DYNAMICS OF MAGNETIC MONOPOLES IN ARTIFICIAL SPIN ICE

by

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Abstract

While finding the elementary magnetic monopole seems not an easy task, recently, scientists have come out an alternative approach by studying emergent particles in spin systems. *Spin ice* is a magnet with frustrated interactions from which we observe emergent magnetic charges. Two typical spin ice materials are Dy$_2$Ti$_2$O$_7$, with tetrahedral lattice structure. Artificial spin ice is an array of magnetic nano-wires with similar frustrated interactions as spin ice.

In this Senior Thesis Project, I studied both meso-scopic behavior and microscopic behavior of the dynamics of artificial spin ice in a honeycomb network of magnetic nanowires made with permalloy. The concept of magnetic charges is introduced for better visualization and interpretation of the magnetization behavior.

Microscopic simulations on a single wire and on a honeycomb junction with high damping are presented in this thesis. The dynamics of magnetic charges is observed in our simulation. By finding the critical field that triggers the reversal process on a junction with respect to the angle of external field, an offset angle $\alpha$ is defined in the system to better estimate the critical field at different angle.
This thesis also includes a detailed discussion on the avalanche length distribution when an external field is applied to a uniformly magnetized honeycomb lattice sheet. We found when external angle $\theta \in (90, 131)$, the avalanche length distribution decays exponentially; and when $\theta \in (132, 180)$, the avalanche length distribution decays as a power law.

This work concludes with the introduction of "inertia" and its characteristic parameter $\varepsilon$ that helped us deal with the case when magnetic charges travels in low damping system.
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Finally, special thanks to my parents, for providing me the chance to explore this world.
Dedication

This thesis is dedicated to those who have a dream in their mind and would try bravely to make it happen no matter how hard the environment is.
## Contents

Abstract ii

Acknowledgments iv

List of Tables x

List of Figures xi

1 Introduction 1

1.1 Emergent Magnetic Monopole . . . . . . . . . . . . . . . . . . . . . . 2

1.2 Spin Ice . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3

2 Background 8

2.1 Artificial Spin Ice . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8

2.2 Experiment Set Up . . . . . . . . . . . . . . . . . . . . . . . . . . . . 9

   2.2.1 Avalanche . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 9

   2.2.2 Experiment description . . . . . . . . . . . . . . . . . . . . . . . 10
## CONTENTS

### 3 Model Description

3.1 magnetic charges .................................................. 13
3.2 Critical Field $H_c$ .................................................. 15
3.3 Things to do .......................................................... 16

### 4 Microscopic Simulation

4.1 Spin reversal on a single link ..................................... 18
4.2 Spin reversal on Kagome Junction and Offset Angle .......... 22
  4.2.1 Basic Construction and Definition ........................... 22
  4.2.2 Simulation Result .............................................. 23
  4.2.3 Offset Angle and experiment verification .................... 24
  4.2.4 Further Discussion ............................................ 26

### 5 Avalanche length distributions

5.1 Case Description .................................................... 33
5.2 120° regime: 90° $\sim$ 131° (Exponential Decay Regime) ........ 35
5.3 150° regime: 131° $\sim$ 178° (Power law decay) ............... 39
  5.3.1 Modeling of the reversal process .............................. 41
  5.3.2 Theoretical derivation of length distribution ............... 41
  5.3.3 Transition angle between 150° regime and 180° regime: .... 46
5.4 180° Regime: 178.4° $\sim$ 180° (power law decay) ........... 47
  5.4.1 Theory on 180° avalanches .................................... 48
List of Tables

4.1 Kagome Junction reversal at different angle of external field . . . . . 29
List of Figures

1.1 The magnetic moments in spin ice reside on the sites of the pyrochlore lattice, which consists of cornersharing tetrahedra. The Ising axes are the local (111) directions, which point along the respective diamond lattice bonds, figures from [1] ........................................ 4
1.2 Emergence of magnetic monopole in artificial magnetic bar system . . 6
1.3 Emergent of Magnetic Monopole in Spin Ice System .................... 7

2.1 Avalanche of a honeycomb artificial spin ice system under external field. The external field is not strong enough to trigger an avalanche in (a), several avalanches has already been triggered in (b), the color code represent the direction of magnetization on each wire. .................. 10
2.2 Experimental Model of 2 dimensional honeycomb ......................... 11
2.3 Experimental model of 2 dimensional honeycomb system ............... 12

3.1 Magnetization of links and magnetic charges ............................ 14

4.1 Initial magnetization ................................................... 18
4.2 Simulation of magnetization reversal on a Single link ................... 19
4.3 Dynamic of magnetic charges on a single link .......................... 21
4.4 Another possible dynamic of magnetic charges on a single link ...... 22
4.5 The ground state magnetization of Kagome Junction (without external field) .................................................. 23
4.6 Simulation by OOMMF of reversal process of a Y junction ............ 28
4.7 Simulation result for H(θ) and the fitting ................................ 30
4.8 Asymmetric behavior in a close view .................................. 30
4.9 Experimental Verification of Offset Angle ............................. 31
4.10 The disc diagram of reversal character verses external field ......... 32

5.1 Initially the honeycomb lattice system is magnetized to one direction, which in this case is right direction, by a very strong external magnetic field ............................................................... 34
LIST OF FIGURES

5.2 Detailed Magnetization for 120° regime reversal .......................... 35
5.3 Detailed Magnetization for 120° regime reversal .......................... 36
5.4 x-axis corresponds to the avalanche length, and y-axis corresponds to
the number of times such avalanche is observed. Red line is the sim-
ulation result with step size 0.01, blue line is the result with step size
0.025 mT. ................................. 38
5.5 The first few avalanches for 170° external field reversal .................. 40
5.6 Comparison between the simulated result (blue line) from a program
provided by Olga Petrova of the Johns Hopkins University and the
predicted result using my model (red line) for a system of size 23 × 37,
for orientation angle equals 150° ................................................ 42
5.7 The loglog plot of the distribution of avalanche length for θ = 170°
(simulation result), neglect the noise in the end of the line, the slope
(power) for avalanche length is around −1, agreed with my predic-
tion(red line) ................................. 44
5.8 Transition angle between 150° and 180° regime ......................... 46
5.9 The loglog plot of the distribution of avalanche length for θ = 180°
(simulation result), neglect the noise in the end of the line, the slope
(power) for avalanche length is around −1.5, agreed with my predic-
tion(red line). The data points for those avalanches longer than 11 are
above system size (W=45,L=11) hence can be ignored ..................... 48
5.10 Fit of the theory on 180° avalanches. The red points is the experi-
mental result of survival probability $C_k$ divided by $k$, the blue points is the
theoretical $C_k$ divided by $k$, the blue line is the bench mark using $k^{-1.5}$,
the red lines is the simulated count of avalanche of a system with size
50×50 and 1000 repetition ................................. 51
5.11 Theoretical Model Check .................................................. 52
5.12 Asymptotic .................................................. 53

6.1 Construction of the sample .................................................. 56
6.2 Finding the ground state .................................................. 57
6.3 Vector field of the initial state’s magnetization I obtained .......... 59
6.4 Inertia helps a magnetic charge to get through Y junctions in honey-
comb lattice .................................................. 61

A.1 Bernal-Fowler periodic model of ice Ih. Covalent and hydrogen bonds
are shown as sticks as an aid to visualization of the bonding network. 66
A.2 Analogues between water ice and spin ice lattice structure .......... 66

B.1 An example of geometric frustration. Three Ising spins coupled anti-
ferromagnetically in a triangle ................................................ 68
B.2 Two possible ground state magnetic configurations of the dipolar kagome
ice that form honeycomb lattice ................................................ 69
Chapter 1

Introduction

Electrically charged particles such as electrons are common in our world. In contrast, no elementary particles with a net magnetic charge have ever been observed, despite intensive and prolonged searches. All magnets, no matter how small they are, no matter how many times you cut them, they always have two poles, the north pole and the south pole.

The magnetic monopole was first hypothesized by Pierre Curie in 1894, but the quantum theory of magnetic charge started with a paper by the physicist Paul A.M. Dirac in 1931. [2] In this paper, Dirac showed that the existence of magnetic monopoles was consistent with Maxwell’s equations only if electric charges are quantized, which is always observed. Since then, several systematic monopole searches have been performed. Experiments in 1975 and 1982 [3,4] produced candidate events that were initially interpreted as monopoles, but are now regarded as inconclusive [5].
CHAPTER 1. INTRODUCTION

Stanford scientists nowadays are still detecting signals that shows evidence of the existence of elementary monopoles.

