Abstract

Cardiac auscultation with a stethoscope has served as the primary method for qualitative screening of cardiovascular conditions for over a century. However, a lack of quantitative understanding of the flow mechanism(s) responsible for the generation of the murmurs and the effect of intervening tissues on the propagation of these murmurs has been a significant limiting factor in the advancement of automated cardiac auscultation. In this study, a multiphysics computational modeling approach is used to investigate these issues.

A previously developed sharp-interface immersed boundary flow solver is upgraded to efficiently tackle internal flow problems which are commonly seen in cardiovascular systems. First, the geometric multigrid pressure Poisson solver is replaced by an efficient biconjugate gradient stabilized method. Second, a graph-partitioning based parallel framework is adopted to reduce computational overhead associated with the large proportion of wasted grid points in internal flow problems.

To understand the role of the shear wave in the propagation of murmurs, a two-dimensional hemodynamic-acoustic study is conducted, wherein a classic vector decomposition is employed to separate the murmurs generated from an arterial stenosis into compression part and shear part. Results show that the shear wave has a pro-
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found impact on source localization and signal characteristics.

Next, we use a one-way coupled hemodynamic-acoustic approach to investigate the generation and propagation of murmurs associated with the aortic stenosis from first principles. Direct numerical simulation is used to explore the hemodynamics of the post-stenotic jets. Subsequently, the propagation of the murmurs through a tissue-like material is resolved by a high-order, linear viscoelastic wave solver. The implications of these results for cardiac auscultation are discussed.

Finally, the effect of the valve on the murmur generation is explored. The fluid-structure interaction between the valve and the flow is modeled through a reduced degree-of-freedom model of the aortic valve. Aortic stenoses with different severities are created by changing the stiffness of the leaflets. This simple valve model is demonstrated to be able to accurately capture the opening/closing motion predicted by a more sophisticated model. Simulations with the aortic valve model provide additional insights into post-valvular flow and the characteristics of the murmur source.

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Introduction

1.1 Background

Heart disease is the most consequential disease in the United States. According to Centers for Disease Control and Prevention (CDC), heart disease is the number one cause of death for both males and females (see Fig. 1.1), accounting for 614,348 death in 2014 [1]. Despite the improvement in diagnosis and treatment options over the past decades, current trends in aging, obesity and diabetes point to a troubling outlook for this disease. Worse still, the diagnosis and treatment of heart disease also imposes significant burden on individuals, families and communities. As a matter of fact, it is the number one medical expenditure in the United States by a large margin as shown in Fig. 1.2.

It is recognized that early detection and treatment of heart disease can effectively reduce mortality and cost. There are many established diagnostic modalities, such as computed tomography (CT), angiography and electrocardiography (ECG), which can help doctors diagnose various heart conditions. However, they are either
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Figure 1.1: Percentage distribution of the 10 leading causes of death by sex in the United States, 2014.

invasive, expensive, labor-intensive, hospital-centric or could expose patients to chemical/radioactive hazards, making them not ideal for early detection and screening. In this context, cardiac auscultation provides an inexpensive, non-invasive alternative. This diagnostic modality is based on the fact that virtually every heart condition produces a distinct abnormal heart sound, called a bruit or murmur, which could be detected on the skin surface through a stethoscope or other sensors.

The idea of diagnosing heart conditions based on the associated murmurs dates back to ancient Greece and Egypt. The modern form of cardiac auscultation has only been established after the invention of the stethoscope by Laennec in 1816. It has since become the primary method for initial screening as well as follow-up for cardiovascular conditions. However, cardiac auscultation with stethoscope suffers from limited sensitivity and specificity. Some of the reasons for this include but are
not limited to:

1. The lack of understanding of causal mechanism(s) between specific heart disease and the associated murmurs. No tools have existed that can study the generation and propagation of heart murmurs from first principles to make one-to-one connections between disease condition and murmur.

2. The human factor. The accurate interpretation of heart murmurs relies heavily on the hearing acuity of the physicians, which requires years of training and practice to develop. Worse still, cardiac auscultation has been a skill in decline, since newer generations of cardiologist and physicians are not being
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trained properly in the art and science of cardiac auscultation [5].

3. Limited information can be extracted from traditional auscultation. Heart murmurs have wide range of characteristics such as existence, timing, intensity, location, duration and pitch. However, auscultation via hearing can only qualitatively dissect very limited information.

4. Signal-to-noise ratio. Noises such as breathing sound, normal heart sound and environmental noise are usually detected concurrently with the murmurs.

