ESSAYS ON STRUCTURAL MODELS AND MICRO
DATA IN CONSUMER FINANCE

by

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Abstract

In the first essay, I analyze how housing and debt affect households’ marginal propensity to consume out of wealth through the lenses of micro data and a structural model. Empirically, using novel Norwegian registry data, I find that after controlling for wealth, households with higher leverage respond more to wealth changes, and this result does not seem to be primarily driven by heterogeneity in household preferences or other unobserved characteristics. Theoretically, I develop a structural model of household behaviors that can account for household balance sheets over the life cycle, and I show that the model-implied relationship between consumption and leverage is quantitatively similar to that in the data. My findings corroborate the view that household indebtedness and leverage matter for consumption dynamics, that a substantial fraction of households are likely to behave in a hand-to-mouth fashion even though their wealth is high, and that the housing market is key to these phenomena.

In the second essay, I study consumption insurance and welfare under progressive taxation through the lens of a life-cycle model that matches the empirical distribution of wealth and labor supply. I find that consumption insurance is achieved through
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one direct channel of shock reductions and two indirect channels of wealth accumulation and labor supply under progressive taxation. In the case of Germany, the direct channel accounts for 22% reduction in consumption response to a permanent wage shock, while wealth and labor account for 2.6% and 15.3% reductions, respectively. The order of quantitative importance in consumption insurance is therefore progressivity, elastic labor supply, and wealth buffer. I also find that the optimal degree of progressivity is one that is closer to proportional taxation than to the current tax system, and the optimal degree of progressivity is similar for households with different preferences and different initial wealth conditions.

In the third essay, I investigate the relationship between China’s urbanization and the persistent increase in its household saving rates. Using data at the provincial level in China, we show that there exists a strong correlation between urbanization and the increase in saving rates: Provinces that experienced more rapid urbanization saw a bigger rise in household saving rates. To quantitatively evaluate the role of urbanization plays in the rise in household saving rates, we develop a simple model of migration that features buffer stock saving behaviors of urban households. Our calibrated model shows that urbanization between 1995 and 2007 contributes to at least an increase of 8 percentage points in household saving rates in China.

Keywords Consumption, Housing, Leverage, Precautionary Saving, Progressive Tax, Life Cycle, Urbanization, High Saving Rate

JEL Classification D12, D14, D81, C61, D91, E21, H31, H24, R23
ABSTRACT

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Professor Jon Faust
Professor Gregory Duffee
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Dedication

This thesis is dedicated to my family.
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Chapter 1

Housing, Debt, and the Marginal Propensity to Consume

1.1 Introduction

Household mortgage debt is ubiquitous. According to the recent wave of the Survey of Consumer Finances, 74.5% of U.S. families have debt and 41.5% have mortgages or home equity loans in 2013. Many economists have argued that high levels of household debt have played a role in suppressing aggregate consumption and thus propagating the Great Recession. [Mian, Rao, and Sufi (2013)], who provide evidence in this direction, find that during the Great Recession, aggregate consumption responded more to wealth losses in ZIP code areas where leverage was high.

The underpinnings of how and why debt affects consumption dynamics, however,
are limited. First, empirically, most of the evidence and the discussion to date has taken place at an aggregate level. Aggregate level data does not establish a direct link between consumption and debt within the same household. It is therefore unclear whether the patterns found actually reflect a link between leverage and consumption responsiveness at the individual level. Moreover, even with individual level data, as in Baker (2015), it is important to be aware that debt might reflect household characteristics rather than constraints. For instance, unobserved household characteristics such as impatience might drive household balance sheets and consumption simultaneously, making it difficult to assess the role of debt despite the use of household level data. Second, on the theoretical side, traditional models of household consumption decisions characterize household balance sheets by total net wealth only, saying nothing about why leverage might matter.

In this paper, we aim to develop a structural model of household behavior that can account for the heterogeneity of household balance sheets in the data and shed light on the relationship between housing leverage and the marginal propensity to consume out of wealth. To this end, we utilize novel Norwegian registry data that contain detailed information on household balance sheets and allow for the construction of imputed consumption. We proceed in two steps. First, we explore if the link between leverage and the marginal propensity to consume out of wealth, which has been documented at the macro level in the U.S. recession episode, also holds at the micro level in normal times. We find that it does. After controlling for wealth, house-
holds with higher leverage have a higher consumption response to wealth changes. More importantly, this finding also holds when we control for individual fixed effects. Hence, the pattern does not seem to be primarily driven by heterogeneity in household preferences or other unobserved characteristics. Second, we proceed to our main objective, which is to develop a structural model that can quantitatively account for the typical life cycle profile of household balance sheets. Leverage is endogenously determined in our model, and by construction, households are identical in preferences and their expectations about the future. We compare the model-implied relationship between leverage and consumption and the relationship seen in the data. Such a comparison will be informative about the balance-sheet channels through which debt affects consumption dynamics.

Canonical consumption theory does not distinguish between different asset classes on household balance sheets. The implicit assumption is that only total wealth affects consumption choice. Debt, in other words, matters only insofar as it affects net worth. In the data, however, there is substantial heterogeneity in household balance sheets. Figure 1.1 compares leverage and net worth in Norwegian households between 2005 and 2011. For any given level of net worth, there is a great deal of variation in leverage. This is what allows us to estimate the role of leverage, over and beyond its relation with wealth. To understand the role of debt and evaluate its policy implications, we must move beyond the benchmark single-asset model of consumption towards a model that incorporates a richer balance sheet. We develop a model that differentiates
between the three main asset classes held by Norwegian households: housing, debt, and other financial assets. We then estimate our model to capture the life cycle profile of balance sheets in the data.

**Figure 1.1: Heterogeneity in Household Leverage and Net Worth**

![Graph showing the relationship between leverage and net worth.]

*Notes:* This figure presents a 0.2% random sample of the household data we use in this paper. Each dot represents a household-year observation. Leverage in this context is defined as the ratio of debt to housing value. Net worth is in millions of Norwegian Krones, indexed to the 2000 price level. The nominal exchange rate between Norwegian Krones and US dollars during our sample period is about 6 NOK per 1 USD.

We argue that housing decisions are key to accounting for the typical life cycle profile of household balance sheets that are seen in the data. Before making discrete house purchases that are largely financed by debt, households typically accumulate financial assets over time. Indeed, they tend to re-balance their portfolio
between housing and other assets very infrequently. For this reason, our model treats
housing decisions in some detail, and it distinguishes homeownership from renting.
Households are subject to uninsurable idiosyncratic labor income risks and borrow-
ing constraints. In each period, renters allocate intratemporal consumption between
non-housing consumption and housing services (rental payments); homeowners make
decisions about non-housing consumption while enjoying the service flow of their current
house. Households also make decisions about next period’s homeownership status.
For instance, renters can decide to become homeowners next period and choose the
house size that is optimal for them. However, there are transaction costs associated
with buying and selling houses.

The existence of transactions costs makes homeowners move infrequently. In addi-
tion, households must hold some equity on their house. As a result, housing wealth,
at least the fraction against which homeowners cannot borrow, is less liquid than
financial wealth. While a house purchase almost does not change a household’s total
wealth, it does imply a shift in the liquidity of its balance sheets. Buyers who are
expanding their housing stock move closer to their borrowing constraint because part
of their liquid wealth has been transformed into housing. Most home buyers finance
their house purchase primarily with debt, and so homeowners who have recently ex-
panded their housing stock have high leverage. The combination of proximity to the
borrowing constraint and high transaction costs raises these households’ marginal
propensity to consume out of wealth. In fact, households’ MPC does not mono-
tonically decrease in wealth: a recent home buyer has a higher MPC than she had immediately before buying the house, even though there is virtually no change in her total wealth. Households that own larger houses and have more debt tend to have higher MPC than those that have smaller balance sheets. Leverage, therefore, measures liquidity, or equivalently the proximity to the borrowing constraint in the presence of housing, which is an aspect that total wealth would not capture. Thus housing decisions are essential both for our model to capture the life cycle evolution of household balance sheets and for its ability to capture how the marginal propensity to consume out of wealth is related to leverage.

The rest of this paper is organized as follows. Section 1.2 reviews related literature. Section 1.3 describes the Norwegian registry data and explores the empirical relationship between leverage and the consumption response to wealth changes. In section 1.4, we develop a full-fledged consumption-saving model with housing, debt, and financial assets. Section 1.5 shows that a calibrated version of this model is able to capture the typical composition of a median household’s balance sheet over the life cycle and generates a reasonable marginal propensity to consume out of wealth. Section 1.6 discusses the policy implications of the model and Section 1.7 concludes.
1.2 Relation to the Literature

Our paper contributes to three strands of literature. The sluggish recovery after the recent Great Recession in the U.S. and elsewhere in the world has raised questions of whether high levels of household leverage impeded consumption growth over and above what the observed wealth changes would imply. In an empirical analysis of household level data, Dynan (2012) find that compared to other homeowners, highly leveraged homeowners had larger declines in consumption between 2007 and 2009. Mian, Rao, and Sufi (2013), who examine ZIP code level auto sales data, find that the consumption response to housing wealth changes was larger in zip codes with poorer and more levered households. Our empirical analysis contributes to this literature by using novel Norwegian registry data at the household level. We focus on the period between 2005 and 2011 because our housing wealth measure is the most accurate for this period. While the U.S. and Europe were greatly hit by the Great Recession, the impact on Norway during this period was relatively small. Thus, the role of leverage that we highlight is not limited to recessions. Most of the theoretical literature that examines leverage and consumption focuses on how a credit crunch reduces consumption for constrained households (for instance, Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2011)). In these models, an exogenous reduction in the debt limit amounts to an increase in wealth; deleveraging is forced and there is no propagating role for debt and leverage. In our model, households that have higher leverage respond more to wealth changes, and thus when wealth declines they would
Our paper is closely related to an old but recently revived literature on excess sensitivity with respect to transitory shocks. Mounting evidence indicates that the marginal propensity to consume out of transitory shocks is well above zero—a finding that contrasts with the implication of off-the-shelf representative agent models. Using a macroeconomic model that matches the wealth distribution in the U.S., Carroll, Slacalek, and Tokuoka (2014) show that the MPCs can be much larger than those implied by off-the-shelf representative agent models. In their model, however, among households that have the same preferences, it is essentially the poor households who exhibit the largest MPCs. As our model shows, even for households that have the same preferences, the MPC might not decline monotonically in wealth. The presence of durable purchases, especially housing, induces a high MPC for rich households. Kaplan and Violante (2014) show that high returns on illiquid assets induce hand-to-mouth behavior among wealthy households. Our model of housing resembles theirs. But in our model households prefer home ownership because it provides more utility than renting. Thus our results do not rely on excess returns on housing. Moreover, in Kaplan and Violante (2014), there are no explicit transitory shocks, but as emphasized in Deaton (1991), the presence of transitory shocks can to a great extent affect wealth accumulation. In this paper we explicitly consider transitory shocks. In our model, transitory shocks give rise to a dispersion of income and wealth for households with the same permanent income at the same stage of their life cycle. The disper-
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

sion is important for the timing of housing transactions. Without transitory shocks, households tend to move together, creating discrete jumps in homeownership rates.

The third strand of literature to which our paper contributes examines life cycle choices. Gourinchas and Parker (2002a) and Cagetti (2003a) estimate the structural preference parameters of life cycle models of consumption and saving. Fernandez-Villaverde and Krueger (2011) study durable and nondurable consumption over the life cycle. Yang (2009) accounts for housing and non-housing consumption profiles over the life cycle. In contrast to these papers, which are centered on consumption and saving patterns over the life cycle, we focus on the heterogeneity in consumption response to wealth changes and its implications. To estimate the structural parameters of our model, we match the life cycle profiles of housing and net worth of a median household and homeownership rates in our data. We show that the estimated model implies reasonable MPCs and a similar propagating role of leverage that resembles what is seen in the data.

To solve our model, we employ a variant of the endogenous grid point method in Carroll (2006a). In contrast to the common practice of using value function iterations to solve dynamic stochastic optimization models, the endogenous grid point method solves the model quickly and accurately and allows us to estimate the structural model within a reasonable amount of time. Because it involves transaction costs, discrete and continuous choices, and occasionally binding constraints, our modified version of endogenous grid point method contains elements adopted from Iskhakov, Jørgensen.
1.3 The Role of Leverage

This section explores the empirical relationship between leverage and consumption dynamics at the household level. This issue cannot be answered without full knowledge of the structure and the dynamics of household balance sheets, which makes the Norwegian registry data ideal for the purpose of our study. We first describe the data and then move on to the empirical analysis.

1.3.1 The Norwegian Registry Data

The Norwegian administrative micro-data on income and wealth reports wealth every year, and not, as in the PSID, every 4 years. Thus, consumption can be estimated as the residual of disposable income and savings without having to estimate wealth as well. Browning and Leth-Petersen (2003) and Koijen, Van Nieuwerburgh, and Vestman (2014) take this approach to impute consumption from the Danish and Swedish registry data, respectively, and they conclude that the results are promising. Following a similar approach, Fagereng and Halvorsen (2015) impute consumption for Norwegian households from 1993 to 2011. We base our study on their consumption measures. In what follows, we provide a brief description of the imputation procedure. A more detailed exposition of the procedure can be found in Fagereng and Halvorsen.
The imputation is based on the household budget constraint, which states that consumption of household $i$ in period $t$ is income minus savings:

$$c_{it} = y_{it} - s_{it}$$

$$c_{it} = y_{it} + r_{it}^f a_{it-1}^f + r_{it}^r a_{it-1}^r - r_{it}^d d_{it} - (\Delta a_{it} - \Delta d_{it}) \quad (1.1)$$

Here the second line first separates between labor income (including pensions and public transfers), $y_l$, and capital income ($r_{it}^f a_{it-1}^f + r_{it}^r a_{it-1}^r - r_{it}^d d_{it}$), and thereafter it separates between savings in terms of financial asset accumulation $\Delta a_{it}$ and savings in terms of debt changes $\Delta d_{it}$. Capital income $r_{it}^f a_{it-1}^f$ is after-tax financial asset income (interest on bank accounts, coupons from bonds, dividends from stocks, and income from stock option contracts). The rate $r_{it}^d$ is the household specific interest rate on debt between $t$ and $t - 1$, and $r_{it}^f$ is the household specific return on the asset portfolio held between $t$ and $t - 1$. Imputed rents on real assets, $(r_{it}^r a_{it-1}^r)$, are included as part of income, but we do not include capital gains on housing. The savings variable is separated into total debt ($d$) and assets ($a$) where $\Delta d_{it} = d_{it} - d_{it-1}$ and $\Delta a_{it} = a_{it} - a_{it-1}$. Financial assets consist of bank accounts, stocks (listed and non-listed), bonds, mutual funds, money market funds, cash value of life insurance, contributions to private pension accounts, and other financial assets. Income that is not invested or used to reduce debt, declines in net asset values, and net increases in

\footnote{All incomes are assumed to be after-tax values. Taxes are computed using tax functions.}
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

debt all translate into higher consumption. The richness of the Norwegian data makes all terms on the right-hand side of equation (1) observable. All amounts are denoted in real terms (with base year 2000), where the deflator is the Norwegian consumer price index.

Appendix 1.8.1 provides further information about the administrative tax records in Norway and the imputation of housing values. For the empirical exercises in this paper, we use a 20% sample of the Norwegian registry data from 2005 to 2011. We focus on this time period because before 2005, individual level house values were substantially underreported. Since 2005, Statistics Norway has estimated house values on the basis of hedonic price regressions, using characteristics such as location, size and number of balconies. We drop observations in the top and the bottom 5% of wealth and the wealth-to-income ratio because the consumption behavior of the extremely wealthy or indebted households is not our primary interest. We also drop those who have non-listed stocks because the imputation of their stock value and hence their consumption is more prone to error. We further drop observations whose housing leverage is greater than 3 because most of their debt probably is related to business or their housing value is undervalued. In the end, we have about 2 million observations.
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

1.3.2 Housing Leverage and Consumption

We now use the Norwegian micro data to explore whether leverage, defined as the debt-to-housing ratio, is related to household consumption response to wealth changes. Our investigation is closely related to that of Mian, Rao, and Sufi (2013), who find that at the ZIP code level, the marginal propensity to consume out of housing wealth changes (MPC hereafter) differed significantly by leverage level during the Great Recession. We address two questions. First, does the relationship between leverage and consumption responses to wealth changes hold at a disaggregated household level in an environment that was more tranquil than the one explored? Second, how does the relationship between leverage and the MPC move from the micro level to the macro level? In other words, we study how aggregation affects the role of leverage.

As a starting point, consider the consumption function in standard buffer-stock saving models with only one asset, such as the type surveyed by Heathcote, Storesletten, and Violante (2009). In these models, labor income uncertainty gives households a precautionary motive. Consumption $C_t$ is an increasing and concave function of wealth $W_t$ (inclusive of current labor income). Wealth, $W_t$, is the state variable that summarizes a household’s balance sheet at time $t$, and it influences the consumption response to wealth changes in the next period. In the presence of permanent income shocks, it is the ratio of wealth to permanent income that summarizes household balance sheet because it measures the wealthiness of households.
in terms of their lifetime income.

To study the role of debt, we define leverage as the ratio of household debt over housing value:

\[ \text{lev}_t = \frac{B_t}{H_t} \]

We then explore whether leverage is related to the marginal propensity to consume out of wealth changes. To this end, we estimate

\[
\Delta C_t = \beta_0 + \beta_1 \Delta W_t + \beta_2 W_{t-1} + \beta_3 \Delta W_t \times W_{t-1} \\
+ \beta_4 \text{lev}_{t-1} + \beta_5 \Delta W_t \times \text{lev}_{t-1} \tag{1.2}
\]

From the standard buffer-stock saving models, we expect that \( \beta_3 < 0 \) because of the concavity of the consumption function (Carroll and Kimball (1996)). We would also expect that \( \beta_5 \) is insignificant, as wealth summarizes household balance sheets and there is essentially no role for leverage. Our key parameter of interest in Equation (1.2) is \( \beta_5 \). After controlling for household wealth, does the composition of a household balance sheets affect its consumption response to wealth changes (\( \beta_5 \neq 0 \))?

Table 1.1a shows that leverage does play such a role. Column (1) of Table 1.1a estimates the concavity of the consumption function in the Norwegian data. The estimated coefficient on the interaction term, \( \beta_3 \), in equation (1.2) is negative and statistically significant, indicating that consumption is indeed concave in wealth, in
## Table 1.1a: Housing Leverage and Consumption Response to Wealth Changes

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<tr>
<th>Dep.Var:</th>
<th>( \Delta C_t )</th>
<th>( \Delta W_t )</th>
<th>( W_{t-1} )</th>
<th>( \Delta W_t \times W_{t-1} )</th>
<th>( lev_{t-1} )</th>
<th>( \Delta W_t \times lev_{t-1} )</th>
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<td>0.445***</td>
<td>0.531***</td>
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<td>(0.106)</td>
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<td>( W_{t-1} )</td>
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<td>( \Delta W_t \times W_{t-1} )</td>
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<td>(0.008)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta W_t \times lev_{t-1} )</td>
<td>0.197***</td>
<td>0.226***</td>
<td>0.375***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.022)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year#</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tilde{Y} # )</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CHAR#</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FEIS</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.281</td>
<td>0.309</td>
<td>0.346</td>
<td>0.231</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>1,346,844</td>
<td>1,346,844</td>
<td>1,346,264</td>
<td>1,191,995</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. This table presents coefficients from regressions that relate the change in household consumption to the change in household wealth at an annual frequency between 2006 and 2011. All regressions are at the household level. \( \Delta \) indicates change in millions of Norwegian Krones that are indexed to the 2000 price level. Leverage is defined as debt over housing value. \# signifies that both the level of the term and its interaction with \( \Delta W_t \) are included. Year includes year dummies. \( \tilde{Y} \) is average household income in the sample. CHAR includes terms of household characteristics. FEIS is fixed effect in the slope of \( \Delta W_t \). Thus, for column (4), we omit the coefficient for \( \Delta W_t \) and report within-household adjusted \( R^2 \). Throughout, standard errors are in parentheses. *** indicates that coefficients are statistically different from 0 at the 1% confidence level.
line with what a standard buffer-stock saving model would predict. Column (2) adds leverage and its interaction with wealth changes. We see that the estimated interaction coefficient, $\beta_5$, in equation (1.2) is positive and statistically significant. This coefficient is both highly statistically significant and economically important. For instance, consider a household that recently bought its first house, which was largely financed by debt as is typical of first-time home buyers. Its wealth level has barely changed but its balance sheet composition has changed dramatically. In particular, this household’s leverage jumps from zero to almost one. Our coefficient estimate implies that this household’s marginal propensity to consume out of a 1 dollar wealth change would then increase by almost 20 cents.

Column (3) adds to the regression income and household characteristics as well as their interaction terms with the change in wealth. We want to examine the possibility that leverage is picking up either the effect of income expectations or observable household characteristics. It is possible that households that expect higher a future income want to take on more leverage, and because their current wealth is low relative to lifetime income, they have a higher marginal propensity to consume out of wealth changes. In column (3), average income serves as a proxy for households’ permanent income. Age polynomials, which are included in household characteristics, together with average income captures the deterministic profile of household income over the life cycle and thus serve as a proxy for households’ income expectations. The

---

2Household characteristics include age polynomials up to the third order, family size, number of children, education status, marital status, family type, and counties where households reside.
estimates in column (3) indicate that expected income is not driving our results, nor is any other observable household characteristics. The coefficient on the interaction term between leverage and wealth changes—\( \beta_5 \) in equation (1.2)—increases slightly and remains highly significant.