1.1 Emergent Magnetic Monopole

While finding the elementary magnetic monopole seems not an easy task, recently, scientists have come out an alternative approach by studying emergent particles in spin system.

To explain this idea, consider a long line of bar magnets Fig 1.2. Initially, when all magnets have the same orientation, every north pole cancels against the adjacent south pole so that the system is free of magnetic charges. However, if we reverse the magnetization of a bar magnets, two magnetic charges appear. The red site is “positively charged” since all magnetic field lines points out of the site, the blue site is “negatively charged”. We can continue to flip the bar besides the first bar. This caused the negative charge to be moved to the left. Similarly, we can keep on flipping the magnetic bars so that the two magnetic charge can be arbitrary positioned at any point. When they are really far from each other, their correlation are nearly negligible and each point can be considered as an effective magnetic monopole [1]. This model can be extended into two dimensions and three dimensions. And we can have more than two charges moving in the plan (or space) as well. Therefore, when the spin system is become larger and larger, these sites with non-zero divergence of
spins will become further away from each other, hence more independent from other charged sites. Ultimately, they can be considered as “magnetic monopoles” that move in the magnetic-bar system.

1.2 Spin Ice

The one dimensional model is a purely artificial system, it is easier to conduct experiments on these artificial systems to study the dynamical behavior of magnetic charges. What is more, similar system is also observed in natural materials, one of them is called the spin ice [6,7].

Spin ice is a magnet with frustrated interactions. Spin ice and water ice shares many remarkable properties, known as ice rule (Appendix A). Briefly, spin ice are magnetic analogs of excitations with fractional electric charge found in the water ice [8], it has several interesting properties:

- Because the ice rules are satisfied by a large portion of the states, the system has large amount of degeneracy of its ground state, hence retains much entropy down to very low temperature [9].

- Low-frequency dynamics in ice is associated with the motion of defects violating the ice rules. In water ice, these defects carry fractional electric charges of ±0.62e (ionic defects) and ±0.38e (Bjerrum defects) [8]. Fractionalization takes an even more surprising form in spin ice: while the original degrees of freedom
are magnetic dipoles, the defects are magnetic monopoles. The low-energy excitations of spin ice are point defects acting as sources and sinks of magnetic field $\mathbf{H}$ [8].

Two typical spin ice materials we consider here are $\text{Dy}_2\text{Ti}_2\text{O}_7$ and $\text{Ho}_2\text{Ti}_2\text{O}_7$, with their magnetic ions form tetrahedra. The magnetic moments in spin ice reside on the sites of the pyrochlore lattice, which consists of corner-sharing tetrahedra. Notice from Figure 1.1 that the center of each tetrahedra forms a diamond lattice, which was closely related to the later part of this paper. The Ising axes are the local [111] directions, which point along the respective diamond lattice bonds.

![Figure 1.1: The magnetic moments in spin ice reside on the sites of the pyrochlore lattice, which consists of cornersharing tetrahedra. The Ising axes are the local [111] directions, which point along the respective diamond lattice bonds, figure from [1]](image)

A simple depiction of how magnetic monopole can be extracted from spin ice system is shown in Figure 1.3. The dumbbell picture (c, d) is obtained by expanding the point-like magnetic moments of the rare-earth ions into long magnetic bars con-
CHAPTER 1. INTRODUCTION

necting the centers of adjacent tetrahedra, and replacing each magnetic bars in a and b by a pair of opposite magnetic charges placed on the adjacent sites of the diamond lattice. In the left panels (a, c), two neighboring tetrahedra obey the ice rule, with two spins pointing in and two out, giving zero net charge on each site. In the right panels (b, d), inverting the shared spin generates a pair of magnetic monopoles (diamond sites with net magnetic charge). This configuration has a higher net magnetic moment and it is favored by an applied magnetic field oriented upward (corresponding to a [111] direction). A pair of separated monopoles (large red and blue spheres). A chain of inverted dipoles (Dirac string) between them is highlighted in white, and the magnetic field lines are sketched.

The energy of a configuration of dipoles is well approximated as the pairwise interaction energy of magnetic charges, given by the magnetic Coulomb law:

\[
V(r_{\alpha\beta}) = \begin{cases} 
\frac{\mu_0 Q_\alpha Q_\beta}{4\pi r_{\alpha\beta}}, & \alpha \neq \beta \\
\frac{1}{2}u_0 Q_\alpha^2, & \alpha = \beta
\end{cases} \tag{1.1}
\]

Where \(Q_\alpha\) and \(Q_\beta\) are magnetic charges of monopoles, which will be discussed in more detail in chapter 3.
CHAPTER 1. INTRODUCTION

(a) Emergence of magnetic charges in 1 dimensional system

(b) Emergence of magnetic charges in 2 dimensional system

Figure 1.2: Emergence of Magnetic Monopole in artificial magnetic bar system, figures from [1]
CHAPTER 1. INTRODUCTION

(a) Imaginary magnetic monopole in spin ice system

(b) Spin ice system and emergent magnetic charges on it

Figure 1.3: Emergent of Magnetic Monopole in Dy$_2$Ti$_2$O$_7$ and Ho$_2$Ti$_2$O$_7$, figures from [1]
Chapter 2

Background

2.1 Artificial Spin Ice

Artificial spin ice [10] is an array of magnetic nano-wires with similar frustrated interactions as spin ice. It used to be considered as a bare replication of natural spin ice at large scale, however, compared with spin ice, artificial spin ice has several peculiar advantages to be considered as a model to study magnetic charges:

- Artificial spin ice is easier to control:

  Because the magnetic moments $\mathbf{m}$ in artificial spin ice are on the order of $10^8$ Bohr magnetons, and the energy scale of the dipolar interactions is $10^5$ K in temperature units [11], both are much larger than the scale of natural spin ice. Therefore, thermal fluctuations of the macro-spins is very small, meaning that the system is never in thermal equilibrium.
CHAPTER 2. BACKGROUND

- Artificial spin ice is “cleaner”:

  In natural spin ice, all of them have more or less impurities, which might affect the quality of our experiments. On the other hand, artificial spin ice generally can be made very pure.

- It is easier to manipulate and observe the movement of magnetic charges in artificial spin ice. Dynamics of magnetization can be induced by an external magnetic field, and observed by magnetic microscopy [10].

2.2 Experiment Set Up

2.2.1 Avalanche

Nice experiments have being carried out aiming at understanding behavior of artificial spin ice under an external magnetic field of varying angles and magnitude [12]. It turns out that the behavior of magnetic nano-wires under external field is analogous to the fluidized granular matter under external gravitation field. Recently, S. Ladak et al observed magnetic avalanches in the process of magnetization reversal [13]. Similar to the avalanches of granular matters.

In artificial spin ice, an avalanche is defined to be a simultaneous magnetization reversal of several consecutive nano-wires, triggered by the magnetization reversal of the first nano-wire. A more visualized example is shown in Fig 2.1.
CHAPTER 2. BACKGROUND

![Image](image.jpg)

Figure 2.1: Avalanche of a honeycomb artificial spin ice system under external field. The external field is not strong enough to trigger an avalanche in (a), several avalanches have already been triggered in (b), the color code represents the direction of magnetization on each wire.

2.2.2 Experiment description

Experiments of artificial spin ice have been carried out in various two dimensional sheet models. In this project we study the two dimensional honeycomb model. The honeycomb lattice is not corresponding to any real natural spin ice, as most natural spin ice lattice lies in three dimensional space. The reason why people are interested in this type of model is it is geometrically frustrated (Appendix B). What is more, if we look at a cross section of the three dimensional Dy₂Ti₂O₇ lattice system, neglect those nanowires pointing out of the sheet, we will obtain a two dimensional honeycomb system with each link representing a spin and each junction carries a net magnetic charge.

Our model is inspired by experiments carried out by Cumings et al. [14] Their artificial spin ice is a connected honeycomb network of permalloy nanowires with
CHAPTER 2. BACKGROUND

Figure 2.2: Experimental Model of 2 dimensional honeycomb

magnetization length $M = 8.6 \times 10^5$ A/m and the typical dimensions: length $l = 500$ nm, width $w = 110$ nm, and thickness $t = 23$ nm. Three nanowires come together at a node in the bulk. The experiment sample made by lithography is shown in Figure 2.2. Figure 2.3(a) is made by Tunneling Electron Microscopy (TEM) in Lorentz mode, the mode that measure the lorentz force generated by the caused by the magnetic field. The location of the dark stripe tells us about the direction of magnetization in the wire. It is on the left edge if M points forward.

