle, which will touch both circles.

If the required circle is to include both circles, draw any radius in each, as Ct L, C2 M.

Produce both these radii.

On the radius Ct, set off from L the radius of the required circle—viz., to X. Diminish this by the radius of circle 2—viz., to G.

On radius M C2 set off L G—viz., M H.
LINEAR DRAWING.

From centres $C_1$ and $C_2$, with radius $C_1 G$, and $C_2 H$, describe arcs cutting each other in $J$.

From $J$, draw a line through $C_1$ to $K$.

With radius $J K$, describe the enclosing circle, which will touch circles 1 and 2.

The Conchoid. (Fig. 122.)

The Conchoid is a curve which always approaches a straight line, but never reaches it, however far the curve and straight line may be produced.

The straight line $A B$ is called the asymptote, $C D$ the diameter, and $P$ the pole.

The asymptote $A B$, pole $P$, and diameter $C$ being given, draw $C P$ at right angles to $A B$.

On each side of $D$, set off any number of equal parts, $1 2 3 4 5 6 7$.

From $P$, draw lines passing through these points.

From $1 2 3, \&c.,$ with radius $D C$, describe arcs cutting these lines in $a b c d, \&c.,$ and through these intersections trace the curve. The curve above the asymptote is called the superior Conchoid. By setting off the same lengths under the line the inferior Conchoid is obtained.

The Conchoid has been used in architecture in drawing the slightly curved line which forms the profile or side of columns, called the Entasis.

The Conchoid was invented by Nicomedes about A.D. 450.
To draw the Cissoid.* (Fig. 123.)

Draw any line A B and C D perpendicular to it.

On C D, describe a circle.

From the extremity D of the diameter, draw any number of lines at any distance apart, passing through the circle, and meeting the line A B in a b c d e f g h and i.

Take the length from i to g, and set it off on the same line on each side from D—viz., to D' D'.

Set off the length h 8 from D—viz., points E E.

Set off the length g 7 from D—viz., points F F.

Proceed thus with all the lines, and trace the double curve through D' D', E E, F F, G G, H H, I I, J J, K K, &c.

* Sometimes called the Cissoid of Diocles, from the name of its discoverer, who flourished about A.D. 190.
IRON WORK

IRON RAILING FOR A BALCONY

1. Draw the framing as shown and shaded in the squares.
DIAPER, OR UNIFORM SURFACE DECORATION.

To begin.—1. Construct the central pentagon A. 2. Construct a pentagon on each of the sides. The leaves of the flower are formed of arcs drawn from the angles of the pentagon.
DIAPER, OR UNIFORM SURFACE DECORATION.

1. To begin.—Construct the central hexagon (shaded in line) divide into triangles. 2. On each side of hexagon construct similar triangles.
will give the point R, which is the centre for striking the arc, caused by the elliptical arm meeting (called penetrating) the elliptical rim.

**Example 2 of the application of the Hexagon in Mechanical drawing.** (Fig. 61.)

In the above drawing of a nut and bolt, the plan, that is, the appearance it would have if your eye were directly over it, and you looked down upon it, is to be drawn firstly. The two largest circles being described, the inner one is to be divided into six equal parts, and a hexagon inscribed in it. Perpendiculars drawn from each of the angles of the hexagon will give the projection of the widths of the sides of the nut.
The rest of the figure may be copied without further
instructions, and the whole subject of evacuation of eleva-

**Acme Use**

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