In column (4), we test whether the role of leverage is mainly driven by unobserved household characteristics. Parker (2015) finds that the propensity to consume is a persistent trait of households and that this trait is highly related to impatience. This poses a challenge to the interpretation of our results because impatient households probably are more indebted than patient households and hence our leverage term could reflect this unobserved heterogeneity in preferences. We therefore add a fixed effect in the slope of wealth changes in order to capture the persistence in households’ propensity to consume.\(^3\) In the results, reported in column (4), we see that the role of leverage does not disappear when we control for these fixed effects. Hence it does not seem that the leverage effect is driven by heterogeneity in impatience. In fact, the estimate of \( \beta_5 \) in equation (1.2) is larger than what is estimated in columns (1)-(3)—a point we will come back to in our aggregation results.

Table 1.1b presents the results of equation (1.2) wherein the level of wealth is replaced by the wealth-to-income ratio. We see that the two sets of estimates are quite similar. In particular, the estimates of \( \beta_5 \) are are comparable to those in Table 1.1a.

\(^3\)That is, we allow \( \beta_0 \) and \( \beta_1 \) in equation (1.2) to be different for different households. We use FEIS to denote a fixed effect in slopes in regression tables. Because house price appreciated in Norway during the sample period, variation in household leverage allows us to identify \( \beta_5 \) even when there is a fixed effect in slopes.
### Table 1.1b: Housing Leverage and Consumption Response to Wealth Changes

<table>
<thead>
<tr>
<th>Dep.Var:</th>
<th>$\Delta C_t$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta W_t$</td>
<td>0.659***</td>
<td>0.521***</td>
<td>0.804***</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.107)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{W_{t-1}}{Y_{t-1}}$</td>
<td>0.001***</td>
<td>-0.013***</td>
<td>-0.012***</td>
<td>0.006***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>$\Delta W_t \times \frac{W_{t-1}}{Y_{t-1}}$</td>
<td>-0.017***</td>
<td>-0.010***</td>
<td>-0.010***</td>
<td>-0.002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>$lev_{t-1}$</td>
<td>-0.201***</td>
<td>-0.253***</td>
<td>-0.864***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta W_t \times lev_{t-1}$</td>
<td>0.167***</td>
<td>0.206***</td>
<td>0.299***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.021)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Notes. | This table presents coefficients from regressions that relate the change in household consumption to the change in household wealth at an annual frequency between 2006 and 2011. All regressions are at the household level. $\Delta$ indicates change in millions of Norwegian Krones that are indexed to the 2000 price level. Leverage is defined as debt over housing value. # signifies that both the level of the term and its interaction with $\Delta W_t$ are included. Year includes year dummies. $\bar{Y}$ is average household income in the sample. CHAR includes terms of household characteristics. FEIS is fixed effect in the slope of $\Delta W_t$. Thus, for column (4), we omit the coefficient for $\Delta W_t$ and report within-household adjusted $R^2$. Throughout, standard errors are in parentheses. *** indicates that coefficients are statistically different from 0 at the 1% confidence level. |
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

The richness of the micro-level data allows us to aggregate households to the municipality and county level. This is interesting for two reasons. First, aggregation is likely to average out heterogeneity in household preferences, and consequently it will help us evaluate whether leverage plays a role because of the true balance sheet effect or because of preference heterogeneity.\footnote{On the other hand, if consumption response to wealth changes is inherently related to leverage in a nonlinear way, aggregation is likely to exaggerate or mitigate the role of leverage.} Second, by aggregating the data we can gauge the extent to which estimates at the macro level, like those in Mian, Rao, and Sufi (2013), are likely to capture effects present at the micro level.

Column (3) in Table 1.2a shows that in our setting aggregation at the municipality level does not reduce the role of leverage. At the macro level, leverage is even more strongly associated with the consumption response to wealth changes. Column (4) shows that at the county level the point estimate of $\beta_5$ is about the same as that at the municipality level, but it is no longer statistically significant. Because there are only 19 counties in the data, the lack of significance is not surprising. Columns (1) and (2) in Table 1.2a restate some of the results that are reported in Table 1.1a. Interestingly, columns (2) and (3) show that adding fixed effects in slope at the household level yields an estimate of $\beta_5$ that is similar to that at the municipality level—both of the estimates are larger than the micro estimate without fixed effects in column (1). It is possible that at the household level the role of leverage is mitigated by differences in household preferences or by the nonlinearity of the relationship between leverage and the marginal propensity to consume. In Table 1.2b, where we replace the level
Table 1.2a: The Role of Housing Leverage at Different Aggregation Levels

<table>
<thead>
<tr>
<th>Dep.Var:</th>
<th>( \Delta C_t )</th>
<th>( \Delta W_t \times lev_{t-1} )</th>
<th>Baseline ( W_{t-1} )</th>
<th>Year#</th>
<th>Age#</th>
<th>CHAR#</th>
<th>FEIS</th>
<th>adj. ( R^2 )</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agg. Level:</td>
<td>Household</td>
<td>Household</td>
<td>Municipality</td>
<td>County</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta W_t \times lev_{t-1} )</td>
<td>0.226***</td>
<td>0.375***</td>
<td>0.348***</td>
<td>0.390</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.022)</td>
<td>(0.054)</td>
<td>(0.595)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline ( W_{t-1} )</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year#</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age#</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>CHAR#</td>
<td>X</td>
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<td>X</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>FEIS</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.291</td>
<td>0.231</td>
<td>0.939</td>
<td>0.950</td>
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<td></td>
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<tr>
<td>N</td>
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<td>1,191,995</td>
<td>2,147</td>
<td>95</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Notes. This table presents coefficients from regressions that relate the change in household consumption to the change in household wealth at an annual frequency between 2006 and 2011. Regressions in columns (1) and (2) are at the household level, and regressions in columns (3) and (4) are at the municipality and county levels, respectively. \( \Delta \) indicates change in millions of Norwegian Krones that are indexed to the 2000 price level. Leverage is defined as debt over housing value. # signifies that both the level of the term and its interaction with \( \Delta W_t \) are included. Year includes year dummies. CHAR includes terms of household characteristics. FEIS is fixed effect in the slope of \( \Delta W_t \). \( \bar{Y} \) is average household income in the sample. Baseline \( W_{t-1} \) refers to four terms involving \( \Delta W_t \), \( W_{t-1} \), \( \Delta W_t \times W_{t-1} \), and \( lev_{t-1} \). Age for columns (3) and (4) are the average age of households at the time and includes interaction with \( \Delta W_t \). Throughout, standard errors are in parentheses. *** indicates that coefficients are statistically different from 0 at the 1% confidence level.
Table 1.2b: The Role of Housing Leverage at Different Aggregation Levels

<table>
<thead>
<tr>
<th>Dep.Var:</th>
<th>( \Delta C_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agg. Level:</td>
<td>Household</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>( \Delta W_t \times lev_{t-1} )</td>
<td>0.206***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
</tr>
<tr>
<td>Baseline ( \frac{W_{t-1}}{Y_{t-1}} )</td>
<td>X</td>
</tr>
<tr>
<td>Year#</td>
<td>X</td>
</tr>
<tr>
<td>Age#</td>
<td>X</td>
</tr>
<tr>
<td>CHAR#</td>
<td>X</td>
</tr>
<tr>
<td>FEIS</td>
<td></td>
</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.335</td>
</tr>
<tr>
<td>N</td>
<td>1,346,264</td>
</tr>
</tbody>
</table>

Notes. This table presents coefficients from regressions that relate the change in household consumption to the change in household wealth at an annual frequency between 2006 and 2011. Regressions in columns (1) and (2) are at the household level, and regressions in columns (3) and (4) are at the municipality and county levels, respectively. \( \Delta \) indicates change in millions of Norwegian Krones that are indexed to the 2000 price level. Leverage is defined as debt over housing value. # signifies that both the level of the term and its interaction with \( \Delta W_t \) are included. Year includes year dummies. CHAR includes terms of household characteristics. FEIS is fixed effect in the slope of \( \Delta W_t \). \( \bar{Y} \) is average household income in the sample. Baseline \( \frac{W_{t-1}}{Y_{t-1}} \) refers to four terms involving \( \Delta W_t, W_{t-1}/Y_{t-1}, \Delta W_t \times W_{t-1}/Y_{t-1}, \) and \( lev_{t-1} \). Age for columns (3) and (4) are the average age of households at the time. Throughout, standard errors are in parentheses. *** indicates that coefficients are statistically different from 0 at the 1% confidence level.
of wealth with the wealth-to-income ratio, the results resemble those of Table 1.2a.

Our results at different aggregation levels and the fixed effects regressions indicate that the role of leverage is not due simply to preference heterogeneity. Moreover, the influence of leverage, over and beyond its correlation with wealth and possibly with preferences, cannot be explained within single-asset buffer-stock models of household consumption. In the remainder of this paper we develop a structural model that can account for the relationship between leverage and consumption dynamics.

1.4 Model

In this section, we develop a full-fledged consumption-saving life-cycle model with housing, debt, and financial assets. Housing leverage naturally emerges in our model and hence enables us to later explore its relation with the consumption response to wealth changes. Although our model is similar to that of Kaplan and Violante (2014), it differs in two main aspects. First, in our model, households buy housing because it is a consumption good and not because it provides a higher return than the risk free rate. Second, in our model debt consists primarily of mortgages and many households are levered. In Kaplan and Violante’s (2014) model, debt is unsecured borrowing, and because the interest rate on this debt is prohibitive, in practice there is little leverage.

We then estimate our model in two steps. First, we estimate one set of the model’s parameters externally. Second, we use the simulated method of moments to estimate
the remaining parameters that govern household preferences, such that our simulated median profiles of housing, debt and financial asset as well as homeownership rates are as close to the data as possible.

1.4.1 A Model of Housing, Debt, and Financial Assets

The economy consists of a continuum of households. Households begin their life cycle at age $a_0$ and exit at age $T$ with certainty.\footnote{In practice, we choose $a_0 = 27$ and $T = 90$ for the life cycle.} At each age $a$, there is risk of death and the conditional probability of survival is $p_a^S$. Households derive utility from consumption and bequest.

Households enjoy a bundle of housing and non-housing consumption, with a constant elasticity of substitution between the two:

$$\tilde{C}_a = \left[ \alpha_a^\theta C_a^{\theta-1} + (1 - \alpha_a)^{\frac{\theta}{2}} S_a^\theta \right]^{\frac{1}{\theta-1}}.$$

Here $\theta$ is the elasticity of substitution, $C_a$ is non-housing expenditure, and $S_a = \zeta H_a$ is the service flow from housing. $H_a$ is the stock of owner-occupied housing.

The weight on non-housing expenditure in the consumption bundle, $\alpha_a$, depends
on the household composition and thus varies with age. Specifically, we assume

\[ \alpha_a \propto \alpha \exp\{f_a N_a^{\text{Adult}} + f_c N_a^{\text{Children}}\} \]

with the normalization that \( \alpha_{a_0} = \alpha \), where \( \alpha \) is the initial weight on non-housing expenditure. \( N_a^{\text{Adult}} \) and \( N_a^{\text{Children}} \) are the number of adults and the number of children in the household at age \( a \), and \( f_a \) and \( f_c \) are parameters that capture their impact on the weight on non-housing consumption. This specification is chosen to capture how household size and household composition affect consumption over the life cycle. Because household composition varies, the flexible formulation of this specification allows us to estimate the equivalence scale in order to capture consumption per capita.\(^{6}\)

Households have a constant relative risk aversion (CRRA) utility function over consumption

\[ u(\tilde{C}_a) = \frac{\tilde{C}_a^{1-\rho}}{1-\rho} \quad \rho > 1. \]

When there is positive probability of death, households derive additional utility from

---

6 As emphasized in Attanasio, Banks, Meghir, and Weber (1999) and Cagetti (2003a), allowing demographics to affect household preferences can generate consumption profiles over age similar to those observed in household data.

7 For instance, Kaplan (2012) lists five equivalence scales that are used in different contexts. We do not take a stand on which one is the better; instead we estimate the influence of household composition on consumption choice.
leaving a bequest. We assume that the utility from bequests follows

\[ u^b(W_{a+1}) = \frac{\varphi W_{a+1}^{1-\rho}}{1-\rho}, \]

where \( W_{a+1} \) is wealth upon death and \( \varphi \) is the relative weight with which households value bequests. Each household therefore maximizes its expected discounted utility from consumption and bequest

\[ u(\tilde{C}_{a_0}) + E_{a_0} \left[ \sum_{a=a_0+1}^{T} \beta^{a-a_0} \left( p_a u(\tilde{C}_a) + (1 - p_a) u^b(W_a) \right) \right], \]

where \( \beta \) is the discount factor.

**Income Process**

Households have a permanent-transitory type of income process:

\[
Y_a = P_a \Xi_a \\
P_a = \Gamma_a P_{a-1} \Psi_a, \quad (1.3)
\]

where \( Y_a \) is after-tax income, \( P_a \) is the permanent component of income and \( \Xi_a \) is the transitory component of income at age \( a \). \( \Gamma_a \) is the deterministic growth rate of permanent income common to all households, and \( \Psi_a \) is the permanent shock to income.
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

We assume that transitory and permanent shocks are log-normally distributed

\[ \xi_a = \log \Xi_a \sim N\left(-\frac{\sigma_{\xi,a}^2}{2}, \sigma_{\xi,a}^2\right) \]
\[ \psi_a = \log \Psi_a \sim N\left(-\frac{\sigma_{\psi,a}^2}{2}, \sigma_{\psi,a}^2\right), \]

where \( \sigma_{\xi,a}^2 \) and \( \sigma_{\xi,a}^2 \) are age-varying variances of transitory and permanent shocks, respectively. Under these assumptions, \( E(\Xi_a) = E(\Psi_a) = 1 \).

Renters and Homeowners

Households can be renters or homeowners. They also make decisions about moving and house size. Renters can decide to remain renters or become homeowners in the next period. Homeowners can become renters, stay in their current house, or buy another house to move into in the next period. For transparency, we denote the five possible types of movements between renters and homeowners as \( rr, rh, hr, hh \) and \( hh' \) respectively. We assume that moving out of or into rented housing has no cost and that changes in owner-occupied housing imply a transaction cost. In particular, we assume that there are proportional transaction costs \( \kappa_p \) and \( \kappa_s \) that accompany housing purchase and sale.

Budget Constraints

When renters decide to remain renters for one more period, they allocate con-
sumption between non-housing expenditure \( C_a \) and housing service \( S_a \) in the current period. Their intertemporal budget constraint is

\[
A_a = M_a - C_a - S_a,
\]

where \( M_a \) is total market resources available at age \( a \) and \( A_a \) is the end-of-period assets. If renters decide to become homeowners, they must finance their housing purchase in addition to their current consumption:

\[
A_a = M_a - C_a - S_a - (1 + \kappa_p)H_{a+1}.
\]

Homeowners, in contrast, enjoy their housing, and if they do not move, all of their expenditure at the age \( a \) is non-housing expenditure. Moving introduces housing transactions to homeowners’ budget constraint. For instance, a homeowner who decides to become a renter (\( hr \)) sells her house, but during the current period she still enjoys the service flow from her current house:

\[
A_a = M_a + (1 - \kappa_s)H_{a+1} - C_a.
\]

\textit{Borrowing}
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

The borrowing rate $r_b$ is higher than the risk free interest rate $r$. There are three types of constraints. First, there is unsecured borrowing, wherein households are able to borrow up to a certain amount of their permanent income

$$A_a \geq -\mu_U P_a .$$

Second, there is a loan to value constraint

$$A_a \geq -\mu_V P^h h_{a+1} .$$

Third, there is a loan to income constraint

$$A_a \geq -\mu_Y PV_a ,$$

where $PV_a = E_t \left[ \frac{Y_{a+1}}{1+r_b} + \cdots + \frac{Y_T}{(1+r_b)^{T-a}} \right]$ is the present value of expected income in the future discounted at the borrowing rate. With respect to a household’s end-of-period assets, the loan to value constraint requires that debt cannot exceed a certain fraction of its current housing value, while the loan to income constraint requires that debt not exceed a certain fraction of the household’s expected future income.

Households’ Optimization Problem

Taking all the aforementioned details into account, we can express the households’
optimization problem as:

\[
\max_{\{C_a, S_a, H_a\}} u(\tilde{C}_a) + E_{a_0} \left[ \sum_{a=a_0+1}^{T} \beta^{a-a_0} \left( p_a^S u(\tilde{C}_a) + (1 - p_a^S) u^b(W_a) \right) \right]
\]

subject to

\[
A_a = \begin{cases} 
M_a - C_a - S_a & rr \\
M_a - C_a - S_a - (1 + \kappa_p)H_{a+1} & rh \\
M_a - C_a + (1 - \kappa_s)H_a & hr \\
M_a - C_a + (1 - \kappa_s)H_a - (1 + \kappa_p)H_{a+1} & hh' \\
M_a - C_a & hh \\
(1 + r)A_a + Y_{a+1} & A_a \geq 0 \\
(1 + r_b)A_a + Y_{a+1} & A_a < 0
\end{cases}
\]

\[
M_{a+1} = \begin{cases} 
(1 + r)A_a + Y_{a+1} & A_a \geq 0 \\
(1 + r_b)A_a + Y_{a+1} & A_a < 0
\end{cases}
\]

\[
W_a = M_a + H_a
\]

Here \(M_a\) is the liquid market resources households have at the beginning of age \(a\) and \(W_a\) is wealth inclusive of income at age \(a\). The only distinction between debt and financial assets in our model is their interest rate. When end-of-period assets, \(A_a\), are negative, households are in debt; when \(A_a\) is positive, households hold financial assets. Both debt and financial assets are liquid in the sense that households can run them up or down without cost, subject to the constraints.
1.4.2 First Step Estimation

Transaction Cost

In Norway, home buyers must pay a “document tax” that is 2.5% of the purchasing price. We therefore set $\kappa_p = 0.025$. The main cost of selling is the honorarium charged by real estate agents. The Financial Supervisory Authority of Norway reports the compensation collected by the main real estate agents in Norway since 2006.\footnote{See http://www.finanstilsynet.no/no/Eiendomsmegling/Informasjon/Statistikk/} Between 2006 and 2014, the average ratio of compensation to transaction value for house sales hovered around 2%. In addition, sellers normally pay for advertisement and sales insurance. Hence, we set $\kappa_s = 0.025$.\footnote{There are likely to be other costs associated with moving, such as the time spent searching for an attractive new house, preparing one’s house for sale, or settling down in a new home. By including only the observable components of transactions costs, we indicate that our parameterization of $\kappa_p$ and $\kappa_s$ is probably best seen as a lower bound.}

Deterministic Component of Income

It is well known that because of collinearity, age, year and cohort effects on income growth cannot be separately identified without making further assumptions (see for example Deaton and Paxson (1994)). Our data only span 7 years—not long enough to cover several business cycles. Thus we cannot assume that year effects are zero on average. Instead of making strong assumptions about the pattern of cohort effects, we choose the growth rates of mean after-tax income over age in the data as the deterministic growth rate of permanent income, $\left\{\gamma_a\right\}_{a=28}^{90}$. Using a third-order polynomial
we then obtain smoothed growth rates, which in the second step estimation we feed into our structural model through $\Gamma_a = 1 + \gamma_a$ in equation (1.3). By using this simple income profile we avoid making assumptions about the timing of retirement and the specification of the household pension scheme, both of which vary by cohorts.

The top left graph in Figure 1.2 displays our estimates of raw as well as smoothed mean (after-tax) labor income growth rates over the life cycle. When households enter the labor force around 27 there is a strong labor income growth, which implies that at the beginning of working life there is a sharp increase in the level of income. Labor income growth declines sharply until age 40 and then it falls moderately toward the end of the life cycle.

**Age-varying Labor Income Risk**

To estimate idiosyncratic labor income risks over the life cycle, we consider the following regression:

$$\log Y_{ia} = f_i + Z_{ia} \beta + y_{ia},$$

where $f_i$ is a household fixed effect and $Z_{ia}$ is a vector of observable household characteristics at age $a$. These include an age dummy, education, family size, family composition, marital status, nationality and geographical region. The object of interest here is $y_{ia}$, which is the unexplained stochastic component of labor income.

The dispersion of income during the early stages of life gives rise to differences in the timing of housing purchases. Allowing for age-dependent variances of permanent
Figure 1.2: Life Cycle Profiles of Household Labor Income and Demographics

Notes: In the top two graphs, the dashed lines are raw estimates from data. The solid line in the top left graph is the approximation by third-order polynomial. Solid lines in the top right graph are approximations by fourth-order polynomials.
and transitory shocks to income is therefore crucial for the ability of our model to match the data. The stochastic component of income follows

$$\Delta y_{ia} = \psi_{ia} + \Delta \xi_{ia}.$$ 

As shown in Blundell, Pistaferri, and Preston (2008a), age-varying variances of permanent shocks are identified by

$$\sigma^2_{\psi,a} = \text{Cov}(\Delta y_{ia}, \Delta y_{ia-1} + \Delta y_{ia} + \Delta y_{ia+1}),$$

and age-varying variances of transitory shocks are identified by

$$\sigma^2_{\xi,a} = -\text{Cov}(\Delta y_{ia}, \Delta y_{ia+1}).$$

Four years of data, from $a - 2$ to $a + 1$, are needed to identify the variance of permanent shocks at age $a$. Three years of data, from $a - 1$ to $a + 1$, are needed to identify the variance of transitory shocks. To estimate our structural life cycle model in the second step, we need age-dependent variances for ages 27 to 90. Because the age of households in the data ranges from 19 to 111, age-dependent variances can be identified.