The $+1$ and $-1$ magnetic charges are observed experimentally in their honeycomb system. However, for some reasons the $+3$ and $-3$ magnetic charges are not observed experimentally (Figure 2.3(b)). Notice the magnetic charges are defined by the divergence of $H$ (rather than $B$). In another word, the presence of magnetic charge can also be seen from the TEM images by counting the flux of $M$ and adding a minus
CHAPTER 2. BACKGROUND

sign.

Figure 2.3: (a) The TEM image of the honeycomb that indicate the magnetization, (b) The emergent charges in the system

Taking multiple Lorentz TEM images over the course of an experiment allows the experimentalists to record a detailed history of magnetization in the sample. This is very helpful in keeping track of the dynamics of magnetic charges in the two dimensional system. For example, one can place the sample in a uniform magnetic field, fix the sample, and gradually increase the field. By observing how many links are reversed at each step we can infer the dynamics of magnetic charges in the plane.
Chapter 3

Model Description

Our model is an idealized version of Cumming’s experiment of artificial spin ice on honeycomb lattice with dimensions $500\,\text{nm} \times 110\,\text{nm} \times 23\,\text{nm}$ and $M = 8.6 \times 10^5 \text{A/m}$. Each link is assumed to have the same length, and each link will reverse at certain external field, which we call it critical field.

3.1 magnetic charges

The concept of magnetic charges has been mentioned for several times in previous sections, following the idea mentioned in Chapter 1.1, here I propose a more systematic definition of magnetic charges $q$ in our model.

Firstly, all nodes are labeled by single variables $i$, hence each wire can be defined uniquely by its two ends $i, j$. We assume the magnetization $\mathbf{M}$ of each wire is parallel
CHAPTER 3. MODEL DESCRIPTION

Figure 3.1: Magnetization of links and magnetic charges

to the long axis of the wire in thermal equilibrium, therefore, $\mathbf{M}$ for each nanowire can be characterized by a two-state variable $\sigma_{ij} = \pm 1$, where $\sigma_{ij} = +1$ when $\mathbf{M}$ points from $i$ to $j$, and $\sigma_{ij} = -1$ when $\mathbf{M}$ points from $j$ to $i$, clearly $\sigma_{ij} = -\sigma_{ji}$. A graphical illustration is shown in Fig. 3.1

Since the magnetic charges are defined by the divergence of $\mathbf{H}$, the dimensionless magnetic charge at node $i$ can be represented by

$$q_i = \sum_j \sigma_{ji}, \quad (3.1)$$

Therefore, the magnetic charge of each node in our model is

$$Q_i = \oint \mathbf{H} \cdot d\mathbf{A} = -\oint \mathbf{M} \cdot d\mathbf{A} = -Mtw \sum_j \sigma_{ji} = -Mtwq_i, \quad (3.2)$$

where $t$ is the thickness of each bar and $w$ is the width. Similar to electrical static energy, the magnetostatic energy of spin ice can be written as Coulomb interaction
CHAPTER 3. MODEL DESCRIPTION

of magnetic charges defined above: [1]

$$E \approx \frac{\mu_0}{8\pi} \sum_{i \neq j} \frac{Q_i Q_j}{|r_i - r_j|} + \sum_i \frac{Q_i^2}{2C}. \quad (3.3)$$

The total magnetostatic energy is dominated by the second term, which is the self-energy of each node, notice this term requires the natural spin ice to minimize the magnetic charge on each node. The first term become significantly important when we are calculating the critical field of reversal, which will be explained in more detail in the next chapter.

In our honeycomb model, the coordination number is 3, hence $q_i = \pm 1$ or $\pm 3$. To minimize the node’s self-energy,

$$q_i = \pm 1 \quad (3.4)$$

are preferred states. Nodes with triple charges are very rare. Branford et al [15] have found nodes with triple charges with a higher amount of quenched disorder in the samples. Here quenched disorder means a random slightly different critical field $H_c$ in individual wires. However, Cumings et al have never observed nodes with $q_i = \pm 3$ in their honeycomb ice experiment. Therefore, in our model we assume the system strictly follows this ice rule (equation 3.4) in its magnetization reversal process.

3.2 Critical Field $H_c$

It is clear that the magnetization of a nanowire will not be affected when the external field is very weak. However, under an external field that is strong enough and
opposite to the original direction of magnetization of the nanowire, the magnetization of that wire will be reversed. Therefore, there must be a critical field $H_c$ such that the wire is just able to reverse when the external field is larger than $H_c$ and opposite to its initial magnetization direction. An analytical theory about calculating $H_c$ will be presented in the next section.

3.3 Things to do

In this thesis I present a model of magnetization dynamics in artificial spin ice subject in an external magnetic field, specialize to the case of kagome spin ice, in which magnetic elements form a connected honeycomb lattice. [16] I will first describe a microscope view of the spin reversal process, which represent the dynamics of magnetic charges on a single link and a single node. Then a little discussion on the concept of avalanche that we think might help us better explain the avalanche process in large system. Finally I will go back to the large system to study the general distribution of avalanches at different angles.
Chapter 4

Microscopic Simulation

In the first step of my project, I studied the detailed magnetization reversal process in a single link by using a simulation software called OOMMF [17]. Given a problem description, OOMMF integrates the Landau-Lifshitz equation

$$\frac{dM}{dt} = -\gamma M \times H_{eff} - \frac{\gamma \alpha}{M_s} M \times (M \times H_{eff}) \quad (4.1)$$

Where:

- $M$ is the point-wise magnetization (A/m)
- $H_{eff}$ is the point-wise effective magnetic field (A/m)
- $\gamma$ is the gyromagnetic ratio (m/(A·s))
- $\alpha$ is the damping coefficient (dimensionless)

The effective field is defined by:

$$H_{eff} = -\mu_0 \frac{1}{\gamma} \frac{\partial E}{\partial M} \quad (4.2)$$
4.1 Spin reversal on a single link

![Initial magnetization](image)

Figure 4.1: Initial magnetization

The arrows in the figure 4.1 represent the direction of local magnetization. Initially the color just represent the direction of the magnetization. If I change the color to represent the divergence of \( \mathbf{M} \), which is the definition of magnetic charges with a minus sign, we can easily observe magnetic charges on each junction. In figure 4.2(a), the left junction is negatively charged and the right junction is positively charged. The magnetic charge on each site is \( Q_i = | \oint \mathbf{H} \cdot d\mathbf{A} | = Mtw \) (equation 3.2). The horizontal link is of our interests, and it is initially magnetized to the right.

Now we apply an external field \( \mathbf{H} \) pointing to the left. As we gradually increase the field, there is a critical point where the magnetic wire start reversing. At first, we assume the wire is *one-dimensional*, in this simplified case the critical field can be represented by

\[
H_c = \frac{H_0}{\cos \theta}
\]

(4.3)
In this equation $H_0$ (the projection of external field on the wires direction) is a constant, which is just big enough to overcome the Coulomb attraction of each magnetic charge on each site. [18] Suppose a node with magnetic charge $q_i = \pm 1$ emits a domain wall with magnetic charge $q_w = \pm 2$.[19,19] Conservation of magnetic charge means that the charge of the site turns to $q_i = \mp 1$. The emission process can thus be viewed as pulling a charge $q_w = \pm 2$ away from a charge of the opposite sign $q_i = \mp 1$. The maximum force between the two charges is achieved when the separation between them is of the order of their diameter $a$, which roughly equal to the width of the wire.
w. Therefore, we have

\[ F_{\text{max}} = \mu_0 Q_i Q_w \frac{4}{\pi a^2}. \]

This coulomb force must be overcome by the Zeeman force acting on the emitted charge caused by the external magnetic field, which equals:

\[ F_{\text{ext}} = \mu_0 Q_w H_{\text{ext}}, \]

Thus the critical field can be estimated by

\[ H_c = \frac{|Q_i|}{4\pi a^2} = \frac{Mt w}{4\pi a^2} \approx \frac{M t}{4\pi w}. \] (4.4)

Apply the parameters used by Cumings et al [14] into the equation above, we get \( \mu_0 H_c \approx 18 \text{ mT} \), while the value observed by Cumings et al [20] is 35 mT, which is in the same order of magnitude.

In our simulation, since the geometric shape of link can be made perfectly symmetric, hence we can see each site will emit a magnetic charge to the center, and the two magnetic charges will meet each other and neutralize in the center of magnetic nanowire. While in reality usually only one wire will emit the charge as there is always a difference between two junctions, one has a higher critical field than the other.