Clinicians and engineers have sought to transform cardiac auscultation with the simple stethoscope into a more objective and quantitative modality that could help to enhance the ability of clinicians to screen for and monitor heart disease. Electronic stethoscopes and computer-based sound analysis (a technique referred to as ‘phonocardiography’) was introduced in the 1970s by Lees and his colleagues [6, 7]. While this method mitigates many of the limitations associated with manual cardiac auscultation, the inability to make one-to-one connections between heart conditions and murmurs, has never allowed this technique to become a reliable tool for diagnosis. Consequently, phonocardiography has not advanced beyond being a novelty in medicine, and has been relegated to use primarily in medical training and education. Many studies have since been conducted in this area [8-13]. But most of them aim to provide better visual representation of the heart murmurs to aid the diagnosis process, and the correlations between disease and murmurs are largely derived
empirically. Thus, the clinical impact of these techniques has been very limited.

In spite of limited previous success, this central-old technique can potentially flourish in modern era with new insights and paradigms \[14\]. The emergence of low-power, low-cost compact acoustic sensors, advanced signal analysis algorithms and powerful portable computational devices in the last decade provides the technological support needed to transform cardiac auscultation into a more accurate automated diagnostic modality that could potentially play a role in traditional medicine as well as in telemedicine \[15,16\]. Fig. 1.3 shows one vision of how automated cardiac auscultation can be incorporated into telemedicine. Patients will measure their heart sound at home through a special device that contains an array of sensors. The signal is sent to remote server for analysis (spectral analysis, source localization, machine learning, etc.), and actions will be recommended based on the results. Such a system would reduce the subjectivity associated with human hearing acuity and training, making it suitable for rapid mass diagnostic screening, longitudinal (tracking

Figure 1.3: Envisioned automated monitoring process for heart conditions.
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Figure 1.4: Illustration of the generation and propagation of the heart murmurs. The insert shows the abnormal flow caused by an arterial stenosis.

over time) at-home monitoring of patient health, and field operations in areas with limited medical access. However, this requires a much better understanding of the causal mechanism(s) between specific medical condition and the associated heart murmurs \[3,11,13\] as well as the propagation characteristics of these murmurs.

1.2 Motivation

The established theory of aero/hydroacoustics \[17\] broadly classifies flow-induced sound sources into monopoles (associated with bulk acceleration of flow), dipoles (associated with unsteady hydrodynamic loading on boundaries) and quadrapoles (associated with flow turbulence). The sound produced by these three mechanisms has distinct characteristics (spectra, radiation efficiency, and directionality) and different correlations with flow variables \[18\]. All cardiac sounds and murmurs can also be classified into one of these categories. For instance, the ‘lub-dub’ sounds of the
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beating heart (also known as the S1 and S2 sounds) are monopole sounds, whereas murmurs associated with valvular insufficiency (prolapse, regurgitation) as well as obstructions (such as those in hypertrophic obstructive cardiomyopathy (HOCM) and valvular stenoses) would produce dipole and quadrupole sounds. However, these ideas of sound source classification have made little inroads into auscultation. Due to our inability to ‘look inside’ the human heart to understand causal mechanisms while simultaneously recording the sounds, correlations between heart sounds and underlying pathology are currently based primarily on deduction, inference and heuristics. A further complication with the analysis of heart sounds is that the sound is sensed on the precordium (portion of the body over the heart and lower chest) and therefore has to propagate through, and around thoracic organs such as the myocardium, lungs, tissue, ribs and epidermis. This propagation induces differential damping, diffraction and dispersion in the acoustic field and there is very little quantitative understanding of this process. The inability to clearly delineate the causal mechanism(s) for the heart sounds as well as to quantify and characterize the effect of thoracic organs on the sound propagation has been a significant limiting factor in the advancement of automated cardiac auscultation.

The understanding of the causal mechanism(s) can be facilitated through studying the multiphysics implicated in the generation and propagation of the murmurs. For example, in arterial stenoses (see Fig. [1.4]), the narrowing of the vessel leads to abnormal local blood flow patterns (post-stenotic jet and/or turbulence), which
induces pressure fluctuations at the lumen wall. These pressure fluctuations subsequently generate elastic waves in the tissue which propagate through various tissues/organs before reaching the skin surface, where they are detected through stethoscopes or other acoustic sensors. A multiphysics modeling approach that models the hemodynamics as well as the murmur propagation could help to delineate the connections between the characteristics of the murmurs and the underlying medical condition.

The generation of stenosis related murmurs has been studied extensively, based on which, it has been generally concluded that the murmurs are generated by the abnormal pressure fluctuations at the vessel wall. Moreover, the source location, where the most intense pressure fluctuations are observed, is found not at the site of stenosis but further downstream.