The top right graph shows that while variation in the transitory component of labor income remains quite stable through life, variation in the permanent component
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of labor income declines sharply during the first decade of working life. At the beginning of working life, great variation in the levels of permanent income leads to a wide distribution of expected lifetime income. Close to age 40, the variance of transitory shocks starts to surpass that of permanent shocks, at which point transitory shocks become the dominant source of income uncertainty.

*Conditional Probability of Survival*

We assume that before the age of 67, which is the official retirement age, the probability of death is zero. Thereafter it is positive. Using official data on death rates, we calculate the conditional probability of survival for males during and after retirement. As characterized in the bottom left graph of Figure 1.2, the conditional probability of survival is averaged over 5 years and thus it appears as a step function. We use this conditional probability of survival over age as $p^S_a$ in our model.

*Household Composition*

Household size and composition vary over the life cycle. This demographic change affects both the shape of consumption profiles and the relative expenditure on non-housing and housing consumption. The bottom right graph of Figure 1.2 profiles how many adults and how many children households typically consist of at various stages of life cycle. Both are hump-shaped. We use these two profiles for $N^\text{Adult}_a$ and $N^\text{Children}_a$ in our model.
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Initial Distributions

We sort the net worth of 26-year-old households in ascending order and divide them into 20 equal-sized groups. For each of the 20 net worth groups, we calculate mean net worth, mean housing, mean income, and homeownership rate. Table 1.5 in Appendix 1.8.4 displays these statistics. In simulating household profiles in our model, we assume that households enter the life cycle with an equal probability of belonging to any given net worth group. Within each group, households start the life cycle with the group’s mean level of net worth and income, and if they are homeowners, their housing size is equal to the mean level of housing. The share of homeowners is equal to the homeownership rate in that group. In short, we calculate in the data a non-parametric joint distribution of net worth, housing, and income at the beginning of the life cycle, and the initial balance sheets of our simulated households are draws from that distribution.

Other Parameters

In Table 2.1 we list other first step estimates of our parameters, including the risk free interest rate, the borrowing rate, housing depreciation rate, and minimum housing.
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1.4.3 Second Step Estimation

In the second step estimation, we employ the simulated method of moments to estimate household preference parameters. These parameters include the coefficient of relative risk aversion \(\rho\), the discount factor \(\beta\), the elasticity between non-housing consumption and housing services \(\theta\), the share of non-housing consumption at the beginning of the life cycle \(\alpha\), the service flow from owner-occupied housing \(\zeta\), the weight on bequest \(\varphi\), and the influence of household composition on the share of non-housing consumption, \(f_a\) and \(f_c\).

Targets in the Data

Our objective is to develop a model that captures the dynamics of household balance sheets over the life cycle seen in the data. Therefore, it is natural to target the age profiles of median net worth and median housing in the data.\(^{[10]}\) Because housing gives rise to potential leverage, and because the relationship between leverage and the consumption response to wealth changes hinges crucially on the housing decision, the distribution of housing across households is important. We therefore include one additional set of moments: the age profiles of the homeownership rate. Altogether, we have 192 moments and 8 preference parameters.

\(^{[10]}\)In our model, there are two types of assets: housing \(H\) and financial assets \(A\). \(A\) captures households’ debt when it is negative and financial assets when it is positive. In the data, households often hold debt and financial assets simultaneously. For this reason, we do not target debt explicitly.
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Initial Values of Parameters

To minimize the cost of computation and increase the probability of finding the global minimum, we start our search of parameters with values that are consistent with macro and micro evidence under some simplifying assumptions of the model. Appendix 1.8.5 shows that under these assumptions $\alpha$, $\theta$, $\zeta$, $f_a$, and $f_c$ can be estimated directly. We use these estimates as the starting point of our estimation.

Estimation Results

Table 2.1 displays our estimation results. All of our preference estimates are in line with the literature. For example, our estimated coefficient of relative risk aversion, $\rho$, is less than 2—a result that is similar to Chetty (2006). Because there is no analytical solution to our model, we briefly discuss the identification of the parameters. The average level of median net worth over the life cycle provides identification of the discount factor $\beta$ because a more patient household would on average hold more wealth. The curvature of the age profile of net worth pins down the coefficient of relative risk aversion $\rho$. Because $1/\rho$ is the intertemporal elasticity of substitution, a higher $\rho$ implies less wealth accumulation at the beginning of the life cycle and more in the middle. Utility from housing $\zeta$ is nailed down by the average level of median housing, and the elasticity of substitution between non-housing and housing consumption $\theta$ is determined by the curvature of the age profile of median housing. The parameter that governs non-housing consumption, $\alpha_a$, is identified by the share
Table 1.3: Parameter Values of the Model Economy

<table>
<thead>
<tr>
<th>Estimates</th>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First Step</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demographics</td>
<td>$T$</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Lifespan</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional probability of survival</td>
<td>${p_{a}^{S}}$</td>
<td>Figure 1.2</td>
<td>SSB*</td>
</tr>
<tr>
<td>Mean number of adults</td>
<td>${N_{a}^{Adult}}$</td>
<td>Figure 1.2</td>
<td>Data</td>
</tr>
<tr>
<td>Mean number of children</td>
<td>${N_{a}^{Children}}$</td>
<td>Figure 1.2</td>
<td>Data</td>
</tr>
<tr>
<td>Income process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permanent income growth rate</td>
<td>${\Gamma_{t}}$</td>
<td>Figure 1.2</td>
<td>Data</td>
</tr>
<tr>
<td>Variance of permanent income</td>
<td>${\sigma_{\Psi,t}}$</td>
<td>Figure 1.2</td>
<td>Data</td>
</tr>
<tr>
<td>Variance of transitory income</td>
<td>${\sigma_{\Xi,t}}$</td>
<td>Figure 1.2</td>
<td>Data</td>
</tr>
<tr>
<td>Borrowing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td>$r$</td>
<td>0.016</td>
<td>Norges Bank</td>
</tr>
<tr>
<td>Borrowing rate</td>
<td>$r_{b}$</td>
<td>0.054</td>
<td>Norges Bank</td>
</tr>
<tr>
<td>Maximum loan to value ratio</td>
<td>$\mu_{V}$</td>
<td>0.90</td>
<td>Norges Bank</td>
</tr>
<tr>
<td>Maximum debt to lifetime income ratio</td>
<td>$\mu_{Y}$</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Housing market</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>2%</td>
<td></td>
</tr>
<tr>
<td>Transaction cost of purchase</td>
<td>$\kappa_{p}$</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>Transaction cost of sale</td>
<td>$\kappa_{p}$</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>Minimum housing</td>
<td>$\bar{h}$</td>
<td>8.2</td>
<td>Data**</td>
</tr>
<tr>
<td><strong>Second Step</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial weight on consumption</td>
<td>$\alpha$</td>
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<td></td>
</tr>
<tr>
<td>Adults’ impact on consumption weight</td>
<td>$f_{a}$</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>Children’s impact on consumption weight</td>
<td>$f_{a}$</td>
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<td>Discount factor</td>
<td>$\beta$</td>
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<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\rho$</td>
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<tr>
<td>Elasticity of substitution</td>
<td>$\theta$</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>Utility of owning</td>
<td>$\zeta$</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>Bequest weight</td>
<td>$\varphi$</td>
<td>12.3</td>
<td></td>
</tr>
</tbody>
</table>

*Statistics Norway.
** We use the 5th percentile of housing value in the data.
of non-housing consumption as well as the homeownership rates. The relative weight with which households value bequest, \( \varphi \), is driven by the level of net worth at the end of the life cycle.

### 1.5 Model vs. Data

We now evaluate how well our quantitative model performs in fitting the life cycle profiles of household balance sheets in the data. Thereafter, we explore the model-implied relationship between leverage and households’ marginal propensity to consume out of wealth changes, and we compare it to the regression results in section 1.3.2.

#### 1.5.1 Life Cycle Profiles

Figure 1.3 shows that under the estimated parameters, our model generates life cycle balance sheet profiles that resemble those in the data.

First, the hump-shaped profile of median net worth, which reflects wealth accumulation before retirement and decumulation after, exists both in the data and in the model. Three factors are at work here. Labor income uncertainty induces precautionary savings early in life. Decreasing income growth rates over the life cycle lead to savings for retirement in midlife. Bequest motives prevent households from depleting their wealth after retirement.
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Figure 1.3: Household Balance Sheet Over the Life Cycle: Model and Data

Notes: Throughout, household profiles are in 100k Norwegian Krones, indexed to the 2000 price level.
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Second, median housing wealth is hump-shaped in the data as well as in the model. Note that the median profile of housing wealth is not tracking a single household. Thus the rise in the level of housing before the age of 60 partly reflects an increasing homeownership rate and partly reflects housing upgrades among existing homeowners. The fall in the level of housing afterwards indicates that households late in the life cycle move into houses of smaller size. Except for the early years of the life cycle, median housing wealth in the model closely tracks the data. In fact, the discrepancy in the early years is probably due to our abstracting from house price dynamics. As Table 1.5 shows, households in the first few net worth deciles have negative net worth and high levels of housing. Perhaps these households hold on to a high level of housing when their net worth is low because they expect house price appreciation in the future. In our model—and in the absence of house price appreciation—owning so much housing wealth when net worth is low is not optimal.

Third, in our model house purchases are mortgage-financed for a median household. Debt is present over most of the life cycle. The median household only pays off debt near retirement. Although we do not directly target the debt in the data, our model fits the life cycle profile of debt quite well. This gives us confidence that the leverage ratio in our model is similar to that of the data, despite the fact that our model summarizes mortgage debt and financial assets in a single variable.

Finally, although our model generates the correct average homeownership rate, it does not capture the shape of the homeownership rate over the life cycle. This
is probably due to the fact that our model does not acknowledge that there is heterogeneity in the preference for owning. In our model, old households rarely rent for two reasons: first, owning provides higher utility than renting per housing unit; second, as a bequest, housing is almost as good as financial assets. Consequently, the homeownership rate in our model rises slightly over the life cycle. Of course, the fact that old households have a disutility from owning a house that they cannot maintain might explain why in the data homeownership declines after retirement.

Regarding the cross-sectional distribution of net worth and leverage, a comparison of Figure 1.4b which is simulated from the model, and Figure 1.4a which displays the Norwegian data, reveals that our model captures the heterogeneity in the data reasonably well.
1.5.2 Leverage and the Consumption Response to Wealth Changes

Next we address our main question, which regards the importance of leverage for the marginal propensity to consume. Focusing on data generated by our model, we repeat the exercises undertaken in Section 1.3.2. To render the sample comparable to that in the actual data, we restrict our analysis to simulated households that are between 30 and 80 years of age. To control for household characteristics, we use age polynomials up to the third order and their interaction terms with the change in wealth. Note that by construction there is no preference heterogeneity among households of the same age in our model.

Table 1.4 presents the regression results from our model along with the key estimates from the Norwegian data that we saw previously in Table 1.1a and Table 1.1b. Again, the main parameter of interest is the interaction effect reported in line 8 of the table. We see in both the simulated data and the actual data that the role of leverage for the consumption response to wealth changes is very similar: that is, the coefficients of the interaction effect are both statistically significant and economically important (the point estimate slightly above 0.2). Given our model’s simplicity and in particular the fact that it relies on limited preference heterogeneity, this result is

---

11 The age distribution is almost uniform in our simulated data but far from uniform in the actual data. As a result, our simulated data have many more young households and old households. In our simulated data, about 20% households are under age 30 or above 80; in the data, these age groups account for less than 10% of households. To mitigate this issue, we drop households at the very beginning and at the very end of the life cycle.
somewhat surprising. Our model generates heterogeneous balance sheets only by carefully modeling the heterogeneity in housing choices. The regression results indicate that this source of balance sheet heterogeneity helps to account for the empirically observed link between leverage and households’ heterogeneous consumption responses to wealth changes.

As is evident in Table 1.4, there are some notable discrepancies between coefficients in the simulated and the actual data. A model that successfully captures the entire distribution of wealth across households at different ages in the data probably would produce similar results, but that is beyond the scope of this paper. We leave it for future research.

In our model household balance sheets and age affect the heterogeneity in the marginal propensity to consume out of wealth. To understand the role of leverage, we focus on households that are of a small age group. Specifically, we consider households whose age is between 30 and 35, which is the time when most of them have leverage. We first sort simulated households by their wealth-to-income ratios and we then divide them into 8 equal-sized groups in ascending order. Within each wealth-to-income-ratio group (WG), we next sort households by their leverage and divide them further into 8 equal-sized groups in ascending order. This partition leaves us with 64 wealth-leverage groups of equal mass. For each of these groups, we calculate the average marginal propensity to consume out of wealth. Figure 1.5 reveals graphically the role played by leverage. Within each WG, the average MPC tends to increase
as the leverage group increases from 1 to 8. The role of leverage is most apparent for groups that have a low wealth-to-income ratio because less wealthy households tend to have higher leverage. There is, however, a notable subtlety in Figure 1.5: for households that have lower leverage (such as those in leverage group 1), there is a greater probability that in the near future their liquid wealth will turn from negative to positive and thus the interest rate on their liquid wealth will change from the high borrowing rate to the low risk free rate. The potential decrease in the interest rate will give a small boost to the MPC out of wealth. As wealth increases, the possibility that the interest rate will decrease also increases. Therefore, we see in the figure that the MPC increases in wealth for the low leverage groups.

1.6 Policy Implications

1.6.1 Abrupt Credit Tightening

In the wake of the Great Recession, many economists have studied how credit crunches affect the economy. In our calibrated model, we focus on the quantitative implication of a particular type of reduction in credit availability: a sudden reduction in loan-to-value (LTV) limit. As Geanakoplos and Fostel (2008) and Geanakoplos (2010, 2014) emphasize, the abrupt change of loan-to-value requirement on new loans is a crucial source of economic crashes.

Our model is well suited at assessing the immediate response of the economy
## Table 1.4: The Role of Leverage in the Model and in the Data

<table>
<thead>
<tr>
<th>Dep.Var:</th>
<th>(1) Simulation Data</th>
<th>(2) Simulation Data</th>
<th>(3) Simulation Data</th>
<th>(4) Simulation Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta W_t$</td>
<td>0.527*** (0.042)</td>
<td>0.531*** (0.106)</td>
<td>0.203*** (0.042)</td>
<td>0.804*** (0.107)</td>
</tr>
<tr>
<td>$W_{t-1}$</td>
<td>-0.004*** (0.000)</td>
<td>-0.096*** (0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta W_t \times W_{t-1}$</td>
<td>-0.000 (0.000)</td>
<td>0.008*** (0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{W_{t-1}}{Y_{t-1}}$</td>
<td></td>
<td>0.034*** (0.000)</td>
<td>-0.012*** (0.000)</td>
<td></td>
</tr>
<tr>
<td>$\Delta W_t \times \frac{W_{t-1}}{Y_{t-1}}$</td>
<td></td>
<td>0.017*** (0.000)</td>
<td>-0.010*** (0.000)</td>
<td></td>
</tr>
<tr>
<td>$lev_{t-1}$</td>
<td>-0.091*** (0.005)</td>
<td>-0.337*** (0.001)</td>
<td>0.147*** (0.005)</td>
<td>-0.253*** (0.001)</td>
</tr>
<tr>
<td>$\Delta W_t \times lev_{t-1}$</td>
<td>0.200*** (0.005)</td>
<td>0.226*** (0.002)</td>
<td>0.259*** (0.004)</td>
<td>0.206*** (0.002)</td>
</tr>
<tr>
<td>Year#</td>
<td>X</td>
<td>X</td>
<td></td>
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<tr>
<td>$\bar{Y}$#</td>
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<td>CHAR#</td>
<td>X</td>
<td>X</td>
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<td>X</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.316</td>
<td>0.346</td>
<td>0.346</td>
<td>0.335</td>
</tr>
<tr>
<td>N</td>
<td>144,246</td>
<td>1,346,264</td>
<td>144,246</td>
<td>1,346,264</td>
</tr>
</tbody>
</table>

Notes. This table presents coefficients from regressions relating the change in household consumption to the change in household wealth in the simulated data. For comparison purposes, column (2) reproduces the result in column (3) of Table 1.1a and column (4) reproduces the result in column (4) of Table 1.1b. Throughout, standard errors are in parentheses. *** indicates that coefficients are statistically different than 0 at the 1% confidence level.
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Figure 1.5: Average MPC by Wealth and Leverage Groups in the Simulated Data

Notes: This graph presents the average marginal propensity to consume out of wealth for households whose age in the simulated data is between 30 and 35. The wealth-to-income ratio is sorted ascendingly into 8 equal-sized groups. Within each wealth-to-income-ratio group, leverage is sorted ascendingly into 8 equal-sized groups. Group 1 on either axis has the smallest values of the corresponding variable.
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to a policy change. Being a partial equilibrium model, it falls short of depicting completely the short and medium run dynamics of consumption and housing choices. Nonetheless, with its close attention to micro-level adjustment, our model provides important quantitative insights into the channels through which the effect of credit crunch takes place.

We consider the immediate response of households when the LTV limit on new loans is reduced from 90% to 80% on new loans. We assume that there are two periods. In period 0, households are at their optimal choice of consumption and housing; the LTV constraint is 90%. At the beginning of period 1, there is a sudden and permanent change in the LTV limit from 90% to 80%. We then compare how households respond with and without the period 1 policy change.

The first three graphs in Figure 1.6 depict the percentage change in some key variables under the low LTV limit and under no policy change. The top left graph in Figure 1.6 shows the main contractionary effect of the policy change on the housing market. Under the policy change the number of housing transactions in our simulated economy falls by 10.2%. Most of the fall is concentrated among young households. The top right graph, in contrast, shows that rather than falling, consumption for almost all households increases by about 1%. Intuitively, a sudden increase in the down payment requirement from 10% to 20% forces some potential buyers, particularly the young, to delay their housing purchases. Young renters who can no longer afford homeownership increase their non-housing consumption. Existing homeowners who
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would otherwise increase their housing stock find themselves constrained by the new policy, which applies to new loans, and they therefore hold on to their existing houses and increase non-housing consumption instead. The implication for debt, however, is different among households. On average, the subdued housing transactions lead, on the one hand, to less mortgage debt among young households (due to delayed housing purchases) and, on the other, to increased borrowing among existing homeowners (due to increased consumption), as is shown in the bottom left graph. The bottom right graph shows that the impact on the average leverage of the economy is limited (a reduction of 5%).

Our results suggest that a realistic reduction in credit availability in mortgages alone leads to a quantitatively moderate consumption response and a large housing transaction response. In fact, consumption increases by a small amount, housing transactions drop abruptly, and the debt-to-housing ratio barely changes. Given the potential general equilibrium effect, reduced activity in the housing market probably places downward pressure on house prices, which in turn decreases households’ wealth and reduces households’ borrowing capacity. The decrease in house prices, when it is sufficiently large, will put highly levered households underwater and have further ramifications for consumption. This recalls features of the onset of the Great Recession: when subprime mortgages in the U.S. turned bad in 2006, down payment requirements for new loans dramatically increased, and housing prices started to fall. However, consumption continued to increase until 2008, when it fell sharply. The
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Figure 1.6: Impact of Tightening in LTV Constraint

Notes. This figure presents the impact of a tightening of loan-to-value constraint from 90% to 80%. We compare households’ behavior under 80% LTV constraint in period 1 to their behavior under 90% LTV constraint in the same period.

debt-to-GDP ratio plateaued in late 2007.

In our policy exercise the channel through which a credit crunch affects the economy differs from those considered in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011). These papers examine how a reduction in credit availability to consumers pushes the economy to the zero lower bound, which exacerbates the fall in consumption and aggregate demand. But the models therein are very stylized...
and do not include leverage. An exogenous reduction in the debt limit in these models forces net-worth-constrained households to repay their debt and thus it reduces consumption. In our model, leverage is endogenous and households hold assets and debt simultaneously. From the perspective of household balance sheets at the micro level, credit availability is quantitatively far more important for housing than is its direct effect on consumption. We conclude that, in addition to the zero lower bound channel, the housing wealth channel might be key to understanding the effect of a reduction in credit availability.

1.6.2 Consumption Responsiveness and the Loan-to-Value Ratio

One widespread narrative of the Great Recession is that negative shocks to household wealth were propagated by high household leverage. Indeed, this is a natural interpretation of the regression results provided by Mian, Rao, and Sufi (2013), Baker (2015), and in this paper. Partly for this reason, policymakers have been urged to implement tighter regulations that restrict the loan-to-value ratios at which banks issue mortgages to households. However, to address the effectiveness of such policies, one cannot simply rely on evidence from the past, but must use structural models. Our framework is well suited to give a partial equilibrium answer to this question, and we ask: Is consumption less sensitive to wealth changes in a world where loan-to-value
limit are low?

To answer this question, we compare the marginal propensity to consume out of wealth in steady states that differ in terms of their LTV-ratios only. We consider three ratios: 70%, 80% and 90%. Figure 1.7 shows the result for each age group separately.

First, we note that the MPC always displays a U-shaped pattern over the life cycle. Three factors are at work here. First, wealth accumulation over the life cycle helps households build up buffers against income risk and hence decreases MPC. Second, as the remaining life horizon shortens, the resolution of uncertainty increases the MPC. These two factors drive MPC in opposite directions over the life cycle. Our calibrated model implies that when households are young wealth accumulation dominates the age-MPC relationship, while thereafter the second effect dominates. The third factor is leverage, and its role differs by age; it is closely related to the timing of when housing is affordable.