From the OOMMF simulation shown above, we can summarize the spin reversal process in the horizontal link by imagine a node \( q_i = \pm 1 \) emits a domain wall of charge \( q_{w1} = \pm 2 \) and change its charge to \( q_i = \mp 1 \), while on the other side of the wire the initial charge is \( q_j = \mp 1 \). The domain wall will move all the way to the other end and get absorbed by node \( j \), changing its charge to \( q_j = \pm 1 \). (Figure 4.3).
In addition to the above mentioned process, there is another possible process of reversal, which is shown in figure 4.4.

In this reversal process, the reversal is triggered when a site has charge \( q_i = \pm 1 \) emits a domain wall of charge \( q_{w1} = \pm 2 \) and change its charge to \( q_i = \mp 1 \), while on the other side of the wire the initial charge is also \( q_j = \pm 1 \). In this case the emitted charge will not be directly absorbed by node \( j \), since that would create an energetically unfavorable node with \( q_j = \pm 3 \), but instead node \( j \) will emit the second domain wall with charge \( q_{w2} = \pm 2 \) to the other side of the node and turns its own charge to \( q_j = \mp 1 \). Now node \( j \) can happily accept \( q_{w1} \) without creating any triple charged node and the domain wall \( q_{w1} = \pm 2 \) is finally absorbed by node \( j \), changing
its charge to $q_j = \pm 1$.

### 4.2 Spin reversal on Kagome Junction and Offset Angle

#### 4.2.1 Basic Construction and Definition

After running simulation on a single link, the second step I took is to understand the behavior of magnetization on a junction and how critical field depend on the angle of external field. To do this, I ran simulation of the dynamics of magnetization under external field on a Honeycomb Junction (a node). The construction of the junction
and the definition of the angle of external field in shown in Figure 4.5.

![Figure 4.5: The ground state magnetization of Kagome Junction (without external field)](image)

4.2.2 Simulation Result

In our simulation, we apply an external magnetic field at the junction constructed above at different angles. At each angle, we start the external field from a very small magnitude that is not able to cause any reversal on any wire. The external field’s strength is gradually increased until the red link is reversed at the critical field $H_c$, the critical field at this angle is recorded and a sample reversal process is shown in
CHAPTER 4. MICROSCOPIC SIMULATION

figure 4.6. The purpose of this simulation is to obtain the critical field of red wire at different angles. The simulation result for the critical field at different angles is tabulated in table 4.1.

4.2.3 Offset Angle and experiment verification

Due to symmetry, it is sufficient to only consider the simulation result of $H(\theta)$ for $-75^\circ \leq \theta \leq 30^\circ$, which is plotted in Figure 4.7. From our numerical simulation result, we found the critical field is not symmetric with respect to the angle of external field, as it is not symmetric to $\theta = 0^\circ$. Instead of our previous one-dimensional link assumption that the critical field can be predicted by the formula

$$H_c = \frac{H_0}{\cos(\theta)}$$

, the simulation result shows that the critical field can actually be fitted very well by the Equation:

$$H_c = \frac{H_0}{\cos(\theta + \alpha)} \quad (4.5)$$

Where $\alpha \approx -19^\circ$ is the offset angle. This phenomena can be explained by the asymmetric behavior of magnetization in Honeycomb Junction. As can be seen from figure 4.8, even though the Honeycomb Junction is symmetric geometrically, however, its magnetization is apparently not symmetric. There is a magnetization defect between link 1 and link 3, however, no such defect is observed between link 1 and link 2. In addition, if we look carefully, we can see that the magnetization in the first
link that is close to the junction are generally tilted to the right.

The experimental group John Cumings have run two experiment to test our model. In both experiments, the sample (figure 2.2) is placed under a very strong external magnetic field at first so that all links point in one direction. After that, the external field is removed and another external field is imposed on the system at a specific angle. Start from a small strength, the external field is gradually increased, and the magnetization of the whole system is recorded after each small increment. The two experiments carried out are, respectively, for $0^\circ$ and $20^\circ$ external field.

The two steps increment in figure 4.9 (a) and (b) corresponds to the reversal of red links (which have lower critical field) and blue links (which have higher critical field).

In the first experiment, $\theta = 0^\circ$, the experimental result shows the ratio of the two critical field

$$\frac{H_{c_2}}{H_{c_1}} = \frac{46\text{mT}}{36\text{mT}} \approx 1.28$$

If we use the previous assumption of one-dimensional wires,

$$\frac{H_{c_2}}{H_{c_1}} = \frac{H_0}{\cos(\theta_2)} = \frac{\cos(\theta_1)}{\cos(\theta_2)} = \frac{\cos(0^\circ)}{\cos(60^\circ)} = 2$$

The critical field should be 2. However, if we add the offset term into the expression of critical field,

$$\frac{H_{c_2}}{H_{c_1}} = \frac{H_0}{\cos(\theta_2 + \alpha)} = \frac{\cos(\theta_1 + \alpha)}{\cos(\theta_2 + \alpha)} = \frac{\cos(19^\circ)}{\cos(41^\circ)} = 1.26$$

25
CHAPTER 4. MICROSCOPIC SIMULATION

The ratio become 1.26, which is almost the same as the experimental result. In the second experiment, $\theta = 20^\circ$, the experimental result shows the ratio of the two critical field:

$$\frac{H_{c2}}{H_{c1}} = \frac{91mT}{35mT} \approx 2.6$$

The one dimensional link assumption gives:

$$\frac{H_{c2}}{H_{c1}} = \frac{\cos(\theta_1)}{\cos(\theta_2)} = \frac{\cos(20^\circ)}{\cos(80^\circ)} = 5.4$$

With the offset angle, we have:

$$\frac{H_{c2}}{H_{c1}} = \frac{\cos(\theta_1 + \alpha)}{\cos(\theta_2 + \alpha)} = \frac{\cos(1^\circ)}{\cos(61^\circ)} = 2.1$$

Again much closer to the experimental result.

4.2.4 Further Discussion

There is several things that we can get from the simulation result above. Firstly, when the angle is from -10 to 90 degrees (for link 1), and from 150 to 240 degrees (for link 2), the Domain Walls are all emitted from the junction, and the critical field can be calculated by

$$H_c = \frac{H_0}{\cos(\theta + \phi)}$$

While $H_c$ for link 1 is slightly smaller than $H_c$ for link 3, because link 3 is a bit narrower than link 1 due to the geometric imperfection. Secondly, when angle is from 90 to 150 degree, no link can be reversed. However, the result for the angle between
255 degrees to 345 degrees is more complicated. Take link 1 as example, when the angle is less than -15 (345) degrees, it is easier for Domain Wall to be emitted from the end of link 1 compared with from the junction, therefore, the critical field cannot be fitted by the previous formula. Same applied to link 3. Finally, when the angle is between 280 to 315 degrees, the Domain Wall can be most easily generated from the 2nd link. The result should be symmetric with respect to 300 degrees, the slight violation of this symmetry is contributed from the geometric asymmetry. The disc diagram of reversal character verses external field is shown in Figure 4.10.
Figure 4.6: Simulation by OOMMF of reversal process of a Y junction
CHAPTER 4. MICROSCOPIC SIMULATION

Table 4.1: Kagome Junction reversal at different angle of external field

<table>
<thead>
<tr>
<th>Angle (degree)</th>
<th>$H_c$ (mT)</th>
<th>Reversed Link</th>
<th>DW emitted place</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>57</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>20</td>
<td>54</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>30</td>
<td>55</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>40</td>
<td>57</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>50</td>
<td>62</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>70</td>
<td>84</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>80</td>
<td>113</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>90</td>
<td>208</td>
<td>1</td>
<td>j</td>
</tr>
<tr>
<td>91 ~ 149</td>
<td>No Reversal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>214</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>160</td>
<td>116</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>170</td>
<td>86</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>180</td>
<td>71</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>190</td>
<td>63</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>200</td>
<td>58</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>210</td>
<td>55</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>220</td>
<td>54</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>230</td>
<td>56</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>240</td>
<td>59</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>250</td>
<td>63</td>
<td>3</td>
<td>j</td>
</tr>
<tr>
<td>260</td>
<td>65</td>
<td>3</td>
<td>f</td>
</tr>
<tr>
<td>270</td>
<td>66</td>
<td>3</td>
<td>f</td>
</tr>
<tr>
<td>280</td>
<td>66</td>
<td>2</td>
<td>f</td>
</tr>
<tr>
<td>290</td>
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<td>2</td>
<td>f</td>
</tr>
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<td>300</td>
<td>73</td>
<td>2</td>
<td>f</td>
</tr>
<tr>
<td>310</td>
<td>69</td>
<td>2</td>
<td>f</td>
</tr>
<tr>
<td>320</td>
<td>65</td>
<td>1</td>
<td>f</td>
</tr>
<tr>
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<td>63</td>
<td>1</td>
<td>f</td>
</tr>
<tr>
<td>340</td>
<td>61</td>
<td>1</td>
<td>f</td>
</tr>
<tr>
<td>350</td>
<td>61</td>
<td>1</td>
<td>j</td>
</tr>
</tbody>
</table>
CHAPTER 4. MICROSCOPIC SIMULATION

Figure 4.7: Simulation result for $H(\theta)$ and the fitting [21]

Figure 4.8: Asymmetric behavior in a close view
Figure 4.9: Experimental Verification of Offset Angle
Figure 4.10: The disc diagram of reversal character versus external field
Chapter 5

Avalanche length distributions

In this chapter, detailed discussion about the mesoscopic behavior of magnetic reversal dynamics in artificial spin ice is presented.