More importantly, we see that economies with different LTVs do not have very different marginal sensitivities of consumption to wealth changes. A tighter LTV policy reduces the MPC for young households only and it raises slightly the MPC for middle-aged and older households. The key intuition here is that households make joint decisions about their housing and financial wealth holdings. When the LTV limit is high (low down payment), young households are able to finance a housing purchase with a large mortgage. When the LTV limit is low (high down payment),
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Figure 1.7: Marginal Propensity to Consume Out of Wealth

young households simply postpone housing purchases until they have accumulated enough wealth. In either case, their optimal choice is to finance their housing with as much mortgage as possible. As a result, young households under a low LTV limit tend to hold more liquid balance sheets and so their consumption responds less strongly to wealth changes. However, when the purchase decision is postponed, the propagation of illiquidity on consumption sensitivity simply shifts to a later point in life. We therefore see that when there is a lower LTV limit, the MPC becomes slightly higher for middle-aged and older households.$^{12}$

$^{12}$The average MPC increases somewhat even for households above 50 years of age is because a tighter LTV limit affects consumption at all wealth levels, although the effect is small for the wealthiest of households.
This can be best understood by examining Figure 1.8, which compares the distribution of wealth and leverage under the highest and lowest LTVs.\textsuperscript{13} The left graph shows that the distribution of wealth in the economy almost does not change under different LTVs. Instead, what changes is the composition of household balance sheets. For a given LTV constraint, households optimally choose their leverage to balance their need for liquidity and housing. The right graph shows, not surprisingly, that under the low LTV ratio the distribution of leverage shifts to the left, which indicates that the average leverage is now smaller. However, the share of households that are close to the LTV constraint is not lower, and this is what really matters for the average MPC of the economy. In fact, calculation of our simulated economy reveals

\textsuperscript{13}We use kernel density estimation to approximate the probability density function of wealth. Leverage is bounded from above by the LTV constraint. However, due to kernel smoothing, the density of leverage that exceeds the LTV ratio in the graph is positive.
that under the 70% LTV constraint, 5.9% of households are within the top 20% of the LTV constraint. Under the 90% LTV constraint, only 4.2% are within the top 20%.

Note that here we take the magnitude of wealth shocks as given. Thus, our exercise does not account for the fact that household wealth itself becomes more sensitive to asset price movements when leverage is high. Our results indicate that this is the channel through which more stringent LTV limits might dampen consumption’s sensitivity to various shocks.

1.7 Conclusion

We provide new empirical evidence at the micro level that the composition of household balance sheets and especially their housing leverage ratio matters for the marginal propensity to consume out of wealth over and above their total wealth. We find that this relationship between leverage and the consumption response to wealth changes is not primarily driven by heterogeneity in observed or unobserved household characteristics. Such balance-sheet effects are not present in the conventional single-asset buffer stock saving models. We therefore develop a model that can quantitatively account for the life cycle profile of household balance sheets. The ability of the model

\[\text{For the economy with 90\% LTV constraint, the top 20\% of the LTV constraint is the share of population whose leverage is between 0.72 and 0.9; for the economy with 70\% LTV constraint, it is between 0.56 and 0.7. The share of population who has positive leverage is similar, 64\% and 63.2\% under 90\% and 70\% LTV constraint, respectively.}\]
to reasonably match housing choices is essential for its capacity to match balance sheet profiles. Our model successfully accounts for the empirical association between leverage and the marginal propensity to consume out of wealth. The key mechanism is that transaction costs in the housing market induce households that are moving along the housing ladder to adjust housing stock infrequently. Housing is therefore less liquid than financial wealth. Housing transactions imply a shift in the liquidity of household balance sheets. Thus, recent home buyers who have increased their housing stock are closer to their liquidity constraint, have excessively high housing relative to non-housing consumption, and have a strong desire to increase their non-housing consumption share. In contrast, recent home buyers who have reduced their housing stock are further from their liquidity constraint, have low leverage and low housing consumption, and have a weak desire to increase their non-housing consumption share. Given the important role that leverage plays in distinguishing between liquid and illiquid assets and in explaining consumption dynamics, we aim in future research to extend our analysis to the marginal propensity to consume out of different types of assets.
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1.8 Appendices

1.8.1 Details of the Norwegian Registry Data

1.8.1.1 Administrative Tax Records

Because households in Norway are subject to a wealth tax, they are required to report every year their complete wealth holdings to the tax authority, and the data are available every year from 1993 up until present time. Each year, before taxes are filed (the year after) in April, employers, banks, brokers, insurance companies and any other financial intermediaries are obliged to send both to the individual and to the tax authority, information on the value of the asset owned by the individual and administered by the employer or the intermediary, as well as information on the income earned on these assets. In case an individual holds no stocks, the tax authority pre-fills a tax form and sends it to the individual for approval; if the individual does not respond, the tax authority considers the information it has gathered as approved. In 2009, as many as 2 million individuals in Norway (60% of the tax payers) belonged to this category. If the individual or household owns stocks then he has to fill in the tax statement - including calculations of capital gains/losses and deduction claims. The statement is sent back to the tax authority which, as in the previous case receives all the basic information from employers and intermediaries and

\[15\] In Norway the individuals in a household are taxed jointly when it comes to the wealth tax, while separately for the income tax.
can thus check its truthfulness and correctness. Stockholders are treated differently because the government wants to save on the time necessary to fill in more complex tax statements. This procedure, particularly the fact that financial institutions supply information on their customer’s financial assets directly to the tax authority, makes tax evasion very difficult, and thus non-reporting or under-reporting of assets holdings are likely to be negligible.

Cars, boats and other motor vehicles are reported in the tax record with standardized list values depending on brand and year of production. The list value in the first year after purchase is about 75% of the market value, thereafter most list values decline on average 10 percentage points each year. Where the depreciation is not already given by declining tax values, we compute an annual depreciation rate of 10 percent.

1.8.1.2 Housing Values

Income from housing in the income tax base was abolished in 2005 in Norway. However, the imputed income was based on tax values for housing that had a weak relation to actual market prices. The same tax values were used as a basis for the wealth tax. Tax values for housing for the period 1993-2009 were on average about 20% of market prices.

Individual variation was primarily linked to the construction year of the house. Old, refurbished villas in attractive neighborhoods could in some cases have tax values
close to zero. Furthermore, the tax values were adjusted irregularly. As a result, the
tax values were not useful as approximations of actual housing values. However,
imputations of housing values based on hedonic price regressions are available from
2005 (see Kostøl and Holiløkk 2010; Thomassen and Melby 2009). From 2010 these
values were also implemented as basis for wealth taxation in the tax records (that
is, the tax value is set to 25% of the imputed market value). In the imputation of
consumption we define one measure using these data from 2005 to 2011. To mitigate
potential measurement errors in household assets we exclude year observations of
households that have reported relocation to the address register, since this is likely
to be years in which the household has traded housing (where we would observe fully
the change in mortgage but not the corresponding purchase or selling price).

The housing stock also depreciates over time, but unlike cars and household
durables, it rarely deteriorates completely. Instead, it is common to undertake ir-
regular major refurbishment in order to get the housing stock in line with modern
standards. This lumping of maintenance costs, often financed by remortgaging, rep-
resents a measurement problem in our data since the market value does not represent
the exact individual housing values. Market housing values, when available, are based
on housing attributes such as location, type, size and age.

Holiday homes, on the other hand, are still reported with tax values that are far
below actual market values. This is why we also choose to exclude year observations
of households who trade vacation homes.

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1.8.2 Details of the Households’ Problem

In this section, we describe in detail the dynamic stochastic optimization problems of households introduced in Section 1.4.

At each age \( a \), renters decide whether to rent or own next year. The decision, although regarding the future, will affect the value function of renters in the current period. Denote the value functions conditional on renting or owning next year \( V_{x}^{rr} \) and \( V_{x}^{rh} \) respectively. The value function of renters at age \( a \) is

\[
V_{a}^{r} = \max \{ V_{a}^{rr}, V_{a}^{rh} \}.
\]

Similarly, the value function of homeowners at age \( a \) is

\[
V_{a}^{h} = \max \{ V_{a}^{hr}, V_{a}^{hh}, V_{a}^{hh'} \}.
\]

where \( hr, hh, hh' \) denote the decision of homeowners to rent, stay, and switch to another house next year. We assume that households have bequest motive only after retirement when there is positive probability of death. In particular, the expected value function of households with bequest motive is

\[
E_{a} V_{a+1}^{bh} = p_{a+1}^{S} E_{a} V_{a+1}^{bi} (M_{a+1}, H_{a+1}, P_{a+1}) + (1 - p_{a+1}^{S}) u^{h}(W_{a+1}), \quad i = r, h.
\]
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where

\[ u^b(W_{a+1}) = \varphi \frac{W_{a+1}^{1-\sigma}}{1 - \sigma}. \]

is the utility of bequests and \( W_{a+1} = A_a + H_{a+1} \) is wealth upon death. \( \varphi \) is the relative weight households value bequests and \( \sigma \) governs the elasticity of bequests with respect to wealth. For simplicity, we assume that \( \sigma = \rho \).

The relationship among different value functions is:

\[
\begin{align*}
V_{rr}^a &= \max_{\tilde{C}_a} u(\tilde{C}_a) + \beta E_a V_{a+1}^{b,r}, \\
V_{rh}^a &= \max_{\tilde{C}_a, H_{a+1}} u(\tilde{C}_a + H_{a+1}) + \beta E_a V_{a+1}^{b,h}, \\
V_{hr}^a &= \max_{\tilde{C}_a} u(\tilde{C}_a + \beta E_a V_{a+1}^{b,r}), \\
V_{hh}^a &= \max_{\tilde{C}_a} u(\tilde{C}_a + \beta E_a V_{a+1}^{b,h}), \\
V_{hh}'^a &= \max_{\tilde{C}_a, H_{a+1}} u(\tilde{C}_a + \beta E_a V_{a+1}^{b,h}).
\end{align*}
\]

We now characterize the first order conditions and the envelope conditions of each type of movements.

Case I: renter to renter (rr)

The intratemporal optimal conditions are:

\[
\begin{align*}
\frac{\partial \tilde{C}_a}{\partial C_a} &= \left( \frac{\alpha \tilde{C}_a}{C_a} \right)^{\frac{1}{\sigma}} = \lambda, \\
\frac{\partial \tilde{C}_a}{\partial S_a} &= \left( \frac{(1 - \alpha)\tilde{C}_a}{S_a} \right)^{\frac{1}{\sigma}} = \lambda.
\end{align*}
\]

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Combined with the budget constraint $C_a + S_a = \tilde{C}_a$, we have

$$C_a = \alpha_a \tilde{C}_a,$$

$$S_a = (1 - \alpha_a) \tilde{C}_a.$$

The intertemporal optimal condition is:

$$u' (\tilde{C}_a) = \beta \left( p_{a+1}^S (1 + r_M) E_a \left[ \frac{\partial V_{t+1}^r}{\partial M_{a+1}} \right] + (1 - p_{a+1}^S) \varphi W_{a+1}^{-\sigma} \right),$$

with envelope condition:

$$\frac{\partial V_{t+1}^{rr}}{\partial M_a} = \beta \left( p_{a+1}^S (1 + r_M) E_a \left[ \frac{\partial V_{t+1}^r}{\partial M_{a+1}} \right] + (1 - p_{a+1}^S) \varphi W_{a+1}^{-\sigma} \right) = u'(\tilde{C}_a).$$

Case II: renter to homeowner ($r^h$)

The first order condition with respect to $\tilde{C}_a$ is

$$u' (\tilde{C}_a) = \beta \left( p_{a+1}^S (1 + r_M) E_a \left[ \frac{\partial V_{t+1}^{h}}{\partial M_{a+1}} \right] + (1 - p_{a+1}^S) \varphi W_{a+1}^{-\sigma} \right).$$

The first order condition with respect to $H_{t+1}$ is

$$\beta \left( p_{a+1}^S E_a \left[ \frac{\partial V_{t+1}^{h}}{\partial M_{a+1}} \right] (- (1 + \kappa_p)(1 + r_M)) + \frac{\partial V_{t+1}^{h}}{\partial H_{a+1}} \right) + (1 - p_{a+1}^S) \varphi W_{a+1}^{-\sigma} (-\kappa_p) = 0.$$
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The envelope condition is:

$$\frac{\partial V^r}{\partial M_a} = u'(\tilde{C}_a).$$

Case III: homeowner to renter ($hr$)

The first order condition is:

$$u'(\tilde{C}_a) \frac{\partial \tilde{C}_a}{\partial C_a} = \beta \left( P_{a+1}^S(1 + r_M)E_a \left[ \frac{\partial V^h}{\partial M_{a+1}} \right] + (1 - P_{a+1}^S)\varphi W^{-\sigma}_{a+1} \right).$$

The set of envelope conditions is:

$$\frac{\partial V^{hr}}{\partial M_a} = \beta \left( P_{a+1}^S(1 + r_M)E_a \left[ \frac{\partial V^r}{\partial M_{a+1}} \right] + (1 - P_{a+1}^S)\varphi W^{-\sigma}_{a+1} \right),$$

$$\frac{\partial V^{hr}}{\partial H_a} = u'(\tilde{C}_a) \frac{\partial \tilde{C}_a}{\partial H_a} + \beta(1 - \kappa_s) \left( P_{a+1}^S(1 + r_M)E_a \left[ \frac{\partial V^r}{\partial M_{a+1}} \right] + (1 - P_{a+1}^S)\varphi W^{-\sigma}_{a+1} \right).$$

Case IV: homeowner staying ($hh$)

The first order condition is

$$u'(\tilde{C}_a) \frac{\partial \tilde{C}_a}{\partial C_a} = \beta \left( P_{a+1}^S(1 + r_M)E_a \left[ \frac{\partial V^h}{\partial M_{a+1}} \right] + (1 - P_{a+1}^S)\varphi W^{-\sigma}_{a+1} \right).$$
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The set of envelope conditions is:

\[ \frac{\partial V_{hh}^a}{\partial M_a} = \beta \left( p_{a+1}^s (1 + r_M) E_a \left[ \frac{\partial V_{a+1}^h}{\partial M_{a+1}} \right] + (1 - p_{a+1}^s) \varphi W_{a+1}^{-\sigma} \right), \]

\[ \frac{\partial V_{hh}^a}{\partial H_a} = \frac{u'}{\partial \tilde{C}_a} \frac{\partial \tilde{C}_a}{\partial H_a} + \beta (1 - \delta) \left( p_{a+1}^s E_a \left[ \frac{\partial V_{a+1}^h}{\partial H_{a+1}} \right] + (1 - p_{a+1}^s) p_{a+1}^h \varphi W_{a+1}^{-\sigma} \right). \]

Case V: homeowner moving \((hh')\)

The set of first order conditions are

\[ u'(\tilde{C}_a) \frac{\partial \tilde{C}_a}{\partial C_a} = \beta \left( p_{a+1}^s (1 + r_M) E_a \left[ \frac{\partial V_{a+1}^h}{\partial M_{a+1}} \right] + (1 - p_{a+1}^s) \varphi W_{a+1}^{-\sigma} \right). \]

\[ \beta \left( p_{a+1}^s E_a \left[ \frac{\partial V_{a+1}^h}{\partial M_{a+1}} \right] - (1 + \kappa_p)(1 + r_M) + \frac{\partial V_{a+1}^h}{\partial H_{a+1}} \right] + (1 - p_{a+1}^s) \varphi W_{a+1}^{-\sigma} p_{a+1}^h (-\kappa_p) = 0. \]

The set of envelope conditions is:

\[ \frac{\partial V_{hh'}^a}{\partial M_a} = \beta \left( p_{a+1}^s (1 + r_M) E_a \left[ \frac{\partial V_{a+1}^h}{\partial M_{a+1}} \right] + (1 - p_{a+1}^s) \varphi W_{a+1}^{-\sigma} \right), \]

\[ \frac{\partial V_{hh'}^a}{\partial H_a} = u'(\tilde{C}_a) \frac{\partial \tilde{C}_a}{\partial H_a} + \beta (1 - \kappa_s) \left( p_{a+1}^s (1 + r_M) E_a \left[ \frac{\partial V_{a+1}^h}{\partial M_{a+1}} \right] + (1 - p_{a+1}^s) \varphi W_{a+1}^{-\sigma} \right). \]

Consumption and housing in the last period
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Households’ value function in the last period is:

\[ V_T(M_T, H_T, P_T) = u(C_T) + \beta u^h(W_{T+1}) . \]

For renters, the optimal composite consumption follows

\[ (C_T)^{-\rho} = \beta \varphi p_T^c W_{T+1}^{-\sigma} = \beta \varphi p_T^c (M_T - p_T^c C_T)^{-\sigma} , \]

and the marginal value of market resources is

\[ \frac{\partial V_T}{\partial M_T} = \beta \varphi W_{T+1}^{-\sigma} = \frac{u'(C_T)}{p_T^c} . \]

For homeowners, their optimal non-housing consumption is

\[ u'(C_T) \frac{\partial C_T}{\partial C_T} = \beta \varphi W_{T+1}^{-\sigma} = \beta \varphi (M_T + p_T^h H_T - C_T)^{-\sigma} , \]

with marginal value of market resources and housing

\[ \frac{\partial V_T^h}{\partial M_T} = u'(C_T) \frac{\partial C_T}{\partial C_T} , \]

\[ \frac{\partial V_T^h}{\partial H_T} = u'(C_T) \frac{\partial C_T}{\partial C_T} + p_T^h u'(C_T) \frac{\partial C_T}{\partial C_T} . \]

For simplicity, we assume that \( \sigma = \rho \). Then for renters, their last period composite
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Consumption is

\[ \hat{C}_T = \frac{M_T}{(\beta_0 p_T)^{1/2} + p_T^*}. \]

Hence \( \hat{C}_T \) is a decreasing function of \( \varphi \).

1.8.3 Numerical Solution to the Households’ Problem

Our structural model involves discrete choices about transitions between renters and homeowners (\( rr, rh, hr, hh, hh' \)), and continuous choices about consumption (\( C \)) and housing (\( H \)). The presence of discrete choices induces multiple solutions to the same Euler equation and kinks in the value function, making it computationally intensive to accurately find solutions to households’ problem. To tackle this issue, we first solve for policy rules and value functions conditional on discrete choices, and then use the upper envelope of conditional value functions as the unconditional value function. Specifically, the unconditional value function of renters is obtained by taking the upper envelope of value functions conditional on \( rr \) and \( rh \), and the value function of homeowners by taking the upper envelope of value functions conditional on \( hr, hh \) and \( hh' \). Conditioning on discrete choices, we use the first order condition(s) to solve for consumption and housing rules. To this end, we extend the endogenous grid point method by Carroll (2006a) to the case of two continuous variables. To account for occasionally binding loan-to-value and loan-to-income constraints, we use...
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a modified version of [Hintermaier and Koeniger (2010)]. To deal with kinks of value functions efficiently, we employ an algorithm similar to that in Iskhakov, Jørgensen, Rust, and Schjerning (2014). Starting from policy rules and value functions in the last period, we iterate backward to obtain a set of age-dependent consumption and housing rules and value functions conditional on discrete choices. We then simulate 5000 households to generate simulated profiles. To further speed up computation, we parallelize the computation of household decisions based on their permanent income and parallelize the simulation based on the number of households.

1.8.4 Initial Distribution of Net Worth, Housing, and Income

Table 1.5 presents the initial distribution of net worth, housing and income in the data. In our simulation we draw from this joint distribution the initial values of household balance sheets.

1.8.5 Initial Values of Preference Parameters

Good initial values will speed up the estimation of our model and increase the probability of finding the global minimum. In this section, we derive moment conditions under some simplifying assumptions of the model. These moments are key to identifying parameters related to preference in our model. We use these parameter
CHAPTER 1. HOUSING, DEBT, AND THE MARGINAL PROPENSITY TO CONSUME

Table 1.5: Initial Distribution by Net Worth Group

<table>
<thead>
<tr>
<th>Group</th>
<th>Net Worth</th>
<th>Income</th>
<th>Housing</th>
<th>Homeownership</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-16.87</td>
<td>3.06</td>
<td>15.64</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>-7.03</td>
<td>3.01</td>
<td>13.33</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>-3.44</td>
<td>2.33</td>
<td>13.33</td>
<td>0.36</td>
</tr>
<tr>
<td>4</td>
<td>-2.27</td>
<td>1.97</td>
<td>13.24</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>-1.52</td>
<td>2.06</td>
<td>13.72</td>
<td>0.20</td>
</tr>
<tr>
<td>6</td>
<td>-0.86</td>
<td>2.10</td>
<td>13.78</td>
<td>0.19</td>
</tr>
<tr>
<td>7</td>
<td>-0.27</td>
<td>1.99</td>
<td>13.76</td>
<td>0.17</td>
</tr>
<tr>
<td>8</td>
<td>0.01</td>
<td>1.25</td>
<td>14.04</td>
<td>0.04</td>
</tr>
<tr>
<td>9</td>
<td>0.24</td>
<td>1.99</td>
<td>13.92</td>
<td>0.16</td>
</tr>
<tr>
<td>10</td>
<td>1.05</td>
<td>2.56</td>
<td>14.24</td>
<td>0.49</td>
</tr>
<tr>
<td>11</td>
<td>2.66</td>
<td>2.82</td>
<td>14.45</td>
<td>0.83</td>
</tr>
<tr>
<td>12</td>
<td>4.68</td>
<td>2.76</td>
<td>15.49</td>
<td>0.95</td>
</tr>
<tr>
<td>13</td>
<td>6.78</td>
<td>2.57</td>
<td>15.70</td>
<td>0.98</td>
</tr>
<tr>
<td>14</td>
<td>8.98</td>
<td>2.53</td>
<td>16.32</td>
<td>0.99</td>
</tr>
<tr>
<td>15</td>
<td>11.27</td>
<td>2.32</td>
<td>16.83</td>
<td>1.00</td>
</tr>
<tr>
<td>16</td>
<td>13.75</td>
<td>2.21</td>
<td>18.07</td>
<td>1.00</td>
</tr>
<tr>
<td>17</td>
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<td>19.70</td>
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<td>18</td>
<td>20.02</td>
<td>2.13</td>
<td>22.40</td>
<td>1.00</td>
</tr>
<tr>
<td>19</td>
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<td>2.12</td>
<td>27.71</td>
<td>1.00</td>
</tr>
<tr>
<td>20</td>
<td>51.66</td>
<td>2.56</td>
<td>43.14</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Notes. Groups are based on 20 net worth quantiles. All levels are the mean of each net worth group in hundreds of thousands of Norwegian Krones that are indexed to the 2000 price level. Housing is the mean level of owner-occupied housing.

estimates as initial values for our second step estimation.

Suppose households are able to smooth housing consumption costlessly. Then the intratemporal optimality condition implies

\[
\frac{C_a}{S_a} = \frac{\alpha_a}{1 - \alpha_a}.
\]
In our model, \( S_a = \zeta H_a \) and \( \alpha_a = \exp(\phi_a)/(1 + \exp(\phi_a)) \) where \( \phi_a = \phi_0 + \phi_1 N_a^{\text{Adult}} + \phi_2 N_a^{\text{Children}} \). Thus we have

\[
\frac{C_a}{H_a} = \zeta \exp \left( \phi_0 + \phi_1 N_a^{\text{Adult}} + \phi_2 N_a^{\text{Children}} \right).
\]

Taking logarithm of both sides yields

\[
\log \left( \frac{C_a}{H_a} \right) = \log(\zeta) + \phi_0 + \phi_1 N_a^{\text{Adult}} + \phi_2 N_a^{\text{Children}}. \tag{1.4}
\]

Thus \( \phi_1 \) and \( \phi_2 \) can be identified from a regression relating the ratio of non-housing and housing consumption to the number of adults and the number of children using the micro data. In addition, such a regression would imply a relation between \( \zeta \) and \( \phi_0 \).

Next we add to our model the relative price of housing service to non-housing consumption, \( p^h_t \), while maintaining the assumption of no transaction costs. We assume that the aggregate non-housing consumption and housing service are \( C_t \) and \( S_t \), respectively, and that the aggregate expenditure share on non-housing consumption in each year is \( \bar{\alpha} \). The intratemporal optimality condition costs implies that

\[
\frac{C_t}{S_t} = \frac{\bar{\alpha}}{1 - \bar{\alpha}} \left( p^h_t \right)^{\theta-1}.
\]
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Taking logarithm of both sides yields

$$\log \left( \frac{C_t}{S_t} \right) = \log \left( \frac{\alpha}{1-\alpha} \right) + (\theta - 1) \log (p^h_t) . \quad (1.5)$$

To obtain the time series on the left-hand side of the above equation, we use aggregate expenditure share on housing and rents at the quarterly frequency and calculate its implied consumption to housing service ratio. To construct the relative price of housing service, we use quarterly data of house price index and CPI from 1999Q1 to 2009Q4 and take their ratio (with mean normalized to one) as a proxy for $p^h_t$. This results in an estimate of $\hat{\theta} = 0.497$ and an estimate of $\hat{\alpha} = 0.725$.

Relating the time series results to cross-sectional results,

$$\alpha \approx \frac{\exp \left( \phi_0 + \phi_1 \bar{N}_{\text{Adult}} + \phi_2 \bar{N}_{\text{Children}} \right)}{1 + \exp \left( \phi_0 + \phi_1 \bar{N}_{\text{Adult}} + \phi_2 \bar{N}_{\text{Children}} \right)} ,$$

where $\bar{N}_{\text{Adult}}$ and $\bar{N}_{\text{Children}}$ are the average number of adults and children per family. Combining with estimates from equation (1.4), we obtain an initial estimates of $\hat{\zeta} = 0.22$ and $\hat{\phi}_0 = 0.12$. 

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Chapter 2

Progressive Taxation and Precautionary Saving Over the Life Cycle

2.1 Introduction

A classical argument for progressive taxation of household labor income is that progressivity provides social insurance against uninsurable wage risks. However, at the same time, progressive taxation distorts precautionary saving and labor supply, through which households obtain self-insurance. This chapter studies the interaction between tax progressivity, precautionary saving and labor supply in a life cycle model.

The objective of this chapter is twofold. First, we decompose consumption in-
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insurance under progressive taxation. This decomposition is informative about the quantitative importance of different channels through which households achieve consumption insurance. Second, we quantify the welfare effects of progressive taxation and search for the optimal progressivity within a class of progressive tax functions.

We address these issues from the perspective of a partial equilibrium model. Specifically, we develop an incomplete-markets life-cycle model that includes progressive taxation explicitly. In our setup, households understand the structure of the tax schedule and choose their consumption and labor supply optimally. Households are heterogeneous in their preferences and in their initial conditions when they begin the life cycle. We estimate the model using data from the German Socio-Economic Panel (GSOEP). The strength of the model is that it is quantitatively plausible. In particular, the model is able to capture the empirical wealth and labor supply distributions over the life cycle.

We identify three channels through which progressive taxation affects households’ consumption insurance. First, progressive taxation reduces shocks to after-tax income mechanically. This is the direct effect of progressive taxation. Due to an increasing marginal tax rate under progressive taxation, positive changes to income will be taxed more than negative changes. As a result, after-tax income has a smaller variance than pre-tax income. Second, progressive taxation distorts labor supply. Because labor income is multiplicative in wage and labor, the extent to which wage shocks pass on to income and eventually to consumption is affected by the endogenous choice of labor
supply under progressive taxation. Third, progressive taxation distorts wealth accumulation over the life cycle. Reduced after-tax income uncertainty under progressive taxation diminishes households’ precautionary saving motive and hence diminishes their lifetime wealth accumulation. Consumption is more susceptible to wage shocks with less buffer stock of wealth. The last two channels are the indirect effects of progressive taxation.

Our first main result is that quantitatively the direct effect of progressive taxation reduces wage shocks that pass on to consumption by 22%, wealth accumulation on average reduces them by 2.6% and labor supply reduces them by 15.3%. The indirect effects are additive while the direct and indirect effects are multiplicative, so the overarching effect is that 64% of permanent wage shocks finally pass on to current consumption.

Progressive taxation distorts wealth accumulation and labor supply. Compared to the current German tax system, households under a revenue-neutral proportional taxation would on average save 32% more, provide 5% more labor, and enjoy 5% more consumption. Consumption insurance in the progressive and proportional taxation environments would be similar, though the direct effect of progressive taxation would almost all be replaced by the indirect effect of wealth.

We then analyze the relationship between progressive taxation and welfare in an environment where total tax revenue is kept constant. There are two types of trade-off under progressive taxation. The intratemporal trade-off is that progressivity
encourages consumption and discourages labor supply within the same period, which
tends to enhance household welfare; but decreased labor supply implies an increased
average tax rate, which tends to reduce monetary resources available for consumption
and hence welfare. The intertemporal trade-off is that progressive taxation encourages
current consumption relative to future consumption, the forgone interest on wealth
that would be saved otherwise reduces lifetime resources for consumption. In light
of these trade-offs, it is not obvious what the best degree of progressivity is. We
therefore use our estimated model to find the optimal progressive taxation.

Our second main result is that the optimal progressive taxation is one that is very
close to proportional tax. We use a parameter $\tau$ to capture the degree of progress-
vity. In the current German tax system, $\tau^{Germany}$ is estimated to be 0.22. In the
US, $\tau^{US} = 0.15$. Our estimated optimal progressivity $\tau_{opt} = 0.06$, suggesting that
limited progressivity is helpful in improving the welfare of the economy but too much
progressivity induces severe distortions and results in welfare loss.

We further analyze the welfare implications of progressive taxation for households
with different preferences and different initial conditions. We find that the optimal
degree of progressivity depends more on the distribution of heterogeneity in house-
hold preferences, especially their disutility of labor, and less on the distribution of
wealth. However, the relationship between welfare and progressivity is broadly robust
to preference heterogeneity and initial conditions.
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Related Literature

The social insurance effect of progressive taxation was investigated already in Mirrlees (1974) and Varian (1980) but not with an explicit focus on precautionary saving. A closely related series of studies on precautionary saving in partial equilibrium models with exogenous labor income, (e.g., Cagetti 2003b; Carroll and Samwick 1998; Gourinchas and Parker 2002b), abstracts from modeling taxation and transfers explicitly and emphasizes the importance of precautionary saving. At the same time, an older literature showed that precautionary behavior is influenced strongly by the tax and transfer system, usually with a focus on transfers (Engen and Gruber 2001; Gruber and Yelowitz 1999; Hubbard, Skinner, and Zeldes 1995). We connect the findings in this literature and provide a comprehensive examination of the interdependencies between taxes, labor supply, and precautionary saving.

Our analysis uses the concept of partial insurance developed in Blundell, Pistaferri, and Preston (2008b) and relates to studies quantifying partial insurance. Heathcote, Storesletten, and Violante (2014a) derive a closed-form solution for consumption and labor supply in general equilibrium and find that 39 percent of permanent wage shocks pass through to consumption. We show that this solution is analytically similar to the one obtained for hand-to-mouth households in our model. One of the emphases of our model is its empirical relevance with respect to wealth distribution.

This paper is related to the study on the design of optimal taxation. Fehr, Kallweit, and Kindermann (2013) find from simulation exercises that the optimal system
involves higher progressivity, since insurance benefits over-compensate additional labor market distortions. Conesa, Kitao, and Krueger (2009) (Conesa and Krueger (2006)) study the tradeoff between efficiency and insurance under progressive taxation and find that a proportional income tax with a constant marginal tax rate of 23 (17.2) percent and a deduction of roughly 7,200 (9,400) dollar is optimal for the US. A further result of Conesa and Krueger (2006) is that a pure flat tax without deduction reduces welfare by about 1 percent. For a study that investigates the optimal progressivity of capital income taxes see Saez (2002). Heathcote, Storesletten, and Violante (2014b) find that the optimal degree of progressivity for the US is $\tau^{US}_{opt} = 0.062$ in a general equilibrium environment where consumption, labor supply, human capital, and public goods shape the optimal degree of progressivity. Interestingly, our estimate is similar to theirs, though we emphasize the match of wealth and labor supply distributions over the life cycle in a partial equilibrium framework.

The following papers do not pursue normative exercises: Ventura (1999) explicitly models the general equilibrium implications of a revenue neutral tax reform in which the status quo in the U.S. is replaced by a flat tax, as proposed by a previous edition of Hall and Rabushka (2007). This study and Castañeda, Diaz-Gimenez, and Rios-Rull (1999) who also compare the status quo progressive system in the US to a flat tax find that labor supply does hardly change due to this reform. Finally, our study is related to Caucutt, Imrohoroglu, and Kumar (2003) who study the effects of progressive taxation using a flat tax as a benchmark.
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The rest of the paper is structured as follows. In Section 2.2 we introduce a standard incomplete markets model with taxes and transfers and discuss the insurance effects and welfare implications of progressive taxation. Section 2.3 briefly describes data, the estimation procedure and results of key parameters and variables. Our main results are reported in Section 2.4. Section 2.5 concludes.

2.2 Model

This section first introduces a standard incomplete-markets model of household consumption and labor supply over the life cycle, and then analyzes the insurance effects and welfare implications of progressive taxation in the context of this model. We discuss three channels through which consumption is partially insured under progressive taxation, and two types of welfare trade-offs under progressive taxation.

2.2.1 A Life-cycle Model

The economy consists of a finite number of households indexed by $i$. We assume that households start their economic life in period $t_0$ and live for $T$ years. Households work for $T^w$ years in their life and then enter the stage of retirement when there is a positive probability of death. During their working life, households derive utility from annual consumption $C_{it}$ and disutility from labor $H_{it}$, while there is no disutility
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from labor after retirement. We assume that households face a positive probability of death after retirement, and all households die at time $T + t_0$ with certainty. Let $p^S_t$ be the conditional probability of survival at time $t$, and $M_t$ be the market resources households leave for bequest. Households maximize expected discounted utility,

$$E_{t_0} \left[ \sum_{t=t_0}^{T_w+t_0-1} \beta_i^{t-t_0} u(C_{it}, H_{it}) + \sum_{t=T_w+t_0}^{T+t_0} \beta_i^{t-t_0} \left( p^S_t u^R(C_{it}) + (1 - p^S_t) u^B(M_{it}) \right) \right] (2.1)$$

where $\beta_i$ is the discount factor for household $i$. We allow $\beta$ to be heterogeneous among households to account for the empirical wealth distribution.

Period utility during working age follows

$$u(C_{it}, H_{it}) = \frac{C_{it}^{1-\rho}}{1 - \rho} - \frac{H_{it}^{1+\sigma}}{1 + \sigma}. $$

The parameter $\rho$ governs the intertemporal elasticity of substitution for consumption and $\sigma$ governs the Frisch-elasticity of labor supply. $\phi_i$ captures the extent of disutility from labor supply in terms of consumption goods. We allow $\phi$ to differ across households to account for the empirical distribution of labor supply.

Period utility after retirement follows

$$u^R(C_{it}) = \frac{C_{it}^{1-\rho}}{1 - \rho}. $$

\footnote{In practice, we set $t_0 = 25$, $T_w = 40$, and $T = 65.$}
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We assume utility from bequest follows a simple function form,

\[ u^B(M_t) = \varphi M_t^{1-\rho} (1 - \rho). \]

where the parameter \( \varphi \) captures the degree of bequest motive.\(^2\)

**Budget Constraint**

Let \( A_{it} \) be the amount of financial assets household \( i \) holds at the beginning of period \( t \), \( W_{it} \) the wage in year \( t \), and \( H_{it} \) the labor supply in year \( t \).\(^3\) The budget constraint follows

\[ A_{i,t+1} = A_{i,t}(1 + r(1 - \tau_A)) + W_{it}H_{it} - TX(W_{it}H_{it}) - (1 + \tau_C)C_{it}. \quad (2.2) \]

\( r \) is the risk-free interest. \( \tau_A \) and \( \tau_C \) are proportional tax rate on capital income and consumption, respectively. \( TX(\cdot) \) is the progressive tax function.

**Progressive Tax**

We specify a parsimonious tax function taken from the public finance literature (see Feldstein (1969)) following Bénabou (2002). Taxes or transfers are given by

\(^2\)In principle, we could allow \( \varphi \) to differ across households, but we find that the heterogeneity in \( \varphi \) only makes the model marginally better in fitting the data. In light of the extra degree(s) of freedom introduced by this heterogeneity, we decide to drop it.

\(^3\)We normalize \( H_{it} \) by the total number of working hours in a year (8 \( \times \) 5 \( \times \) 52 = 2080 hours for a single worker) so \( W_{it} \) should be interpreted as annual wage.
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\[ TX(Y) = Y - \lambda Y^{1-\tau} \]  

(2.3)

With this tax function, disposable (post-government) income \( \tilde{Y} \) is a function of pre-government income \( Y \) and the parameters \( \tau \) and \( \lambda \).

\[ \tilde{Y} = \lambda Y^{1-\tau}. \]  

(2.4)

The parameter \( \tau \) determines the degree of progressivity of the tax system.\(^4\) For \( \tau = 0 \), the tax and transfer function becomes a proportional tax system with the tax rate \( 1 - \lambda \); for \( \tau = 1 \), the tax and transfer function becomes so progressive the after-government income equals \( \lambda \) whatever the pre-government income. The parameter \( \lambda \) is related to tax revenue. Holding tax revenue constant, \( \lambda \) increases as the degree of progressivity \( \tau \) increases.

\textit{Wage Process}

We assume that wage follows a permanent-transitory type of process

\[ \log Z_{it} = \gamma_t + \log Z_{it-1} + \eta_{it}, \quad (2.5) \]

\[ \log W_{it} = \log Z_{it} + \epsilon_{it}, \quad (2.6) \]

\(^4\) The age index \( t \) is omitted for legibility.

\(^5\) The standard definition of a progressive tax-transfer function in applied in public economics is that the marginal tax rate is larger than the average tax rate for every level of pre-government income. \( TX'(Y) > TX(Y)/Y \). Applying this definition to our specific tax function implies \( 1 - \lambda (1-\tau)Y^{-\tau} > 1 - \lambda Y^{-\tau} \), which is true when \( \tau > 0 \).
where $Z_{it}$ is the permanent component of the wage process, $\gamma_t$ is the deterministic growth rate common for all households, and $\eta_{it}$ and $\epsilon_{it}$ are normally distributed permanent and transitory shocks to wages, respectively,

$$
\eta_{it} \sim N\left(-\frac{\sigma_{\eta t}^2}{2}, \sigma_{\eta t}^2\right),
$$

$$
\epsilon_{it} \sim N\left(-\frac{\sigma_{\epsilon t}^2}{2}, \sigma_{\epsilon t}^2\right).
$$

We allow variances of permanent and transitory shocks to wage to vary by age because this feature is present in the data and important for the rate of wealth accumulation over the life cycle.

**Borrowing Constraint**

The borrowing constraint can have a large impact on consumption and labor supply choices of households with low wage or wealth. We assume that the maximum amount that households can borrow depends on their permanent component of wage. In particular,

$$
A_{i, t+1} \geq -aZ_{it}.
$$

(2.7)

This constraint can be interpreted as banks’ decision to lend on the basis of households’ annual income: banks assess the average wage of households, calculate their annual income if they work full time, and then decide what fraction of that income could be lent.
2.2.2 Insurance Effects of Progressive Taxation

The key question we want to address is how much of a permanent shock to wage transmits to consumption response under progressive taxation. In other words, how much of a permanent shock is insured against under progressive taxation.

There are three channels through which consumption is partially insured against permanent shocks under progressive taxation. The first is reduction of shocks to after-tax income, which we call the direct effect of progressive taxation. Intuitively, because marginal tax rate is increasing under progressive taxation, households expect to pay a higher (lower) tax rate than their current average when there are positive (negative) changes to their income. As a result, after-tax income has a smaller variance than pre-tax income\(^6\) and part of shocks to pre-tax income are insured against.

The second channel is the distortion of labor supply by progressive taxation. As taxation becomes more progressive, there is less incentive for households to provide labor; when households work less, less of a shock to wage passes on to income and further to consumption.

The third channel is the interaction of progressive taxation with precautionary wealth accumulation. Because progressive taxation reduces the uncertainty of after-tax income in the future, households will choose to increase current consumption relative to future consumption when there is more progressivity. As such, there is

\(^6\)To be precise, after-tax income here refers to not the level of after-tax income, but the log level of it. Proportional tax also reduces the variance of the level of after-tax income, but it does not reduce the log level of after-tax income.
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less precautionary wealth accumulation over the life cycle; with less buffer stock of wealth, consumption is more susceptible to shocks to income. We call the last two channels the *indirect* effects of progressive taxation.

The optimality conditions of households’ problem make it clear about the three channels of the insurance effects under progressive taxation. We focus on consumption and labor supply choices before retirement. The first order conditions follow

\[ C_{it}^r = \frac{(1 + \tau_c)\phi_i}{\lambda(1 - \tau)} H_{it}^{\sigma + \tau} W_{it}^{-1} \]

(2.8)

\[ C_{it}^r = \beta(1 + r(1 - \tau_A)) E_t [C_{it+1}^r] + (1 + \tau_c)\mu_t \]

(2.9)

where \(\mu_t\) is the Lagrange multiplier on the borrowing constraint (2.7).

Equation (2.8) is the intratemporal optimal condition for labor supply. To see how shocks to wage affect consumption and labor supply decisions, we take the logarithm of both sides of equation (2.8) and re-arrange terms to obtain

\[
(1 - \tau) \log W_{it} = \rho \log C_{it} + (\sigma + \tau) \log H_{it} + \log \left( \frac{(1 + \tau_c)\phi_i}{\lambda(1 - \tau)} \right). \tag{2.10}
\]

The lefthand side of Equation (2.10) is the direct effect of progressive taxation. It is the amount of shocks to wage that has to be absorbed by change in consumption
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and labor supply. Note from equations (2.5) and (2.6),

\[ \log W_{lt} = \gamma_t + \log Z_{l,t-1} + \eta_{lt} + \epsilon_{lt}, \]

At time \( t \), \( \gamma_t \) and \( \log Z_{l,t-1} \) are known. The impact of permanent shock, \( \eta_{lt} \), or transitory shock, \( \epsilon_{lt} \), on consumption and labor supply is reduced to \( (1 - \tau) \) of its original size under progressive taxation. When tax is proportional, \( \tau = 0 \) and households bear the full impact of shocks to wage; on the other extreme, when tax is fully progressive, \( \tau = 1 \) and no shocks will pass on to households.

The progressivity parameter, \( \tau \), appears twice on the righthand side of equation (2.8). The middle term shows that progressive taxation distorts labor supply decision of households by making it less elastic than under proportion taxation.\(^7\) However, the relative strength of that distortion depends on the degree of progressivity compared to the elasticity of labor supply. The last term relates to tax revenue. In the exercises that follow, we will adjust \( \lambda \) in order to keep tax revenue constant when changing the degree of progressivity.

Equation (2.9) is the intertemporal optimal condition for consumption. It determines how the impact of shocks to wage is allocated between consumption and labor supply with uncertainty in future wages.\(^8\) Because wealth serves as a buffer against uncertainty in future wages, it plays an important role in determining the

\(^7\)Remember that \( 1/\sigma \) is the elasticity of labor under proportional taxation (\( \tau = 0 \)).

\(^8\)In other words, consumption and labor supply responses upon shocks to wage are indeterminate in equation (2.8).
level of current consumption and hence consumption response to wage shocks. However, wealth in the model is endogenous: the degree of progressivity affects the rate at which wealth is accumulated. It is therefore the third channel through which progressive taxation affects consumption insurance. This also highlights the importance for the model to match empirical wealth distributions in our subsequent quantitative analysis of the three channels of progressive taxation.