5.1 Case Description

Following the experimental setup of Cumings group [22], a honeycomb lattice was initially magnetized to one specific direction by placing it in a large external field. Figure 5.1 depicts a portion of our model, magnetic charge number $q$ on each node is denoted by $\pm 1$.

Subsequently, the initial strong field is switched off, and a reversal field $H$ is later applied to the system, its orientation is set by fixing the angle between $H$ and $x$-axis to be $\theta$, while $x$-axis is defined to be the direction of initial magnetization, in Fig 5.1
Initially the honeycomb lattice system is magnetized to one direction, which in this case is right direction, by a very strong external magnetic field. Starting from a relatively low field such that no links can be reversed under such field, we gradually increase the strength of the external field at a step size of 0.025 mT. The step size is small enough so that in most times only one avalanche happens in one step size. As the external field increased, more and more links reversed to line up with the field. The external field is increased up to a point such that all links have a positive projection of their magnetization on the field. A reversal curve $M(H)$ was determined and from which we can deduce the length of avalanche that happened in each step. Finally, a probability distribution of avalanche length is obtained.

In our simulation, we set the critical field of each link to follow a Gaussian distribution with mean value 57.5 mT and standard deviation 2.875 mT, these two values
are used in order to match the experiment result. [23]

Due to symmetry (Fig 4.10), it is sufficient to only discuss the external field angles ranging from $90^\circ$ to $180^\circ$. The reversal process changes style at different $\theta$. There are 3 different types of reversal process, characterized by $120^\circ$ regime, $150^\circ$ regime and $180^\circ$ regime.

### 5.2 120° regime: $90^\circ \sim 131^\circ$ (Exponential Decay Regime)

![Figure 5.2: Detailed Magnetization for 120° regime reversal](image)

In this regime, all blue wires in fig 5.2 can reverse independently without violating the spin ice rule, recall the spin ice rule we imposed in our system is no node can have
triple charges. Taking link 1 as an example, the reversal of link 1 will not cause the
reversal of link 2 or link 3 because the domain wall emitted in link 1 will be absorbed
by the junction, after absorbing or emitting a domain wall, the junctions charge will
turn from $-1$ to 1 or from 1 to $-1$, hence no further domain wall will be created.

However, the demagnetization field fluctuation caused by the reversal of link 1
affects the local field on link 4 and link 5 and may cause further reversal of these two
links, which is discussed in detail below:

When link 1 is reversed, it is equivalent as adding a $+2$ magnetic charge on the
upper junction and a $-2$ magnetic charge on the lower junction.

![Detailed Magnetization for 120° regime reversal](image)

To determine the behavior of link 4 after link 1 is reversed, we need to calculate
the change in the total demagnetization field on node $A$ contributed by the additional
$+2$ and $-2$ charges. Visually, this field can be calculated by summing up the two
vectors $AC$ (field caused by $+2$ charged node) and $CD$ (field caused by $-2$ charged
node). The demagnetization field from each charge is obtained by

$$AC = \frac{2Q}{4\pi l^2}$$
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

and

\[ CD = \frac{2Q}{4\pi \left( \frac{1}{\sqrt{3}} \right)^2}, \]

note the analogy to Coulomb interaction. Therefore, the net additional field on site A:

\[ AD = \sqrt{(CA)^2 + (CD)^2 - 2CA \cdot CD \cos \theta} \approx 0.73 \cdot \frac{2Q}{4\pi l^2}, \]

Finally, by sine rule, we get:

\[ \frac{AD}{\sin \theta} = \frac{CD}{\sin \phi} \rightarrow \sin \phi \approx 0.228 \rightarrow \phi \approx 13.2^\circ > 11^\circ \quad (5.1) \]

Therefore, as AD has a positive projection on \( \vec{n}_1 \), thus the total demagnetization field do not contribute to the reversal of link 4.

On the other hand, we study site B to determine whether link 5 will tend to reversed following the reversal of link 1. In fact, the projection of demagnetization field on link 5 (\( \vec{n}_2 \)) is negative thus is able to cause its reversal if link 5’s natural critical field is close enough to link 1’s.

\[ H_{\text{Projection}} = H_{\text{demag}} \cdot \vec{n}_2 = -0.86 \cdot \frac{Q}{4\pi l^2} \quad (5.2) \]

Since the standard deviation of \( H_c \) is

\[ \Delta H = 0.05H_c = 2.875 \cdot \frac{Q}{4\pi l^2}, \]

which means \( H_{\text{Projection}} = 0.3\sigma \). Hence in order to let link 5 to reverse, its critical field must lies in the range \( H_{c1} < H_{c5} < H_{c1} + 0.3\sigma \). If \( H_{c1} \) is the mean value, the probability for \( H_{c5} \) to lie in this range is around 11%. Therefore the probability for
k neighboring links to reverse consecutively should follow exponential distribution, which can be approximated by:

\[ P_{k_1} = C \cdot P^{k-1} = C \cdot 0.11^{k-1} = C \cdot 10^{-0.9(k-1)} \]  \hspace{1cm} (5.3)

Since both link 5 and link 6 can reverse, and they are independent of each other. The actual distribution of the avalanche is the sum of two exponential distributions. (Mean value doubled)

\[ P_k = P_{k_1+k_2=k} = \frac{C}{2} \cdot 10^{-0.45(k-1)} = C' e^{-1.04(k-1)} \]  \hspace{1cm} (5.4)

Where C is the normalization constant.

This approximation fit the slope of our numerical simulation well (Figure 5.4).

Numerical Simulation result:

Figure 5.4: x-axis corresponds to the avalanche length, and y-axis corresponds to the number of times such avalanche is observed. Red line is the simulation result with step size 0.01, blue line is the result with step size 0.025 mT.

For \( \theta = 120^\circ \), after link 1 reversed, link 3 can reverse at a much higher field \( H \).
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

There is an abrupt avalanche style change from exponential decay to power decay. That is when the critical field for link 3 exceeds the critical field for link 1. In such case, the reversal of link 3 will activate link 1 (which is previously inhibited from reversal by spin ice rule) hence cause link 1 to reverse (since link 1 has a lower critical field). The reversal of link 1 will create a domain wall that drive the next adjacent black link to reverse.

It is not hard to find that $\theta_c = 131^\circ$

### 5.3 150° regime: $131^\circ \sim 178^\circ$ (Power law decay)

Avalanches in the regime when external magnetic field is between $131^\circ$ and $178^\circ$ have several interesting properties. Firstly, as one can see from figure 5.5, in this regime, all avalanches run diagonally along external field’s direction, therefore two avalanches will never run into each other, under this circumstances the two dimensional problem can be simplified to a one-dimensional problem. Secondly, the avalanches almost always start from the boundary and run into the bulk. This is caused by the offset angle, as I will discuss it below.

*Reason:* Looking at figure 5.2, we found that horizontal links (black) in the bulk
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

Figure 5.5: The first few avalanches for 170° external field reversal has lower critical field than tilted links (blue):

\[ H_{\text{horizontal}} = \frac{H_c}{\cos 30^\circ} \leq H_{\text{tilted}} = \frac{H_c}{\cos (30^\circ + \alpha)} \]  \hspace{2cm} (5.5)

However, in the bulk the reversal of black links are inhibited by spin ice rule, only those links on the boundary can reverse at \( H_{\text{boundary}} = \frac{H_c}{\cos 30^\circ} \). Thus avalanches always start at the boundary.
5.3.1 Modeling of the reversal process

Since two avalanches never run into each other, we can model each avalanche separately. The modeling scheme is as follows:

- Firstly, since the reversal of a horizontal link in bulks always trigger the reversal of the next adjacent tilted link, I combine these two links as one step.

- Secondly, in one avalanche, the avalanche should keep going if the critical field of the next tilted link $H_c$ is less than the projection of the applied field on $\hat{n}_1$, which can be calculated by $H_{app} = \cos(30^\circ - \alpha)$, where the extra term $\alpha$ is the offset angle.