### 2.2.3 Decomposition of Insurance Effects

To further disentangle the insurance effects of labor supply and wealth accumulation, we start with a special case where there is no wealth accumulation and households are hand-to-mouth. While the intratemporal optimal condition still holds, the borrowing constraint is binding

\[
C_{it} = \frac{1}{1 + \tau_C} (W_{it}H_{it} - TX(W_{it}H_{it})) = \frac{\lambda}{1 + \tau_C} (W_{it}H_{it})^{1-\tau} \tag{2.11}
\]

Equations (2.8) and (2.11) together fully characterize households’ consumption and labor choices. In particular,

\[
\log C_{it} = (1 - \tau) \frac{\sigma + 1}{\rho + \sigma + (1 - \rho)\tau} \log W_{it} + C^* \tag{2.12}
\]

\[
\log H_{it} = (1 - \tau) \frac{1 - \rho}{\rho + \sigma + (1 - \rho)\tau} \log W_{it} + H^* \tag{2.13}
\]

\footnote{We assume $\varrho = 0.$}
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where

\[
C^* = \frac{\sigma + 1}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\lambda}{1 + \tau_C} - \frac{1 - \tau}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\phi_i}{1 - \tau}
\]

\[
H^* = \frac{1 - \rho}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\lambda}{1 + \tau_C} - \frac{1}{\rho + \sigma + (1 - \rho)\tau} \log \frac{\phi_i}{1 - \tau}
\]

The coefficient before the wage term in equation (2.12) captures the consumption insurance effects of progressive taxation in the absence of wealth. \((1 - \tau)\) is the direct effect. \(\frac{\sigma + 1}{\rho + \sigma + (1 - \rho)\tau}\) is the indirect effect through distortion of labor supply\(^{10}\) The extent to which labor supply is distorted under progressive taxation is captured by the coefficient before the wage term in equation (2.13). The parameter \(\rho\) determines the sign of labor supply change in response to an increase in wage: if \(\rho > 1\), the income effect dominates the substitution effect and labor supply decreases upon positive changes of wage.

Equations (2.10), (2.12), and (2.13) show how wage shocks are absorbed in the absence of wealth. A fraction \(\tau\) is absorbed by the direct effect of progressive taxation. If we define

\[
\chi = \frac{(1 - \rho)(\sigma + \tau)}{\rho + \sigma + (1 - \rho)\tau},
\]

then a fraction \(\chi\) of the remaining impact of wage shocks falls on labor supply, and \(1 - \chi\) falls on consumption. Due to intertemporal elasticities of substitution, current labor supply will change by \(\chi \frac{1}{\sigma + \tau}\) and current consumption will change by \((1 - \chi) \frac{1}{\rho}\).

\(^{10}\)It is exactly the same pass-through coefficient as in Heathcote, Storesletten, and Violante (2014a) (cf. Proposition 1) in a general equilibrium setting.
That is,

\[ \{\tau, (1 - \tau)(1 - \chi), (1 - \tau)\chi\} \]

characterizes the allocation of wage shocks in the absence of wealth.

To see the indirect effect of labor supply on consumption insurance, we need to compare consumption response to wage shocks when labor is elastic and when it is inelastic. Let \( \sigma \to \infty \) so that labor is completely inelastic, we have \( \chi_\infty = 1 - \rho \). Thus the effect of labor supply is \( (\chi - \chi_\infty)/\rho \).

Because wealth changes the allocation of wage shocks between consumption and labor supply, but not the total amount of shocks to be absorbed by them\(^{11}\) the allocation of wage shocks in the presence of wealth follows

\[ \{\tau, (1 - \tau)(1 - \chi - \chi'), (1 - \tau)(\chi + \chi')\}, \]

where we have used \( \chi' \) to denote the impact of wealth on the allocation of wage shocks to consumption. It should be noted that \( \chi' \) is an average, because the impact of wealth depends on the level of wealth, which differs across households and changes over time for the same household. Uncertainty in wages makes \( \chi' \) intractable, we therefore resort to numerical methods for estimation.

The elasticity of consumption with respect to a wage shock is \( (1 - \tau)(1 - \chi - \chi')^{\frac{1}{\rho}} \).

\(^{11}\)Equation 2.10 holds with or without wealth.
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It follows that

\[(1 - \tau)(1 - \chi - \chi')\frac{1}{\rho} = (1 - \tau)(1 - \frac{\chi'}{\rho} - \frac{\chi - \chi_\infty}{\rho}).\]

This makes it clear that the consumption insurance is obtained through direct effect of progressive taxation ($\tau$), indirect effect of labor supply ($\frac{\chi\chi_\infty}{\rho}$) and wealth ($\frac{\chi'}{\rho}$).
Without progressive taxation, wealth accumulation, or elastic labor, consumption respond one for one to wage shocks.

2.2.4 Welfare and the Optimal Progressive Taxation

We define the welfare of household $i$, $\tilde{W}(\lambda, \tau, A_{it_0})$, as the value of maximized expected discounted utility over the life cycle:

\[
\tilde{W}(\lambda, \tau, A_{it_0}) = \max_{\{C_{it}\}, \{H_{it}\}} \mathbb{E}_{t_0} \left[ \sum_{t=t_0}^{T+t_0-1} \beta^{t-t_0} u(C_{it}, H_{it}) + \sum_{t=T+t_0}^{T+w+t_0} \beta^{t-t_0} u^R(C_{it}) \right].
\]

Welfare depends on the tax system ($\lambda$ and $\tau$) and initial market resources of household $i$ ($A_{it_0}$). To help with intuition, we will work with the following transformation,

\[
W = \left((1 - \rho)\tilde{W}\right)^\frac{1}{1-\rho}, \quad (2.14)
\]
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which is simply welfare measured in consumption terms.

It is important that in the discussion of welfare implications of progressive taxation one keeps (expected) tax revenue constant. Outside of the model described above, one might think that the government redistributes the net tax revenue (after progressive taxation) to households in the form of public goods. Once we assume that the utility of public goods is additive to our current specification of utility, then this redistribution does not change the welfare level as long as tax revenue is constant.

There are two types of welfare trade-off under progressive taxation. The first type is intratemporal. As taxation turns more progressive (an increase in $\tau$), on the one hand there is less incentive for households to work. On the other, less labor supply implies not only that the total pre-tax income will be smaller, but that the average tax rate must increase to maintain the same tax revenue, which further decreases resources available for consumption. Because less labor supply increases households’ utility while less consumption decreases it, welfare is not necessarily monotonic in the degree of progressivity.

The second type is intertemporal. Because progressive taxation reduces future after-tax wage uncertainty, it tends to increase households’ current consumption relative to their future consumption. However, a shift toward current consumption reduces lifetime wealth accumulation, which implies that households forgo part of interest that would otherwise be earned on their wealth. The curvature of utility function, households’ time preference rate, and the interest rate affects this type of
In light of the trade-offs, it is natural to ask what progressive taxation is optimal within the class of tax functions in equation (2.3). This question is not trivial because the degree of progressivity and the total tax revenue to be collected are exogenous to households. At one extreme, taxation can be so progressive that households barely want to work. Almost all of their income are taxed to reach the revenue target, so consumption is close to zero and utility approaches negative infinity. In this case welfare is undoubtedly less than welfare under proportional taxation.

Because there is no closed-form solution to household welfare, we will again resort to numerical methods to determine the relationship between welfare and progressivity under fixed tax revenue.

### 2.3 Estimation

In this section, we estimate the parameters of our model. Throughout, we use household survey data from the German Socio-Economic Panel (SOEP). This dataset is comparable to Panel Study of Income Dynamics (PSID). In the appendix, we compare our results from German data with results from U.S. data. It would be interesting to see how much insurance effects of progressive taxation differ in these two advanced economies.

The key dimensions of the model include progressive taxation, wage process, and
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household preference. We estimate the progressive tax function directly from information on pre-tax and after-tax income in the data. To estimate the wage process, we follow the literature on income process and use the variance-covariance restriction of residual wages to identify the variances of permanent and transitory shocks. Finally, we use the method of simulated moments to estimate parameters related to household preference by matching empirical wealth and labor supply distribution over the life cycle.

2.3.1 Progressive Taxation

The tax function given in equation (2.3) is parsimonious in its parametrization, but gives a remarkably good representation of the actual tax and transfer system in Germany. To estimate the two parameters, we use data on household pre-government and post-government income labor income from the SOEP. Pre-government household income includes labor earnings, private transfers like alimony, pension incomes, and income from interests, dividends, and rents. Post-government income equals pre-government income minus income taxes (including solidarity surcharge, social security contributions, etc.), plus public transfers (unemployment benefits, unemployment assistance etc.).

The left graph in Figure 2.1 shows our estimates and is constructed as follows. We collapse all waves of our data into 50 quantiles. We associate each mean of a quantile of pre-government income to the mean post-government income across the
observations in that same quantile. These points are shown as circles in the figure. Then we estimate equation (2.4) in logs using OLS. Because transfers have a different slope and intercept, we interact log $Y_{it}$ with a dummy indicating whether transfers have been received or not. The point estimates are $\tau = 0.22$ and $\lambda = 7.87$ for the region where no transfers are paid.\footnote{In the region where transfers are paid, $\tau = 0.82$ and $\lambda = 3296.41.$} This is well above the estimate of $\tau^{US} = 0.15$ that Heathcote, Storesletten, and Violante (2014b) find for the USA. This simple model fits the data well with $R^2 = 0.996.$ The left hand graph presents the model fit as a solid kinked line and points where pre- and post-government income are identical as the 45 degree line.

The right hand graph in Figure 2.1 displays the implied marginal and average tax rates which we obtain from our estimates of $\tau$ and $\lambda$. The solid line shows the marginal tax rate, while the dashed line is the average tax rate. Marginal tax rates
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are high for transfer recipients and increase to about 80 percent because of transfer withdrawal. Once a household does not receive transfers anymore, the marginal tax rate drops to about 30 percent. For high income households, the marginal tax rate increases to 53 percent. This was the top tax rate from 1990 to 1999 in Germany. The red dashed line displays the average tax rate.

2.3.2 Wage Process

We use all waves available to estimate the wage process given in equation (2.6). Dating back to Lillard and Willis (1978), Lillard and Weiss (1979), MaCurdy (1982), and Abowd and Card (1989), there is a history of fitting ARMA models to wages from panel data.\(^\text{13}\) We assume that the persistent part of idiosyncratic wages (equation (2.5)) follows a random walk with drift \(\gamma_t\).\(^\text{14}\)

Figure 2.2 presents average growth rates of wages \(\gamma_t\) in the left hand graph and the variance of \(\log W_{it}\) in the right hand graph. The measure of wage growth is the linear prediction from a cross-sectional regression of the first difference in \(\log W\) on a third polynomial in age, an education indicator, and interactions between the education indicator and the age polynomial. The residuals from this regression are used as measure of idiosyncratic wage risk. Wage growth is about 12% at age 25.

Wages increase decreasingly quick to age 35 and remain roughly constant to age 50.

---

\(^{13}\) For estimates based on German data, see e.g. Biewen (2005); Myck, Ochmann, and Qari (2011).

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Figure 2.2: Growth Rate of Wages by Age and Variance of Log Wages

Then, wages increase less and less until retirement age. Wage inequality measured by the variance of log wages increases steadily up to age 45, then remains roughly constant and increases after age 60 because of retirement effects.

The permanent component of the wage process depends on the age-varying parameters $\sigma_{\eta,t}^2$. To decompose log $W_{it}$ further, we estimate the age-varying variances of the transitory component $\epsilon_{it}$. We allow this component to follow an ARMA(1,1) process. E.g. [Blundell, Graber, and Mogstad (2014)] showed that it is important to take the age pattern of permanent and transitory shocks into account. We estimate these variances using the measure of idiosyncratic wage risk from the cross-sectional regression.

For estimation, we use a GMM procedure with identity weight matrix\(^{15}\). Given a vector of 81 parameters $\Theta$, we can generate a (theoretical) variance-covariance matrix of the vector $(\Delta y_t, \ldots, \Delta y_{t+40})$, denoted by $\Omega$. We compute the empirical variance

\(^{15}\)We use equally weighted minimum distance for reasons explained in [Altonji and Segal (1996)].
covariance matrix of the same vector and denote it by $\hat{\Omega}$. Then our objective is to find the set of parameters that minimize $(\Omega - \hat{\Omega})'(\Omega - \hat{\Omega})$. The estimation procedure is identical to Hryshko (2012), where identification is proved.

Figure 2.3 illustrates how permanent (left hand graph) and transitory variances (middle graph) vary over the life cycle. Permanent uncertainty is decreasing: it is high in young years, possibly increasing towards the end of the life cycle. This leads to a flat profile of transitory uncertainty over age. The right hand graph shows that the model provides a good fit to the average autocovariance function.

### 2.3.3 Preference

As mentioned in Section 2.2.2, progressive taxation provides consumption insurance through a direct channel of shock reduction and indirect channels of wealth and labor supply. It is therefore natural to ask our model to match empirical wealth and labor supply distributions when estimating parameters related to household prefer-
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We choose the 25th, 50th, and 75th percentiles of wealth and labor supply at each age from 26 to 60 as our targets. That is, we estimate the parameters such that the model generated profiles of wealth and labor supply over the life cycle can be as close to the data as possible.

Formally, let \( \theta = (\rho, \beta, \phi, \sigma, a) \) be the parameters to be estimated. Let \( \pi_{iA,t}(\theta), i = 1, 2, 3 \) be the 25th, 50th, and 75th percentiles of wealth at age \( t \), respectively, in the simulated data from the model, and \( \pi_{iH,t}(\theta), i = 1, 2, 3 \) be the percentiles of labor supply. The estimator of \( \theta \) solves the following problem:

\[
\min_{\theta} \sum_{t=26}^{60} \sum_{i=1}^{3} \left( L_A(\pi_{iA,t}(\theta)) + L_H(\pi_{iH,t}(\theta)) \right). 
\]

\( L_A(\pi_{iA,t}(\theta)) \) and \( L_H(\pi_{iH,t}(\theta)) \) are loss functions of wealth and labor supply profiles,

\[
L_A(\pi_{iA,t}(\theta)) = E \left[ (A_{j,t} - \pi_{iA,t}(\theta))(\pi^i - 1(A_{j,t} < \pi_{iA,t}(\theta))) \right],
\]

\[
L_H(\pi_{iH,t}(\theta)) = E \left[ (H_{j,t} - \pi_{iH,t}(\theta))(\pi^i - 1(H_{j,t} < \pi_{iH,t}(\theta))) \right],
\]

where \( \pi^1 = 0.25 \), \( \pi^2 = 0.5 \), and \( \pi^3 = 0.75 \).

Figure 2.4 shows that our model is able to capture wealth and labor supply distributions over the life cycle reasonably well. Table 2.1 displays the parameter estimates

\footnote{16}{To treat the heterogeneity in household disutility of labor, we assume half of households have high disutility and half have low disutility. Essentially, we use a binomial distribution of \( \phi^1 \) and \( \phi^2 \) to approximate the true distribution of \( \phi \).}

\footnote{17}{We also include the borrowing constraint \( a \) because it cannot be directly estimated from data.
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**Figure 2.4:** Profiles Over the Life Cycle: Model vs. Data

Notes: For each age, red, blue, and black circles are the 75th, 50th, and 25th percentiles of data, respectively, while solid lines of the same color are the corresponding percentiles of the simulated data from the model.

of the model. Most of the estimates are in line with estimates from the previous literature. The heterogeneity in discount factor (or equivalently, time preference rate) and in disutility of labor is key to the model’s empirical success. Without these two types of heterogeneities, it is impossible for the model to generate the wide dispersion of wealth and labor supply in the data. The variance of discount factor suggests that even very small difference in the degree of impatience can result in a big difference in wealth accumulation; the variance of disutility of labor suggests that households have very distinct preference for the trade-off between consumption and labor.
2.4 Results

In this section, we use our estimated structural model to answer questions brought up previously. In particular, we address the relative strength of different channels through which progressive taxation affects consumption insurance, and the optimal progressive taxation within the class of functions in equation \((2.3)\).

2.4.1 Insurance Effects under Progressive Taxation

Direct effect

The degree of progressivity, \(\tau\), measures the direct effect of progressive taxation. Section \([2.3.1]\) shows that the point estimate of \(\tau\) is 0.22. Thus progressive taxation reduces shocks to after-tax income by 22%.

Indirect effect
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Indirect effects of progressive taxation cannot be directly measured in the data. However, with the help of our estimated structural model, we are able to quantitatively evaluate their magnitude. Section 2.2.3 has shown that the elasticity of consumption with respect to a permanent shock to wage is the result of both direct and indirect effects:

\[
cov(\log C_{it}, \eta_{it}) = (1 - \tau)(1 - \chi - \chi') \frac{1}{\rho}
\]

where

\[
\chi = \frac{(1 - \rho)(\sigma + \tau)}{\rho + \sigma + (1 - \rho)\tau}
\]

With our point estimates of the parameters, \(\chi = -1.09\). Using the simulated data from our model, \(cov(C_{it}, \eta_{it}) = 0.64\). Thus, we obtain \(\chi' = 0.065\) and \(\frac{\chi'}{\rho} = 0.026\). Recall that \(\frac{\chi'}{\rho}\) measures the effect of wealth on current consumption response to after-tax wage shocks. This estimate implies that wealth reduces consumption response by 0.026%. In comparison, labor supply’s response to after-tax wage shocks \(\frac{\chi + \chi'}{\sigma + \tau} = -0.185\). That is, a 1% permanent after-tax wage shock would result in a 0.185% decline in labor supply.

Table 2.2 summarizes the decomposition of the consumption insurance effects under progressive taxation. The indirect effect is a little smaller than the direct effect \(-\tau\): the combined effect of labor and wealth reduces consumption response to a permanent wage shock by 18%; in contrast, the direct effect of progressive taxation
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reduces it by 22%. The indirect effect through labor is much larger than the indirect effect through wealth, with the reduction of consumption response 15.3% for labor and a mere 2.6% for wealth. Although the effect of labor supply is large, its actual adjustment is very small: A 1% positive permanent wage shock only results in 0.14% decrease in labor supply. This is because labor supply is quite inelastic given our estimates of \( \sigma = 5.34 \).

<table>
<thead>
<tr>
<th>Table 2.2: Decomposition of Insurance Effects under Progressive Taxation.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total</strong></td>
</tr>
<tr>
<td><strong>Wealth</strong></td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Estimates</td>
</tr>
</tbody>
</table>

Response to 1% Permanent Shock

<table>
<thead>
<tr>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.64%</td>
</tr>
</tbody>
</table>

2.4.2 Impact of Progressive Taxation on Wealth and Labor Supply

The above decomposition reveals the relative strength of the three channels affecting consumption insurance, but the three channels are not independent of each other. On the contrary, the speed of wealth accumulation and the adjustment of labor supply depend on the degree of progressivity. It is therefore natural to ask: How big is the distortion of progressive taxation quantitatively? Does it affect the
consumption insurance

To answer these questions, we compare mean life-cycle profiles and consumption insurance under progressive taxation and under a revenue-neutral proportional taxation. Figure 2.5 and Table 2.3 display the results.

In spite of the same overall tax burden, there is on average 32% more wealth accumulation, 5% more labor supply, and 5% more consumption under proportional taxation over the working life. Though the overall consumption insurance is similar in these two environments, the relative strength of channels is different. In particular, the direct effect of progressive taxation is almost all replaced by the indirect effect through wealth under proportional taxation. The strength of labor supply channel remains unchanged, because \( \tau \) is much smaller \( \sigma \). Essentially the distortion of progressive taxation on labor supply is making it less elastic. But since labor supply is already very inelastic, the distortion of progressive taxation is very small.

**Table 2.3:** Consumption Insurance under Progressive and Proportional Tax.

<table>
<thead>
<tr>
<th>Model</th>
<th>Total</th>
<th>Direct</th>
<th>Indirect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1 − ( \tau ))(1 − ( \frac{\chi'}{\rho} ) − ( \frac{\chi-\chi_{\infty}}{\rho} ))</td>
<td>( -\tau )</td>
<td>( -\frac{\chi'}{\rho} )</td>
</tr>
<tr>
<td>Progressive</td>
<td>0.64</td>
<td>−0.22</td>
<td>−0.026</td>
</tr>
<tr>
<td>Proportional</td>
<td>0.68</td>
<td>0</td>
<td>−0.132</td>
</tr>
</tbody>
</table>
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Figure 2.5: Mean Profiles under Progressive and Proportional Tax

![Graphs showing wealth, labor supply, consumption, and tax rate profiles under progressive and proportional tax systems over the life cycle.](image-url)
2.4.3 Welfare Implications

We calculate welfare defined in equation \((2.14)\) under different progressivity, holding labor income tax revenue constant. Specifically, we first use our structural model to calculate the net tax revenue for households aged 26-60 under the current German tax system. Then as we vary the degree of progressivity \(\tau\), we numerically estimate \(\lambda\) such that the net tax revenue is fixed. Finally, we calculate welfare under various combinations of \(\tau\) and \(\lambda\).

Figure 2.6 contrasts welfare against the degree of progressivity, holding tax revenue constant. As shown in the figure, the relationship is not monotonic: as the degree of progressivity increases, welfare increases at first but then declines sharply, suggesting that a limited progressivity is desirable but too much progressivity severely distorts labor supply and reduces lifetime resources for consumption. When we numerically solve the problem of maximizing welfare by choosing \(\tau\), we find that the optimal progressivity \(\tau_{opt} = 0.06\). As a comparison, our estimate of the degree of progressivity of the tax system in Germany is \(\tau^{Germany} = 0.22\) and in the U.S. is \(\tau^{US} = 0.15\).

Heathcote, Storesletten, and Violante (2014b) find that the optimal value for the US is \(\tau_{opt}^{US} = 0.062\) in a general equilibrium environment where consumption, labor supply, human capital, and public goods shape the optimal degree of progressivity. Interestingly, this estimate is similar to ours, though we emphasize the match of wealth and labor supply distributions over the life cycle in a partial equilibrium framework.
Notice that we have assigned equal weight in our welfare definition to households with different preferences and different initial wealth. This definition might have masked the impact of household heterogeneity on welfare. It is therefore natural to investigate the relationship between welfare and progressivity for households with different preferences and different initial wealth.

In the estimated version of our model, we have assumed that there are two types of discount factors and two types of disutility of labor. Figure 2.7 plots the relationship between welfare and progressivity for these four types of households. Comparing the two graphs in the left column with the two in the right column, we see that disutility of labor drives the shape of the relationship. However, it seems the preferred degrees of progressivity for households of different types are similar.
Figure 2.7: Progressivity and Welfare for Different Preferences

Figure 2.8 plots the relationship between welfare and progressivity for households at different quintiles of initial wealth distribution. The shape of the relationship is generally preserved regardless of initial wealth, though wealthy households prefer a slightly less progressivity than those who are less wealthy.

Figure 2.7 and 2.8 together emphasize that the optimal degree of progressivity depends more on the distribution of heterogeneity in household preferences, especially their disutility of labor, and less on the distribution of wealth.
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2.5 Conclusion

This paper studies consumption insurance and welfare under progressive taxation through the lens of a life-cycle model that matches the empirical distribution of wealth and labor supply. We find that consumption insurance is achieved through one direct channel of shock reductions and two indirect channels of wealth accumulation and labor supply under progressive taxation. In the case of Germany, the direct channel accounts for 22% reduction in consumption response to a permanent wage shock, while wealth and labor account for 2.6% and 15.3% reductions, respectively. The order of quantitative importance in consumption insurance is therefore progressivity, elastic labor supply, and wealth buffer. We also find that the optimal degree of progressivity is one that is closer to proportional taxation than to the current tax system, and the
optimal degree of progressivity is similar for households with different preferences and different initial wealth conditions.
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2.6 Appendices

2.6.1 SOEP Data and PSID Data

This analysis is based on data from the German Socio-Economic Panel (SOEP), a representative annual household panel survey with about 20,000 observations per year in Germany that started with roughly 6000 households in 1984. Wagner, Frick, and Schupp (2007) provide a detailed description of the data. We use all waves available (1983-2012) to estimate permanent income and income uncertainty measures. We use wealth data that are available in the 2002, 2007 and 2012 waves. The SOEP questionnaire for these waves included a special module that collected information about private wealth. The surveys asked about the market value of personally owned real estate (owner-occupied housing, other property, mortgage debt), financial assets, tangible assets, private life and pension insurance, consumer credit, and private business equity (net market value; own share in case of a business partnership). The wealth balance sheets referred to the personal level, so in the case of jointly owned assets, the survey explicitly asked about each person’s individually owned shares.

The PSID data set is based on a sample of roughly 5000 households that were interviewed in 1968. From 1996 on data are only available every second year (1998, 2000, etc.). Of these, about 3000 were sampled to be representative of the nation as a whole and about 2000 were low-income families that had been interviewed previously.

\footnote{For a more detailed description, see The Panel Study of Income Dynamics (2016).}

We use the Cross-National Equivalent File (CNEF) that contains equivalently defined variables for the Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (SOEP). In each year, we aggregated data on hours worked, labor income, and wealth to the household level and deflated to 2005 purchasing power parity prices in Euro using the consumer price index provided by the Federal Statistical Offices.

Our measure of labor income, available from 1984 to 2012 in the SOEP and from 1970 to 2012 in the PSID, combines annual household pre-government labor income that this household received in the calendar year previous to the survey year. More specifically, labor income is the sum of income from primary job, secondary job, self-employment, service pay, 13th month pay, 14th month pay, Christmas bonus pay, holiday bonus pay, miscellaneous bonus pay, and profit-sharing income. Wages are calculated as household labor income divided by household hours of work.

We define as household heads the individual that has highest earnings and highest wage in each year. If two individuals have the identical values on these variables, we assign the person that is defined by the SOEP or PSID as head as household head.

\(^{19}\)For a more detailed description, see Lillard, Giles, Grabka, and Nargis (2002) for the CNEF.
We drop individuals with non positive hours of work, negative wages or wages larger than 150 Euro. Moreover, we drop individuals that have the same age in two different years. After this, our SOEP sample includes 3,696 in 1983 and 5,565 in 2012. The PSID sample includes 2,782 households in 1970, and 5,456 in 2012.

### 2.6.2 Numerical Solution to the Households’ Problem

**Working life**

Let the value function before retirement be $V_{it}(A_{it}, Z_{it}, \epsilon_{it})$, then we have the recursive relation

$$V_{it}(A_{it}, Z_{it}, \epsilon_{it}) = u(C_{it}, H_{it}) + \beta E_tV_{it+1}(A_{it+1}, Z_{it+1}, \epsilon_{it+1})$$
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Figure 2.10: Growth Rate of Wages by Age and Variance of Log Wages

(a) Growth Rate of Wages  (b) Variance of Log Wages

Figure 2.11: Permanent and Transitory Variances by Age

(a) Permanent Variances  (b) Transitory Variances  (c) Average Auto-Covariance Function
The first order conditions with respect to $C_t$ and $H_t$ are

\[ u^C(C_t, H_t) = (1 + \tau_c)(\beta E_t V_{it+1}^A + \mu_t) \]
\[ -u^H(C_t, H_t) = (W_t - TX'(W_t H_t)W_t)(\beta E_t V_{it+1}^A + \mu_t) \]

where $\mu_t$ is the Lagrange multiplier on the constraint of $A_{it+1} \geq -aZ_{it}$. The envelope condition is

\[ V_{it}^A = \beta(1 + r(1 - \tau_A))E_t V_{it+1}^A \quad (2.15) \]

As such, we have

\[ u^C(C_t, H_t) = \beta(1 + r(1 - \tau_A))E_t u^C(C_{it+1}, H_{it+1}) + (1 + \tau_c)\mu_t \]
\[ u^C(C_t, H_t) = -\frac{(1 + \tau_c)u^H(C_t, H_t)}{W_t - TX'(W_t H_t)W_t} \]

Given the specific forms of the utility function and the tax function, we have

\[ u^C = C^{-\rho} \]
\[ u^H = -\phi H^\sigma \]
\[ TX'(W_t H_t) = 1 - \lambda(1 - \tau)(W_t H_t)^{-\tau} \]
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The key equations for solving the households’ problem are

\[ C_{it}^{-\rho} = \beta (1 + r (1 - \tau_A)) E_t \left[ C_{i,t+1}^{-\rho} \right] + (1 + \tau_c) \mu_t \]

\[ C_{it}^{-\rho} = \frac{(1 + \tau_c) \phi}{\lambda(1 - \tau)} H_{it}^{\gamma+\tau} W_{it}^{\tau-1} \]

Retirement

After retirement, households have no labor supply choices. Let \( V^R_{it}(M_{it}) \) be the value function of households after retirement, then we have the recursive formulation:

\[ V^R_{it}(M_{it}) = u^R(C_{it}) + \beta \left( p_t^S V^R_{i,t+1}(M_{i,t+1}) + (1 - p_t^S) u^R(M_{i,t+1}) \right) \]

The first order condition follows

\[ (u^R)'(C_{it}) = \beta (1 + r (1 - \tau_A)) \left( p_t^S (u^R)'(C_{i,t+1}) + (1 - p_t^S) (u^B)'(M_{i,t+1}) \right) \]

Given the specific function forms of \( u^R(\cdot) \) and \( u^B(\cdot) \),

\[ C_{it}^{-\rho} = \beta (1 + r (1 - \tau_A)) \left( p_t^S C_{i,t+1}^{-\rho} + (1 - p_t^S) \phi M_{i,t+1}^{-\rho} \right) \]

In the last possible period of the life cycle, because households die with certainty, the
budget constraint follows

\[ M_{i,T+1} = (1 + r(1 - \tau_A))(M_{i,T} - C_{i,T}). \]

Combined with the first order condition

\[ C_{iT}^\rho = \beta (1 + r(1 - \tau_A)) \phi M_{i,T+1}^\rho, \]

we obtain the last period’s consumption function

\[ C_{iT} = \frac{1 + r(1 - \tau_A)}{(1 + r(1 - \tau_A)) + (\beta \ast (1 + r(1 - \tau_A)) \ast \phi) / 1/\rho} M_{iT}. \]

It is transparent that \( \phi \) affects the marginal propensity to consume. The higher \( \phi \) is, the more bequest households would leave. One subtlety here is that in the model we have assumed that it is possible for households to leave bequest every period after retirement. This is important for the model’s capability to generate the empirical wealth accumulation over the life cycle, especially the level of wealth close to retirement. Otherwise, had we assumed households could only leave bequest in the very last period (\( T \)), because of discounting, the bequest motive would have limited impact on the level of wealth close to retirement and the model would not be able to capture the empirical wealth profile over the life cycle.
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**Endogenous Gridpoints Method**

We use backward policy function iteration to solve the households’ problem: Given the policy rules $C^{*}_{t+1}(\cdot)$ and $H^{*}_{t+1}(\cdot)$ in period $t + 1$, we calculate the policy rules in period $t$. To speed up the numerical solution, we use a variant of the endogenous gridpoints method first proposed by Carroll (2006b). For example, for policy rules before retirement, given $A_{t+1}, Z_t, \epsilon_t$, and the optimal decision $C^*_t(A_{t+1}, Z_{t+1}, \epsilon_{t+1})$, we will be able to calculate $E_t[\rho_t C^{-}\rho_{t+1}(A_{t+1}, Z_{t+1}, \epsilon_{t+1})]$ and hence $C_t$ and $H_t$ through the first order conditions. The endogenous grid is then calculated by $A_t = \frac{1}{(1 + r(1 - \tau))}(A_{t+1} + (1 + \tau)C_t - (W_t H_t - TX(W_t H_t)))$. With $C_t(A_t, Z_t, \epsilon_t)$ and $H_t(A_t, Z_t, \epsilon_t)$ on the endogenous grid, we can interpolate them to obtain their values on the pre-specified exogenous grid.
Chapter 3

The Human Factor: China’s Urbanization and the Saving Puzzle

3.1 Introduction

Ever since the Open Door Policy was announced by Deng Xiaoping in 1978, China has taken great strides in economic growth. Averaging around 10% for the past 35 years, China’s real GDP growth is one of the longest and greatest in modern economic history. In the meantime, China has witnessed a phenomenal yet puzzling macroeconomic trend: its household saving rate rose persistently from 15% in the early 1990s to 30% in 2012. This trend is at odds with standard off-the-shelf economic
CHAPTER 3. THE HUMAN FACTOR: CHINA’S URBANIZATION AND THE SAVING PUZZLE

theories. Under such theories, high growth rates of income imply low saving rate, as households would want to borrow from their future income to smooth consumption.

Many theories have been put forth to explain China’s high and rising saving rate, but very few focus on China’s labor market, especially the role of internal migration in the process of urbanization. In this paper, we document a strong correlation between the degree of urbanization and the increase in urban household saving rates across provinces in China: Provinces that experienced more rapid urbanization saw a bigger rise in urban household saving rates. We highlight one channel through which urbanization affects household saving rates: Urban and rural households differ in their wealth targets because of the difference in idiosyncratic uncertainty of income and thus urbanization accompanies wealth accumulation of new urban dwellers as they transition from rural to urban jobs. Quantitatively, at least 8% of the increase in household saving rates can be attributed to urbanization.

Figure 3.1 shows that there is a strong correlation between changes in agricultural share of employment and changes in urban household saving rates across provinces in China between 1995 and 2010. Agricultural share of employment is arguably a good proxy for the degree of urbanization. Although urban household saving rates across all provinces in China have increased, those with faster urbanization saw a bigger rise. The negative relationship implies that wealth accumulation of new urban dwellers plays a key role in explaining the cross-sectional differences in household saving rate changes at the provincial level.
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Figure 3.1: Urbanization and Saving Rate across Provinces in China

The dramatic increase in household saving rates during periods of high growth, however, is not limited to the case of China. In fact, when Japanese and Korean economies took off, there were also puzzling increases in household saving rates that sparked a huge literature offering various explanations. Figure 3.2 plots the relationship between urban share of population and private saving rates in Korea, Japan, China, and India.\(^1\) There is a strong association of the degree of urbanization and sav-

---
\(^1\)Private saving rates and household saving rates are highly correlated in these countries. As we do not have a long time series of household saving rates, we use private saving rates in Figure 3.2 to show the broad trend of saving rates. Appendix shows that household saving rates parallel private saving rates in Japan and Korea, with a correlation of at least 60%.
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Figure 3.2: Urbanization and Saving Rate across Countries
CHAPTER 3. THE HUMAN FACTOR: CHINA’S URBANIZATION AND THE SAVING PUZZLE

Saving rates. Japan and Korea’s saving rates almost parallel their paths of urbanization: Saving rates rose as urban share of population increased and plateaued when urbanization finished. China and India are still on their path of urbanization: Both have seen a rise in saving rates that follow the increase in their urban share of population.

The cross-sectional pattern in Figure 3.1 and longitudinal pattern in Figure 3.2 are consistent with the idea that when people leave rural areas for non-agricultural jobs in urban areas, they have a higher saving target and thus their saving rate increases. Considering that social and economic conditions vary tremendously across regions, the relationship between urbanization and household saving rates seems very robust. In this paper, we focus on China because its rise in household saving rates is the most recent and most dramatic in East-Asian countries.

There are various reasons why urban households might have a higher wealth target than rural households in China. We explore the extent to which difference in idiosyncratic income risks contributes to the increase in household saving rates in the process of urbanization. Rural households, who typically work in the agricultural sector, face little idiosyncratic uncertainty of their labor income while urban households, who typically work in manufacturing and service sectors, face a much higher uncertainty. Higher uncertainty results in a higher wealth target of urban households. When rural households move to jobs in urban areas, their saving rate would increase as they save toward the higher wealth target. Urbanization, therefore, can be thought of as waves of new arrivals in urban areas who start accumulating wealth and have a
high saving rate. The continuous flow of households from rural to urban areas leads to a persistent rise in saving rates, as more and more people join the group of high saving rate.

To quantitatively assess the contribution of urbanization to the rise in household saving rates, we develop a simple model that features two sectors, urban and rural, and three groups of households, native urban households, rural households, and migrants. Perhaps for an abuse of language, we use migrants and new urban residents interchangeably to refer to rural households who newly become urban residents. We calibrate our model such that it matches some of the stylized facts from micro and macro data in China. As a conservative result, our model suggests that at least 8% increase in household saving rates between 1995 and 2007 can be attributed to urbanization.

While we put emphasis on the relationship between urbanization and household saving rates, we recognize that a number of other factors might have played a role in contributing to the rise in household saving rates. In particular, habit in household consumption behaviors is a good candidate for explaining the relationship between saving and growth. In fact, habit is consistent with our finding that regions with faster urbanization saw a bigger rise in household saving rates, as regions with greater increase in urban population are also regions with more people who stick to their old consumption habit but have more income. Still, our model result is useful in that it serves as a baseline for the association between urbanization and rise in household
saving rates and provides a lower bound of the role of urbanization.

The rest of the paper is organized as follows. Section 3.2 discusses related literature. Section 3.3 presents some statistical evidence about the relationship between urbanization and household saving rates. Section 3.4 describes our model and section 3.5 calibrates it to some stylized facts from micro and macro data. In section 3.6, we present simulated results from our calibrated model. Section 3.7 concludes.

### 3.2 Related Literature

Our paper is mainly related to three strands of literature. First, we contribute to a big literature that offers various explanations of high and rising household saving rate in China. Chamon, Liu, and Prasad (2013) find that rising income uncertainty and pension reforms account for two-thirds of the increase between 1989 and 2009. Song and Yang (2010) emphasize that flattening of age-earning profiles is responsible for the increase, especially among young households. Chamon and Prasad (2010) attribute China’s high urban saving rate to increasing private burden of education, health and housing. Wei and Zhang (2011) find that half of the increase in China’s household saving rate comes from increased pressure in the marriage market due to rising sex ratio imbalance. Coeurdacier, Guibaud, and Jin (2015) show that the interaction between growth differentials and household credit constraints explains 40% of the divergence between aggregate saving rates in China and the United States. Curtis,
Lugauer, and Mark (2015) show that demographic changes alone accounts for over half of the increase in household saving rate between 1955 and 2009. While these explanations focus on China-specific factors, it should be noted that most of them were explored when Japanese and Korean household saving rates were rising in their high growth periods (see for instance, Hayashi (1986), Park and Rhee (2005)). In this paper, though our model is calibrated to the Chinese economy, it can be readily applied to other economies that experience similar transitions that involves rapid urbanization.

Second, our paper is connected to the literature on the relationship between saving and growth since Houthakker (1961) and Modigliani (1970). A large body of empirical evidence suggests that growth causes saving, but it is difficult for growth to cause saving in standard growth models. Carroll, Overland, and Weil (2000) shows that a standard endogenous growth model with habit formation can generate growth-to-saving causality qualitatively similar to that observed in the data. While we believe that habit is an important mechanism underlying the growth and saving relationship, it is interesting to examine which group among rural, urban, and migrant households contribute more to the increase in household saving rates. In fact, the existence of habit might reinforce the role of migrants because they experience the most dramatic increase in income when moving to urban areas. We focus on a model without habit to highlight the quantitative strength of our mechanism: In

\(^{2}\)For instance, Deaton and Paxson (1994); Carroll and Weil (1994) and more recently, Attanasio, Picci, and Scorcu (2000); Loayza, Schmidt-Hebbel, and Servén (2000); Hausmann, Pritchett, and Rodrik (2005); Gourinchas and Jeanne (2013).
our model, wealth accumulation of a continuous flow of labor from rural to urban areas contribute to the increase in household saving rates. [Horioka and Wan (2007)] find that China’s household saving rate is consistent with the existence of inertia or persistence. We show that this unobserved persistence factor could be the persistent process of urbanization.

Finally, our paper is related to an emerging literature that examine the role of migrants in structural transformation. [Garriga, Tang, and Wang (2015)] show that relocation of workers from rural to urban areas accounts for 80% of housing price movements in China. [Tombe and Zhu (2015)] studies the contribution of reductions in migration costs to aggregate productivity growth in China. Though our paper has a different focus on China’s saving rate, the strength of our mechanism relies on the continuous reallocation of households to urban areas where income is high. Urban-to-rural income ratio has almost doubled in China over the past three decades. New urban dwellers, whose saving rates are among the highest, therefore take up an increasingly big share of the economy and keep increasing aggregate household saving rates.

### 3.3 Statistical Evidence

Over the past several decades, China has experienced many social, economic, institutional, and demographic changes. These changes tend to be incremental and
often co-move with each other. For instance, Garriga, Tang, and Wang (2015) show that house price grows faster in cities with larger inflow of rural migrants from 1998 to 2007 in China. Precautionary saving of migrants and saving for down-payment of young households could both drive up household saving rate. Hence it is difficult to disentangle different factors at play without a comprehensive structural model that builds into these mechanisms. Nevertheless, in this section we perform a simple regression analysis that shows urbanization can be important in explaining the cross-sectional variation of the increase in household saving rates.

We combine several data sources and construct an unbalanced panel at the provincial level from 1995 to 2010 that includes urban household saving rate and a number of potential explanatory variables. Specifically, we use agricultural share of employment, rather than urban share of population, as a proxy for the degree of urbanization, because it not only reflects the nature of jobs in rural areas but is a measure of people who probably actively save. To capture the effect of income and demographics, we include urban household disposable income, dependent ratio of the young and the old from China’s Statistical Yearbook. We also include the share of labor force that work for state-owned-enterprise (ssoe) and sex ratio (srt) constructed in Wei and Zhang (2011). The former is intended to capture the effect of precautionary motive on saving: employees of state-owned-enterprise used to have ”iron bowl” and their jobs were more secure than jobs in the private sector. The latter is used to capture

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3Dependent ratio of the young is defined as the number of people aged 0-14 over those aged 15-65, and dependent ratio of the old the number of people aged 65 and above over those aged 15-65.
the competitive saving motive for marriage. To capture the effect of rising house
price, we use average residential house price, measured as price per square meters,
from CEIC.

Table 3.1 presents results from regressions relating urban household saving rates
to the potential explanatory factors mentioned above. We add year fixed effect and
province fixed effect to control for aggregate shocks and unobserved idiosyncratic
factors at the provincial level. Column (1) shows that urban household saving rates
are highly correlated with agricultural share of employment. The coefficient indicates
that 1% drop in agricultural share of employment is associated with 0.16% increase
in urban household saving rate. This number, however, is not to be interpreted
as the effect of urbanization on saving rate. Clearly other factors are at play and
agricultural share of employment might pick their effects. Perhaps more importantly,
Figure 3.1 shows that all provinces in China have increased urban household saving
rate from 1995 to 2010. Identification in Table 3.1 comes from variation around the
average national trend and thus the coefficient in column (1) only reflects the role
of urbanization in explaining cross-sectional variation of the increase in household
saving rate across provinces in China.