- Finally, the projection of the applied field on the boundary link $H_{app} \cos 30^\circ$ is less than the projection of the same applied field on links in the bulk if $\alpha > 0$, therefore we need a larger applied field to start an avalanche, under such large field the rest links in the bulk can be easily reversed, as a result for $150^\circ$ case the avalanches tend to run all the way diagonally across the system.

A comparison between our model and the avalanche simulation result is presented in figure 5.6.

5.3.2 Theoretical derivation of length distribution

Up to now, I have set up my model, and I’m going to present a pure theoretical calculation that predicted the length distribution of avalanches in one dimensional
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

Figure 5.6: Comparison between the simulated result (blue line) from a program provided by Olga Petrova of the Johns Hopkins University and the predicted result using my model (red line) for a system of size $23 \times 37$, for orientation angle equals $150^\circ$ systems.

When the critical field of the boundary link is the same or lower than the critical field of bulk links, a very neat result can be derived. This applied to the circumstances when the orientation angle of the applied field $\theta$ is in the regime $170^\circ \sim 178^\circ$. Define:

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$  

(5.6)

Which is the Gaussian distribution that described the distribution of critical field for each link. and let:

$$P(x) = \int_{-\infty}^{x} G(x)dx$$  

(5.7)

For a path with finite length $m$, the probability of an avalanche of length $k (k < m)$
start at the \( n^{th} \) \((n < m - k)\) link, which has a critical field \( x \) can be calculated by:

\[
A(x, n, k) = P(x)^{n-1+k-1}(1 - P(x))
\]  

(5.8)

Besides, when \( n = m-k \), \( A(x, m-k, k) \) is different, the probability of an avalanche of length \( k \) \((k < m)\) start at the \((m-k)^{th}\) link, which has a critical field \( x \) can be calculated by:

\[
A(x, m-k, k) = P(x)^{m-k-1+k-1} = P(x)^{m-2}
\]  

(5.9)

Since there is no link after the \( m^{th} \) link.

Therefore, to find the expectation value of the number of avalanches that have length \( k \), we should sum over all possible \( n \) \((n < m - k)\) and integrate through all \( x \) (which satisfy the Gaussian distribution \( G(x) \)).

\[
E_k = \int_{-\infty}^{\infty} G(x) \left[ \sum_{n=1}^{m-k} A(x, n, k) \right] dx
\]  

(5.10)

But

\[
\sum_{n=1}^{m-k} A(x, n, k) = \sum_{n=1}^{m-k-1} P(x)^{n-1+k-1}(1 - P(x)) + A(x, m-k, k)
\]

\[
= \sum_{n=1}^{m-k-1} P(x)^{n-1+k-1} - \sum_{n=2}^{m-k} P(x)^{n-1+k-1} + P(x)^{m-2}
\]

\[
= P(x)^{k-1}
\]

Therefore, we have:

\[
E_k = \int_{-\infty}^{\infty} G(x)P(x)^{k-1}dx = \int_{-\infty}^{\infty} P(x)^{k-1}dP(x) = \frac{1}{k}
\]  

(5.11)
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

which satisfy the inverse proportional decay property. This predicted result agree with the simulation result for $\theta = 170^\circ$ (Fig 5.7).

Remark: Notice that the result of my derivation does not depend on $m$, in fact, the power decay behavior does not change if I change the total length $m$. Therefore, it is worth to point out that the power law decay equation also apply to very large system ($m \to \infty$).

Figure 5.7: The loglog plot of the distribution of avalanche length for $\theta = 170^\circ$ (simulation result), neglect the noise in the end of the line, the slope (power) for avalanche length is around $-1$, agreed with my prediction (red line). The data points for those avalanches longer than 23 are above system size ($W=37$, $L=23$) hence can be ignored.

The second case is when the critical field of the boundary link is higher than the critical field of the bulk links ($131^\circ < \alpha < 170^\circ$). In this case, the formula derived
above will be slightly varied which leads to a less elegant result.

Without loss of generality, let the mean critical field of the boundary link be $c$ times larger than the critical field of the bulk links (for offset angle equals 19°, $c=1.1335$), we have:

$$E_k = E_k(\text{boundary}) + E_k(\text{bulk})$$

(5.12)

Where $E_k(\text{boundary})$ is the expectation value of number of avalanches that start from the boundary link and have length $k$, $E_k(\text{bulk})$ is the expectation value of number of avalanches that start from the bulk links and have length $k$. We have:

$$E_k(\text{boundary}) = \int_{-\infty}^{\infty} G(x) P(cx)^{k-1} (1 - P(cx)) dx$$

$$E_k(\text{bulk}) = \int_{-\infty}^{\infty} G(x) \left[ \sum_{n=2}^{\infty} P\left(\frac{x}{c}\right) P(x)^{n-2+k-1} (1 - P(x)) \right] dx = \int_{-\infty}^{\infty} G(x) P\left(\frac{x}{c}\right) P(x)^{k-1} dx$$

Unlike the previous case, here $E_k(\text{boundary})$ and $E_k(\text{bulk})$ cannot be simplified. The predicted result agree with the simulation result for $\theta = 160^\circ$, and reasonably agree with $\theta = 150^\circ$ (Fig 5.11).

As $k$ become larger, the length distribution $E_k$ behave more and more like an inverse proportional decay with respect to $k$. 
5.3.3 Transition angle between 150° regime and 180° regime:

As I have discussed previously, in 150° regime, only the black and blue links can reverse, therefore two avalanches never clash. However, when the angles of external field gradually increase, the last link (red link in figure 5.2) will start to reverse before the blue links. As long as the red link reversed, clashes between avalanches started to happen. At certain transition angle, when there is a significant possibility that the red link can reverse, we step into the last regime – 180° regime.

To find this transition angle, we need to find the angle such that there is a significant possibility that \( \frac{H_{c2}}{\cos \beta} < \frac{H_{c3}}{\cos \alpha} \), or

\[
\frac{H_{c3}}{\cos \beta} - \frac{H_{c2}}{\cos \alpha} < \frac{2\sigma_H}{\cos \alpha},
\]

so that there is at least 5% of red links will reverse before blue links. To solve this
inequality, we need:

\[
\frac{H_c}{\cos \beta} - \frac{H_c}{\cos \alpha} < \frac{2H_c \cdot 0.05}{\cos \alpha} \rightarrow \cos \beta > \frac{10}{11} \cos \alpha
\]

But \(\alpha + \beta = 120^\circ\) We need:

\[\alpha > 58.4^\circ\]

Therefore the transition angle between 150° regime and 180° regime is 178.4°.

5.4 180° Regime: 178.4° \(\sim\) 180° (power law decay)

In this regime, most avalanches goes horizontally, the mechanism for a single avalanche is the same as the case we discussed in 150° regime. If we dont consider one avalanche bump into another avalanche, the length distribution should follow the equation I derived in the first case of 150° regime, that is, \(E_k = \frac{1}{k}\).

However, in 180° regime, two parallel avalanches can clash into each other, as long as one avalanche bump into another avalanche, it stops. This mechanism make one avalanche easier to stop (like a frictional force), hence increased the length decay rate. Thus intuitively we can guess the length distribution of avalanches in this regime follows the equation:

\[
E_k = k^{-n(\alpha)}
\]
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

Where $1 < n(\alpha) < n(180^\circ)$ depend on the orientation angle of the external magnetic field $\alpha$.

![Loglog plot of avalanche length distribution for $\theta = 180^\circ$](image)

Figure 5.9: The loglog plot of the distribution of avalanche length for $\theta = 180^\circ$ (simulation result), neglect the noise in the end of the line, the slope (power) for avalanche length is around $-1.5$, agreed with my prediction (red line). The data points for those avalanches longer than 11 are above system size ($W=45, L=11$) hence can be ignored.

A complete analytical calculation for $n(\alpha)$ is given as follows.

### 5.4.1 Theory on 180° avalanches

Similar to what I did in the avalanche distribution for 150° regime case, Define:

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

Which is the Gaussian distribution that described the distribution of critical field for each link. and let:

\[ P(x) = \int_{-\infty}^{x} G(x) dx \]

For a path with finite length \( m \), the probability of an avalanche of length \( k \) (\( k < m \)) start at the \( n^{th} \) \((n < m - k)\) link, which has a critical field \( x \) is defined as \( A(x, n, k) \).

Notice: The difference between the 180° regime case and 150° regime case is the expression for \( A(x, n, k) \).

Instead of computing \( A(x, n, k) \) as in equation 5.8, in 180° regime \( A(x, n, k) \) is computed as follows:

\[ A(x, n, k) = P(x)^{n-k-1} \prod_{i=1}^{n-1} B_i (1 - P(x) \cdot B_n) \quad (5.14) \]

Where \( B_n \) is the probability that an avalanche at level \( n \) can safely proceed to level \( n + 1 \) without clash into other avalanches, conditioned on the external field is larger than the critical field of the next link. To find \( B_n \), we introduce another parameter \( C_n \), where:

- \( C_n = \) the probability of a path (can be the sum of several avalanches) with length at least \( n \), without clashing into any other path in its previous steps.

Therefore, we have

\[ B_k = \frac{C_k}{C_{k-1}}. \quad (5.15) \]
The recursive relationship for \( C_k \) can be computed by:

\[
C_k = C_{k-1} - 2p(1-p)C_{k-1}E(P^{k-1}) = C_{k-1} - 2p(1-p)\frac{C_{k-1}}{k}, \quad \forall k \geq 2.
\]

Where \( E(P^{k-1}) = E_k = \frac{1}{k} \) is the probability for an avalanche to be at least \( k \) steps long if only considering critical field restriction. This recursive relationship would approximately give a differential equation if the step size is very small:

\[
\frac{dC}{dk} = -2p(1-p)\frac{C}{k},
\]

which further yields:

\[
C_k = \frac{1}{k^{2p(1-p)}} \tag{5.16}
\]

by considering the initial condition \( P_{1,1} = 1 \). Recall Equation 5.17, together with Eqn. 5.14, Eqn. 5.15 and Eqn. 5.16, at large system limit \( n \to +\infty \) we have:

\[
E_k = \sum_{n=1}^{\infty} \int_{-\infty}^{+\infty} G(x) \left[ P(x)^{n-1+k-1}C_{k+n-1} - P^{n-1+k}C_{n+k} \right] dx
\]

\[
= \sum_{n=1}^{\infty} \left[ \int_{0}^{1} \frac{P(x)^{n-1+k-1}}{(k+n-1)2p(1-p)} dP - \int_{0}^{1} \frac{P(x)^{n-1+k}}{(k+n)^{2p(1-p)}} dP \right]
\]

\[
= \sum_{n=1}^{\infty} \left[ (k+n-1)^{-(2p(1-p)+1)} - (k+n)^{-(2p(1-p)+1)} \right]
\]

Hence we have:

\[
E_k = \frac{1}{k^{-2p^2+2p+1}} = \frac{1}{k^a} \tag{5.17}
\]

It is clear that \( a = 1 + 2p(1-p) \). The probability of choosing the upper link \( p \in [0,1] \). In particular, \( p = 0.5 \) yields \( a = 1.5 \) as in the 180 degree case, which
CHAPTER 5. AVALANCHE LENGTH DISTRIBUTIONS

Figure 5.10: Fit of the theory on $180^\circ$ avalanches. The red points is the experimental result of survival probability $C_k$ divided by $k$, the blue points is the theoretical $C_k$ divided by $k$, the blue line is the bench mark using $k^{-1.5}$, the red lines is the simulated count of avalanche of a system with size $50 \times 50$ and 1000 repetition.

maximizes $a$. Further, $p = 0$ yields $a = 1$ for 170 degree case, which matches the no clash scenario and minimizes $a$. Therefore $E_k = \frac{1}{k^a}$ and $a \in [1,1.5]$. This theory is checked separately using a MATLAB program provided by Cencheng Shen of Johns Hopkins University. (Fig 5.10)

*Remark*: We can again interpret that the expected number of size $k$ avalanche is the expected number of a size at least $k$ avalanche from the start. Such power law phenomenon occurs in our general model simulation with enough starting links by law of large numbers.
Figure 5.11: (a) Comparison (semilogy plot) between the simulated result using my model (blue) and the predicted result using the formula obtained in our theory, when $c=1.1335$, which correspond to $\theta = 170^\circ$ external field. The mismatch at the right hand is due to the finite length effect. (b) A similar comparison (semilogy plot) when $c=1.05$, which correspond to $\theta = 160^\circ$, finite length effect is not significant in this case since most avalanche die out before they reach length=400.
Figure 5.12: (a) loglog plot of $E_k$ vs $k$ (b) Plot of the slope $n(k)$ of figure (a) vs $k$, $E_k = C_k k^{n(k)}$, $n(k)$ is the power decay rate, which converge to -1
Chapter 6

Inertia of Magnetic Monopole

In all of our previous discussion, we rest on an implicit assumption that the critical field for a new domain wall is not affected by whether the node that emits this domain wall has just absorbed another domain wall or not. This assumption is reasonable when the system has high damping factor \((\alpha > 0.5)\), when the dynamics of domain walls is strongly dissipative and the energy generated during the absorption process is quickly dissipated as heat. However, experiments with domain walls in nanowires indicate that they possess non-negligible inertia. [19] Therefore our assumption that the system is always strongly overdamped may not be fully justified. In fact, the damping factor for permalloy is \(\alpha = 0.008\). In this chapter, I present a concept named \textit{inertia} and its characteristic parameter \(\varepsilon\) that may help us deal with the underdamped case, with damping factor less than 0.1.
6.1 Definition of Inertia and $\varepsilon$

In the case when the system is not overdamped, once a magnetic charge passes a junction, the spins in the junction will be disturbed (or activated) by the incoming charge, the energy injected by the magnetic charge makes it easier for the next wire to overcome the energy barrier of being reversed and hence might have a lower critical field. This result in the reversal of the next wire under an external field that is lower than its original critical field. In a chain of Y junction lattice, this phenomena makes the magnetic charge traveling in the chain appears to have “inertia” that helps them to get over the barrier on their way. (figure 6.4)

In this thesis, the inertia is characterized by a factor $\varepsilon$ such that given all other conditions the same, the critical field of an “activated” site is $\varepsilon$ times the critical field of an “inactivated” site. Earlier simulations [21] shows that $\varepsilon \approx 0.5$.

6.2 Basic Idea

I used OOMMF to conduct simulation test on inertia. The basic idea is:

- A magnetic charge is created on a wide wire first at low external field (equation 4.4).
- The magnetic charge that travel along the wide wires and then “squeezes” into a narrower wire, where the critical field for each junction is supposed to be
CHAPTER 6. INERTIA OF MAGNETIC MONOPOLE

higher.

- The vector field of the system at the time when the magnetic charge has just got into the narrower wire is saved as the initial state.

- The initial state is tested on different magnitude of external field and the smallest field that enable the magnetic charge to pass through the next junction is taken as $H_{\text{new}}$.

- The critical field of the narrow wire when there is no incoming magnetic charge $H_{c2}$ is found separately.

- The inertia coefficient is calculated as the ratio between $H_{\text{new}}$ and $H_{c2}$.

6.3 Shape Construction

The shape that I used in my simulation is constructed as shown in figure 6.1. The mask shape I used in OOMMF simulation is drew by Dr. Paula Mellado, the arc in the junction has the diameter the same as the width of the narrower wire, and the

Figure 6.1: Construction of the sample
connection between the wider wire on the left and the narrower wire on the right follows a curve which is defined by polynomial interpolation to make the transaction smooth. Notice instead of the \( Y \) junctions in figure 6.4, the magnetic charge in our model go through \( T \) junctions, which we thought have the same physics as the \( Y \) junction model.

### 6.4 Finding the Ground State

![Figure 6.2: Finding the ground state](image)

To find the ground state (initial condition), the initial magnetization pointing to one direction (Figure 6.2(a)). When there was no external applied, the magnetization
CHAPTER 6. INERTIA OF MAGNETIC MONOPOLE

will automatically saddle to the ground state, such that the magnetization direction is parallel with the long axis. The ground state is obtained as shown in figure 6.2(b). In this case, initially the T junction has the total charge $-1$.

6.5 Simulation of Reversal Dynamics

After the wires have achieved their ground state, an external magnetic field pointing to the right is applied. Since the left wire is wider than the right wire, at certain critical field $H_{c1}$, Domain Wall will be generated at the left end without the help of inertia (there is no initial charge coming in). Since the right wire is narrower than the left wire, if the critical field to generate a Domain Wall (without the help of inertia) from the right junction is $H_{c2}$, then apparently $H_{c1} < H_{c2}$.