Moving to column (2), an increase in urban household income is associated with
an increase in urban household saving rate, although the correlation seems weak.
Columns (3)-(6) show that other factors play an even weaker role in explaining the
cross-sectional variation of the increase in household saving rate. Notice that we
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Table 3.1: Urban Household Saving Rate at the Provincial Level 1995-2010

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>Urban Household Saving Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agricultural Share of Employment</td>
<td>-0.160*** (0.048)</td>
</tr>
<tr>
<td>Urban Income (log)</td>
<td>0.071* (0.037)</td>
</tr>
<tr>
<td>ssoe</td>
<td>0.026 (0.045)</td>
</tr>
<tr>
<td>srt</td>
<td>0.104 (0.095)</td>
</tr>
<tr>
<td>Young Dependent Ratio</td>
<td>0.069 (0.058)</td>
</tr>
<tr>
<td>Old Dependent Ratio</td>
<td>-0.023 (0.027)</td>
</tr>
<tr>
<td>Residential House Price (log)</td>
<td>-0.000 (0.019)</td>
</tr>
</tbody>
</table>

Year fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
Province fixed effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
adj. $R^2$ | 0.710 | 0.707 | 0.622 | 0.624 | 0.700 | 0.753 | 0.735 |
N | 463 | 463 | 369 | 369 | 453 | 379 | 275 |

Notes. This table presents coefficients from regressions relating urban household saving rates to a number of potential explanatory factors. Throughout, standard errors are in parentheses. *** indicates that coefficients are statistically different than 0 at the 1% confidence level.
have fewer observations as we move along columns as data of some of the variables are not available after 2007. The last column suggests that the correlation between urbanization and urban household saving rate remains similar when controlling for other factors.

Given various econometric problems related to small sample, endogeneity, and omitted variables, Table 3.1 is only suggestive. However, it points out that urbanization is closely related to the increase in household saving rates both cross-sectionally and over time. To fully assess the impact of urbanization on aggregate household saving rate, we use a structural model, to which we now turn.

3.4 Model

In this section, we develop a simple dynamic stochastic general equilibrium model of China’s dual labor markets. The economy has two sectors, rural and urban, and three groups of households: rural households, migrants, and native urban households. All types of households work for $J$ years and then retire. Households continue to live for $J'$ years after retirement before they die with certainty.
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3.4.1 Retirement

We assume that pension is deterministic and equal to a fraction $\lambda$ of households’ permanent income in the last year of their working life. This assumption simplifies numerical solution and $\lambda$ can be interpreted as the replacement ratio. $\lambda$ differ for rural and urban households. We use $\lambda^u$ and $\lambda^r$ to denote the replacement ratio for urban and rural households, respectively.

Because of the deterministic nature of retirement, we are able to write down analytically the value function and consumption of retirees. For a household receiving its first pension payment $P$ in year $t$ with beginning-of-year bank balance $B_t$, it solves the following optimization problem

$$V_t^R(B_t, P) = \sum_{i=0}^{J-1} \beta^i u(C_{t+i})$$

subject to

$$M_t = B_t + P$$

$$A_t = M_t - C_t$$

$$B_{t+1} = RA_t$$

where $M_t$ is money resources available for consumption in year $t$ and $A_t$ is end-of-year
balance. It is straightforward to show that

\[ V_t^R(B_t, P) = \frac{1 - \left(\frac{\beta(R \beta)^{1-\rho}}{\rho}\right)^{J_r}}{1 - \frac{1}{\rho} \frac{\beta(R \beta)^{1-\rho}}{\rho}} u(C_t), \]

where

\[ C_t = \frac{1 - \left(\frac{(R \beta)^{1/\rho}}{R}ight)}{1 - \left(\frac{(R \beta)^{1/\rho}}{R}ight)^{J_r}} \left(1 - \frac{1}{\rho} \frac{1}{R} P + B_t\right). \]

### 3.4.2 Urban Households

Households working in urban areas face idiosyncratic income risks. In particular, income for urban households follows a permanent-transitory type process,

\[ Y_t = P_t \Theta_t, \]
\[ P_t = \Gamma_t P_{t-1} \Psi_t. \]

where \( Y_t \) is household disposable income at time \( t \), \( P_t \) is permanent income, \( \Gamma_t \) is the deterministic growth rate of permanent income, \( \Theta_t \) and \( \Psi_t \) are transitory and permanent shocks to income, respectively. We assume that permanent shocks are i.i.d. log normally distributed,

\[ \log \Psi_t \sim N\left(-\frac{1}{2} \sigma_p^2, \sigma_p^2\right), \]
and transitory shocks include unemployment shock and are conditionally log normally distributed,

\[ \Theta_t = \begin{cases} 
0, & \text{with probability } p \\
\Xi_t, & \text{otherwise} 
\end{cases} \]

where \( \Xi_t \) is i.i.d. log normally distributed,

\[ \log \Xi_t \sim N\left(-\frac{1}{2}\sigma^2_\xi, \sigma^2_\xi\right). \]

For a native urban household of cohort \( t_0 \) (who begins working in year \( t_0 \)), its optimization problem at time \( t \) can be summarized as follows,

\[
V^{u}_{t,t_0}(M_t, P_t) = \max_{C^{u}_{t+1}} \left[ \sum_{s=t_0}^{t+J-1} \beta^{s-t} u (C^{u}_s) + V^{R}_{t_0+J}(B_{t_0+J}, \lambda^u P_{t_0+J-1}) \right]
\]

subject to

\[
\begin{align*}
M_s &= B_s + Y_{s,s-t_0}, \\
A_s &= M_s - C^{u}_s, \\
B_{s+1} &= RA_s, \\
Y_{s,s-t_0} &= P_{s,s-t_0} \Theta_{s,s-t_0} \\
P_{s,s-t_0} &= \Gamma_{s,s-t_0} P_{s-1,s-1-t_0} \Psi_{s,s-t_0}
\end{align*}
\]

if \( s \leq t_0 + J - 1 \),

where we have used two subscripts for income to indicate time and where households
are in their life cycle. For instance, $Y_{s,s-t_0}$ is the income at year $s$ of a household who is working in $(s-t_0)$th year.

### 3.4.3 Rural Households

We assume that households working in rural areas face no idiosyncratic risks in their income. In other words, income is deterministic over their life cycle and therefore rural households solve a perfect foresight problem,

$$ V_{t_0}^r(M_t) = \max_{C_t} \sum_{s=t}^{t_0 + J - 1} \beta^{s-t} u(C_s^r), $$

subject to

$$ M_s = B_s + Y_{s,s-t_0}, $$

$$ A_s = M_s - C_s^r, $$

$$ B_{s+1} = R A_s, $$

$$ Y_{s,s-t_0} = \Gamma_{s,s-t_0}^r Y_{s,s-t_0} \quad \text{if } s \leq t_0 + J - 1, $$

$$ Y_{s,s-t_0} = \lambda_r Y_{s,J-1} \quad \text{if } s > t_0 + J - 1, $$

$$ A_s \geq 0. $$
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3.4.4 Urbanization

In each period $t$, rural households receive a shock about whether staying in rural areas or moving to urban areas. However, this shock is unanticipated; rural households do not take the distribution of the shock into account when they make their lifetime optimization choices about consumption. By this assumption, urbanization is a process beyond the control of rural households. In reality, we think of it as an expansion of urban areas: places where most people engage in agricultural production become cities where people switch to jobs in industrial production and services.

3.5 Calibration

In this section, we calibrate our model to the Chinese economy. Income process, idiosyncratic uncertainty, and the distribution of mobility cost are at the heart of the calibration because together they determine the rate of urbanization and consequently its contribution to the increase in saving rate. We assume that households work for $J = 36$ years (from age 25 to 60) and retire for $J^r = 15$ years before they die with certainty. $J$ and $J^r$ are chosen to be consistent with the fact that official retirement age of males in China is 60 and life expectancy is between 70 and 75 from 1990 to 2010.
3.5.1 Income

The deterministic growth rate of income in our model, \( \{ \Gamma_{t,s} \}_{s=0}^{J-1} \) for urban households and \( \{ \Gamma_{t,s}^r \}_{s=0}^{J-1} \) for rural, has an aggregate component that varies with time \( t \) (year effect) and a life-cycle component that varies with the progression of an individual household’s experience \( s \) (age effect). We assume that the aggregate component and the life-cycle component are multiplicative:

\[
\Gamma_{t,s} = \Gamma_t G_s \\
\Gamma_{t,s}^r = \Gamma_t^r G_s^r
\]

where \( \Gamma_t \) and \( \Gamma_t^r \) indicate aggregate growth rates of urban and rural income, respectively, at time \( t \); \( G_s \) and \( G_s^r \) denote age-specific growth rates of urban and rural income, respectively.

Figure 3.3 depicts the age profiles of average urban and rural income in 1995, 2002, and 2008, using survey data from China Household Income Project (CHIP).

Despite the noise in the survey data, it is clear that for either rural or urban households, average income at every age increases from 1995 to 2008 roughly at the same pace, although income of urban young households seem to have increased a little bit faster during this period, which is consistent with the finding of the flattening of age-earning

\[\text{eeb8cdd1}\]
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profiles in Song and Yang (2010). As our primary focus is how much urbanization contributes to the increase in household saving rates, we assume that \( \{G_s^{(j)}\}_{s=0}^{J-1} \) and \( \{G_r^{(j)}\}_{s=0}^{J-1} \) do not vary with cohorts. In other words, we do not allow the change of age-income profiles to drive up household saving rates.

Figure 3.4 shows that urban real income increases at a faster rate than rural real income from 1995 to 2008, using data from China Statistical Yearbook. The average growth rate of urban real income is 8.06% while it is 6.72% for rural real income during this period. Thus we use \( \Gamma_t = \Gamma = 1.0806 \) and \( \Gamma_r = 1.0672 \). To be consistent with the age profile in the micro data from CHIP, we assume that the life-cycle component of income is quadratic:

\[
G_s = \exp\{as^2 + bs\},
\]
\[
G_r = \exp\{a^r s^2 + b^r s\},
\]

where \( s = 0, 1, \cdots, 35 \). We then calculate what average income by age would have been had income grown only at the aggregate growth rate \( \Gamma_t \) and \( \Gamma_r \) from 1995 to 2008. The difference across ages identifies that \( a = -0.00007309, b = 0.00333734, a^r = -0.00004556, \) and \( b^r = 0.00244922 \). Figure 3.5 plot the implied life cycle profiles (on top of aggregate growth rates) of urban and rural household income.
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Figure 3.3: Urban and Rural Average Income by Age
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Figure 3.4: Aggregate Growth Rate of Real Income

Figure 3.5: Life Cycle Growth Rate of Real Income
3.5.2 Uncertainty

To calibrate uncertainty in urban household labor income, we use results from previous literature using micro data. [Meng (2003)] uses the survey of Urban Household Income, Expenditure and Employment (UHIEE) to estimate the probability of unemployment in urban areas to be 0.09 0.14 from 1995-1999. [Feng, Hu, and Moffitt (2015)] use newly available micro data from Urban Household Survey (UHS) and find unemployment rate to be 6.6% during 1995-2002 and 10.9% during 2002-2009, much higher than official rates. We therefore choose $p = 0.09$ as the probability of unemployment in our model.

[Chamon, Liu, and Prasad (2013)] estimate variances of permanent and transitory shocks in urban labor income using China Health and Nutrition Survey (CHNS) from 1989 to 2009. Their estimates of variances of permanent shocks range from 0.012 in 1993 to 0.043 in 2006 and of transitory shocks range from 0.061 in 1991 to 0.181 in 2006. We choose $\sigma_p^2 = 0.02$ as the variance of permanent shocks and $\sigma_\xi^2 = 0.06$ as the variance of transitory shocks. We use a low estimate of transitory shocks in light of the measurement error that might be picked up by transitory shocks.

3.5.3 Preference

There are two parameters that govern wealth accumulation of households facing income uncertainty: the coefficient of relative risk aversion $\rho$ and the discount factor
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<table>
<thead>
<tr>
<th>Year</th>
<th>Y</th>
<th>$W_1$</th>
<th>$\frac{W_1}{Y}$</th>
<th>$W_2$</th>
<th>$\frac{W_2}{Y}$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>14329</td>
<td>12889</td>
<td>0.90</td>
<td>28389</td>
<td>1.98</td>
<td>5420</td>
</tr>
<tr>
<td>2002</td>
<td>23768</td>
<td>37916</td>
<td>1.60</td>
<td>121945</td>
<td>5.13</td>
<td>5364</td>
</tr>
<tr>
<td>2008</td>
<td>56672</td>
<td>80594</td>
<td>1.42</td>
<td>539355</td>
<td>9.52</td>
<td>3622</td>
</tr>
</tbody>
</table>

Notes. Summary statistics are calculated for households aged 25-60 using data from China Household Income Project. $W_1$ and $W_2$ are household wealth excluding and including housing value, respectively. Income and wealth are in nominal values (yuan).

We set $\rho = 2$, a common value estimated in the literature, and use the method of simulated moments to estimate $\beta$ by targeting the average wealth-to-income ratio of working-age urban households. Average wealth-to-income ratio, however, has been changing over time. This is partly because of active saving by households and partly because of the substantial appreciation of housing value in China. Table 3.2 presents urban income and wealth summary statistics from CHIP. It is clear that while the ratio of non-housing wealth over income has nearly doubled from 1995 to 2008, the ratio of total wealth (including housing) over income has more than quadrupled. China started a housing market reform in 1998 when housing became substantially privatized and housing value began increasing rapidly. For this reason, we choose to ask the model to match the wealth-to-income ratio in 2002 when housing value had not increased substantially. This gives us $\beta = 1.03$.  

\footnote{For a description of China’s policies, see Garriga, Tang, and Wang (2015).}
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3.5.4 Urbanization

We assume that the annual percentage of rural population moving to urban areas is 2.3%, which is in line with the path of urbanization in China where rural population has declined from 71% in 1995 to 54% in 2007.

3.5.5 Other Parameters

Real interest rate

Average real interest between 1995 and 2008 in China is 2.7%. We therefore choose gross interest rate $R = 1.027$.

Replacement ratio

Urban replacement ratio $\lambda^u$ is set at 60%, a number that is in line with the results in Song and Yang (2010), and Chamon, Liu, and Prasad (2013). We set rural replacement ratio $\lambda^r$ at 0, partly because there is no reliable estimate for rural replacement ratio other than common recognition that it is very low in China (Herd, Hu, and Koen (2010)), partly because we want to downplay the contribution of urbanization to the increase in urban household saving rates in order to obtain a low estimate – migrants need to accumulate less wealth to reach their target wealth level in urban areas if their wealth is higher upon moving. The replacement ratio for migrants is also set at $\lambda^r$, because migrants typically do not enjoy the same benefits that native
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Table 3.3: Model Parameters

<table>
<thead>
<tr>
<th>Estimates</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Aversion</td>
<td>$\rho$</td>
<td>2</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>1.03</td>
</tr>
<tr>
<td>Real interest Rate</td>
<td>$r$</td>
<td>0.27</td>
</tr>
<tr>
<td>Length of Working Life</td>
<td>$J$</td>
<td>36</td>
</tr>
<tr>
<td>Length of Retirement</td>
<td>$J_r$</td>
<td>15</td>
</tr>
<tr>
<td>Urban Income Growth Factor</td>
<td>$\Gamma$</td>
<td>1.0806</td>
</tr>
<tr>
<td>Rural Income Growth Factor</td>
<td>$\Gamma_r$</td>
<td>1.0672</td>
</tr>
<tr>
<td>Urban Probability of Unemployment</td>
<td>$p$</td>
<td>0.09</td>
</tr>
<tr>
<td>Variance of Permanent shock</td>
<td>$\sigma^2_p$</td>
<td>0.02</td>
</tr>
<tr>
<td>Variance of Transitory shock</td>
<td>$\sigma^2_\xi$</td>
<td>0.06</td>
</tr>
<tr>
<td>Life-cycle Growth of Urban Income</td>
<td>$a$</td>
<td>$-7.309 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>$3.337 \times 10^{-3}$</td>
</tr>
<tr>
<td>Life-cycle Growth of Rural Income</td>
<td>$a^r$</td>
<td>$-4.556 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>$b^r$</td>
<td>$2.449 \times 10^{-3}$</td>
</tr>
<tr>
<td>Urban Replacement Ratio</td>
<td>$\lambda_u$</td>
<td>0.6</td>
</tr>
<tr>
<td>Rural Replacement Ratio</td>
<td>$\lambda_r$</td>
<td>0</td>
</tr>
<tr>
<td>Rural Population Growth Rate</td>
<td></td>
<td>-2.7%</td>
</tr>
</tbody>
</table>

urban households have due to the household registration system.

Table 3.3 summarizes the parameters values in our simple model.

3.6 Results

In this section, we present results from our calibrated model and discuss the role of urbanization in the increase in household saving rates.
3.6.1 Household Saving in Urban and Rural Areas

Figure 3.6 compares between urban and rural households the wealth accumulation and saving rates over the life cycle. The top graph shows that because of uncertainty in urban income, urban households save more than rural households due to precautionary motives. But with the progress of life cycle, saving for retirement become the dominant motive for saving. And because rural households have a much lower replacement ratio (zero under our assumption) than urban households, their wealth accumulation is more aggressive in the later part of their working life. In fact, the bottom graphs shows that in our calibrated model rural households’ saving rate is higher than urban households after age 37.

What happens when a rural household move to urban areas? There are two important forces at work: The first is urban-rural income differential and the second is uncertainty in urban income. That urban income is higher than rural income implies that new urban residents would have a wealth-to-income ratio lower than desired even if there is no income uncertainty. Income uncertainty raises the target wealth-to-income ratio of these new urban residents and further increases their saving motive. As such, rural households who recently become urban residents have a strong desire to save, which contributes to the increase in the aggregate saving rate in the economy.

Figure 3.7 displays the saving rates over the life cycle of rural households who move in various ages. Migrants accumulate wealth aggressively at the start of their
Figure 3.6: Saving and Wealth of Urban and Rural Households
migration, but as their wealth-to-income ratio approaches that of urban households, their saving rate also converges to that of urban households. Migrants who move earlier tend to contribute more to the aggregate saving rate because the difference in saving rates of migrants and rural households decrease with age. Urbanization consists waves of rural households becoming urban residents. Each wave contributes to the increase in aggregate saving rates or maintain it at a high level.

Figure 3.7: Contribution of New Urban Residents to Saving Rate
3.6.2 Urbanization and Household Saving Rates

We now use our calibrated model to explore the relationship between urbanization and household saving rates. To that end, we conduct a numerical exercise that mimics China’s urbanization path from 1995 to 2007. The economy starts with 75% of rural population and 2.3% of rural households become urban residents annually.

We assume that the economy is at its stationary distribution of age, income, and wealth at $t = 0$. Then each year 3.35% of working-age rural households (which is equal to 2.3% of total rural households in our model), uniformly distributed across age, move to cities. This process lasts for 10 years. The results from such an exercise are displayed in Figure 3.8.

At time $t = 0$, the economy has an aggregate saving rate slightly above 10%. As rural households start migrating to cities, there is a sharp increase in the aggregate saving rates, which lasts for about 6 years. The contribution of urbanization amounts to 8% of the aggregate saving rate. As time goes on, saving rates decline despite the continuous inflow of rural households to urban areas, because consumption of existing migrants who have retired outweighs the saving of new migrants.

Our exercise obviously has made many simplified assumptions which potentially lowers the contribution of urbanization to the increase in household saving rates. One assumption is that the economy starts from a stationary distribution of age that is uniform. In reality, the distribution of age tends to be hump-shaped for emerging economies and therefore it takes much longer than the model simulation suggests for
the consumption of retirees to outweigh the saving of new urban residents. In other
words, the increase in household saving rates would perhaps be more prominent and
prolonged in reality. Another assumption that we make is that the age distribution
of migrants is uniform whereas in reality it is mostly young households who move to
cities. This also tends to lower the contribution of urbanization to the increase in
household saving rates. Other assumptions that make our result a likely lower bound
of the effect of urbanization on the increase in household saving rates include a low re-
placement ratio for rural households, the nature of urbanization being unanticipated,
and expectation of high income growth rates that last forever.

For completeness, Figure 3.8 displays the results from our model where urban-
ization continues for 40 years and stops. As is shown in the graph, the sudden stop
of urbanization at time $t = 40$ results in a decline in household saving rates, which
eventually climbs back to its long run level. Again, the temporary decline is because
of the lack of saving of new migrants to counter the consumption of the old migrants.
The long-run implication of urbanization for household saving rates hinges crucially
on expectations of growth rate of income. The simulation in Figure 3.8 maintains the
assumption that income growth rates are constant (8.06% for urban and 6.72% for
rural) over time. This assumption is extremely optimistic. If expectation of income
growth rates falls over time, the increase in saving rate would last longer than the
model suggests. As mentioned earlier, our model is very stylized and we focus only
on the transition of rural households to urban areas. There are a number of factors
that might interact with urbanization and influence household saving rates, including change of expectation of income growth rates in urban areas, which we leave out for future research.

### 3.7 Conclusion

In this paper, we document a strong correlation between the degree of urbanization and the increase in urban household saving rates across provinces in China: Provinces that experienced more rapid urbanization saw a bigger rise in urban house-
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Figure 3.9: Urbanization and Household Saving Rates in the Long Run
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hold saving rates. To investigate this relationship, we develop a very simple model that features two sectors, rural and urban, where income and idiosyncratic risks differ. As urbanization proceeds, migrants save toward a higher-than-before wealth target and therefore contributes to the increase in household saving rates. Our calibrated model suggests that urbanization in China between 1995 and 2007 contributes at least 8% increase in household saving rates.

The analysis in this paper can be extended in several directions. This paper makes the simplifying assumption that urbanization occurs exogenously. It would be interesting to endogenize the migration choices of rural households because anticipated migration to urban areas would cause rural households to save more, which changes the timing of the increase in aggregate household saving rates. This paper also abstracts from changes of demographics and expectation of income growth over time. The effect of the interaction between these changes and urbanization on household saving rates is worth exploring as urbanization, population aging, and declining income expectation tend to happen simultaneously for emerging economies.
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