However, in the above configuration, after the domain wall was generated at the left end first at $H_{c1}$, when the Domain Wall (+2 charged) reaches the right junction, it will first be absorbed by the junction, so the charge of the junction will turn from $-1$ to $+1$. The junction will be excited after it absorbed the Domain Wall, the absorbed Domain Wall will make it easier for the junction to re-emit another Domain Wall to the right under the external field. Suppose the critical field now for the junction to re-emit the Domain Wall is $H_{\text{new}}$, we have $H_{\text{new}} < H_{c2}$. Define

$$\varepsilon = \frac{H_{\text{new}}}{H_{c2}}$$

After reaching the ground state (figure 6.2(b)), since $H_{c1} < H_{c2}$, I apply a field
CHAPTER 6. INERTIA OF MAGNETIC MONOPOLE

that is sufficiently large to generate a Domain Wall from the left end, but still not as large as $H_{c2}$. After the Domain Wall is generated, the Domain Wall can move in the wire even though the external field is very weak, and the function of the wider wire is served. The vector field of the system when magnetic charge is still sufficiently far away from the $T$ junction is saved as the initial state, which is shown in Figure ??

![Figure 6.3: Vector field of the initial state’s magnetization I obtained](image)

Saving the above vector field as initial state, I run multiple separate simulations by applying external field at different magnitude (but the same direction). Start from a quite strong external field, we observed the re-emission of Domain Walls in the $T$ junction. The external field is then decreased gradually, and found the re-emission of Domain Wall stopped when the external field is decreased to $H \approx 58$ mT, then this $H$ should be the $H_{new}$ as I mentioned earlier in this section. Finally, after finding $H_{c2} \approx 121$ mT explicitly by run a simulation on only the narrow part. Therefore, the $\varepsilon$ can be calculated by:

$$\varepsilon = \frac{H_{new}}{H_{c2}} = 0.48$$

This method is approved by other group members and the result matches our expectation and the simulation result.
6.6 Summary

To summarize this chapter, we introduced a concept named *inertia* and its characteristic parameter $\varepsilon$ that helped us deal with the case when magnetic charges travels in underdamped system, where the critical field of an “activated” site is $\varepsilon$ times the critical field of an “inactivated” site. Our simulation method shows $\varepsilon \approx 0.48$ for the $T$ junction.

However, no direct experiment have been carried out to find the inertia parameter $\varepsilon$, and we are still not sure how much weight does inertia played in the dynamics of magnetic charges in spin ice. Further investigation on inertia can be done in the future.
Figure 6.4: Inertia helps a magnetic charge to get through Y junctions in honeycomb lattice
Chapter 7

Conclusion

While finding the elementary magnetic monopole seems not an easy task, recently, scientists have come out an alternative approach by studying emergent particles in spin system. A spin ice is a magnet (i.e. a substance with spin degrees of freedom) with frustrated interactions from which we observe emergent magnetic charges. Two typical spin ice materials we consider here are Dy$_2$Ti$_2$O$_7$, with tetrahedral lattice structure. A two dimensional cross section of the tetrahedral lattice without the wire pointing out of the plane gives a Kagome lattice structure, the Kagome lattice can be transferred to honeycomb lattice if we replace the spin points (sites) in Kagome lattice by spin islands (wires) in honeycomb lattice. The honeycomb spin ice is highly frustrated and recently physicists have raised special interests on the dynamics of magnetization in artificial honeycomb spin ice.

In this senior thesis Project, we studied both meso-scopic behavior and microscopic
behavior of the dynamics of magnetization of a two dimensional Honeycomb lattice, made with permalloy. The concept of magnetic monopole is introduced for better visualization and interpretation of the magnetization behavior. Our model consists of a two dimensional honeycomb sheet made with permalloy, and each wire has the following dimensions: 500nm × 110nm × 23nm.

We did microscopic simulations on a single wire and on a honeycomb junction respectively, using OOMMF simulation software. The dynamics of magnetic monopole is observed in our simulation. By finding the critical field that triggered the reversal process on a junction with respect to the angle of external field, we found there exist an offset angle (approximately 19°) in the system. The critical field that triggers reversal of the magnetization on a wire can be calculated with:

\[ H_c = \frac{H_0}{\cos(\theta + \alpha)} \]

We also studied the avalanche length distribution when an external field is applied to a uniformly magnetized honeycomb lattice sheet. We found when external angle \( \theta \in (90, 131) \), the avalanche length distribution decays exponentially as

\[ E_k = \frac{C}{2} e^{-1.04(k-1)} \]

; and when \( \theta \in (132, 180) \), the avalanche length distribution decays as a power law:

\[ E_k = \frac{1}{k^{1+2p(1-p)}} \]

Detailed analytic derivation is provided.
Finally, we introduced the concept of *inertia* and its characteristic parameter $\varepsilon$ that helped us deal with the case when magnetic charges travels in underdamped system, where the critical field of an “activated” site is $\varepsilon$ times the critical field of an “inactivated” site. Our simulation method shows $\varepsilon \approx 0.48$ for the $T$ junction.
Appendix A

Ice Rule

In chemistry, ice rules are basic principles that govern arrangement of atoms in water ice. They are also known as Bernal–Fowler rules, after British physicists John Desmond Bernal and Ralph H. Fowler.

The rules state [24] each oxygen is covalently bonded to two hydrogen atoms, and that the oxygen atom in each water molecule forms up to two hydrogen bonds with other oxygens.

In other words, in ordinary $Ih$ ice every oxygen is bonded to the total of four hydrogens, two of these bonds are strong and two of them are much weaker. Every hydrogen is bonded to two oxygens, strongly to one and weakly to the other. The resulting configuration is geometrically an ordered lattice periodic lattice, but the distribution of weak vs. strong bonds is disordered. A nice figure of the resulting structure can be found in Fig. A.1
Spin ice and water ice shares many remarkable properties. For example, both of them possess a very large number of low-energy, nearly degenerate configurations satisfying the Bernal-Fowler ice rules. In water ice, an $O^{2-}$ ion has two $H^+$ ions nearby and two $H^+$ slightly further away from it. Similarly, in spin ice, two spins point into and two away from the center of every tetrahedron of magnetic ions (Fig. A.2).

Figure A.1: Bernal-Fowler periodic model of ice Ih. Covalent and hydrogen bonds are shown as sticks as an aid to visualization of the bonding network. [25]

Figure A.2: Analogues between water ice and spin ice lattice structure
Appendix B

Geometric Frustration

Geometric frustration usually arises in magnets where spins with antiferromagnetic interactions on a lattice with triangular or tetrahedral units. Three spins on a triangle cannot all be antiparallel. Instead, they may have multiple ground state as shown in Fig. B.1, or keep fluctuating down to the lowest temperatures.

In generally, frustration corresponds to the situations in which a system contains multiple interactions such that these interaction energy cannot be simultaneously minimized.

The study of frustration in physics began when people started dealing with antiferromagnets. The two-stage electron spins on the triangular lattice with spins pointing up or down with a common axis was the first exactly solvable model exhibiting strong frustration. In 1950, Wannier showed [26] that the system remains disordered down to zero temperature and has a high degeneracy of the ground state. Theoretically,
APPENDIX B. GEOMETRIC FRUSTRATION

Figure B.1: An example of geometric frustration. Three Ising spins coupled antiferromagnetically in a triangle. The ground-state degeneracy is a defining characteristic of frustration. In the triangular Ising antiferromagnet, the number of ground states grows exponentially with the number of spins.

Same frustration applies to our honeycomb lattice too, since the system has an odd coordinate number \( n = 3 \), there is no way to reduce the total magnetic charge on each node to zero. Hence the ground state of the system became degenerated. Figure B.2 shows two possible ground state magnetic configurations of the dipolar kagome ice that form honeycomb lattice. Apparently there are more other ground states.
APPENDIX B. GEOMETRIC FRUSTRATION

(a) One possible ground state

(b) Another possible ground state

Figure B.2: Two possible ground state magnetic configurations of the dipolar kagome ice that form honeycomb lattice
Bibliography


BIBLIOGRAPHY


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Vita

Yichen Shen was born in Hangzhou, a beautiful city on the east coast of China in 1989. After graduating from Hangzhou Foreign Language School in 2007, he started his bachelor’s degree in Physics at Nanyang Technological University in Singapore. In 2009, he enrolled at The Johns Hopkins University in Baltimore, Maryland. In 2010, Yichen received the Provost’s Undergraduate Research Award (PURA) which funded him to conduct one semester of research in Dr. Oleg Tchernyshyov’s research group. In the summer of 2010, he was funded by the National Science Foundation to conduct 10 weeks of intensive research at the Materials Research Science and Engineering Centers Research Experience for Undergraduates. Yichen has attended various physics conferences and given presentations, including the American Physics Society (APS) April Meeting 2010 in Washington DC, and the APS March Meeting 2011 in Dallas. In May 2011, Yichen will graduate from JHU.
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