The propagation of the elastic waves inside the thorax has also been studied by many researchers. The analysis mainly focuses on two aspects of the murmurs; the first one is the spectral characteristics of the signal, which serve as the ‘fingerprints’ of the related medical condition. Fredberg used analytical solutions to predict how the propagation inside the tissue would alter the source spectrum. He concluded that the change in spectrum measured at the skin surface was not a consequence of volume absorption, but due to the superposition of pressure waves generated at different locations along the axial direction of the stenosed vessel. It is noted that shear waves were not included in the calculation. Duncan et al.
conducted *in-vivo* measurements of murmurs from patients with carotid stenosis and identified the break frequency \( f_b \), a critical value around which the slope of the spectrum changed dramatically. They were able to relate the break frequency to the severity of the stenosis through \( D_j = U/f_b \), where \( D_j \) is the diameter of the lumen at the stenosis, and \( U \) is the peak systolic velocity at the healthy part of the artery. The break frequency was also observed in the numerical study by Mittal *et al.* [22] and Seo & Mittal [23].

The second focus of many studies has been on localization of the murmur source. Since the source location usually lies in the vicinity of the stenosis, correctly identifying the source location only using the murmurs can provide valuable diagnostic information. Owsley & Hull [25] built an experimental set-up where a straight tube with a constriction was buried inside a tissue-mimicking gelatin, and a sensor array was used to measure the acoustic signals at the surface. Then, a near-field beam-forming process was employed to image the shear wave energy distribution inside the gelatin to noninvasively determine the source location. They found that the source could be accurately located when there was no obstacle inside, but when dog ribs were included in the gelatin, the accuracy varied based on the frequency band. Cooper *et al.* [26] applied finite element analysis to study the wave propagation inside a two-dimensional cross-section of a physiologically accurate human thorax model. In order to obtain the transfer function between the murmurs and the source signal, they conducted a series of simulations to study the surface signals generated by point sources.
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placed at different locations of the thorax, such as left coronary artery, right coronary artery and ascending aorta. They claimed that the obtained transfer function could be useful in separating multiple co-existing acoustic sources. It should be noted that only compression waves were included in the study. Other studies \[27,29\] also examined auscultation-based source localization but focused on sounds generated from the respiratory system.

Despite the aforementioned studies, there remain some outstanding issues regarding murmur propagation. The first one is the unclear role of shear waves in murmurs. Conventional analysis of wave propagation in elastic media separates the roles of compression and shear waves. In biological materials, for frequencies in the $O(100)\text{Hz}$ range, the wavelength of the compression wave is usually $O(10)\text{m}$, while that of the shear wave is $O(0.01)\text{m}$ \[30,32\]. Since the characteristic length of human thorax is $O(0.1)\text{m}$, the long wavelength of the compression wave makes it less sensitive to structures such as lungs and bones. In fact, researchers have developed microphones that can separate the compression component from the signal and used it to achieve better directionality \[9,33\]. On the other hand, the relatively short wavelength of the shear wave makes it suitable for source localization. In the experimental study conducted by Owsley & Hull \[25\], the shear wave energy field was used to locate the murmur source. Royston \textit{et al.} \[34\] used analytical models to study the compression/shear wave generated by a dipole source embedded in a half-space. However, the contribution of the compression/shear component was approximated by doubling
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the solution obtained in an infinite space, which did not accurately account for the
effect of the surface. Seo & Mittal [23] used the flow in a modeled stenosed artery
as the murmur source, and studied the propagation of the compression wave in the
tissue layer. All of these studies point to the need to gain further insights into the
propagation mechanism of the compression and shear wave components of murmurs
in biological tissues.

The second major issue is that there is a disconnect between the study of the
generation of the murmurs and the study of the propagation of these murmurs, since
these are mostly studied as separate problems. Take the study of murmurs caused
by the arterial stenosis for example. Many studies of the post-stenotic flow identified
a critical frequency in the spectrum of the wall pressure fluctuations [21, 35]. This
critical frequency has the same definition as the break frequency in the murmur
signal, which is the frequency where the slope of the spectrum changes dramatically
[21, 22, 36]. Nevertheless, the relationship between the hemodynamic break frequency
of the flow and the acoustic break frequency of the murmurs is not understood. In
some studies of the murmur propagation inside the thorax, an artificial point source
is used [28, 31, 37], but this ignores the reality that the source may be distributed over
a relatively large area. In experimental studies where the murmurs are generated
from modeled stenoses [25, 38], access to the flow information is greatly restricted
due to the inherent limitations of the experiments, making it difficult to draw the
connections between the post-stenotic flow and the murmurs.
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Thus, the primary motivation of the current study is to use a coupled hemodynamic-acoustic (termed ‘hemoacoustic’) computational approach to provide clear physical insights into the generation and the propagation of the murmurs.

1.3 Scientific Objectives and Thesis Outline

In view of the eventual goal of gaining better understanding of the physics behind the generation and propagation of the heart murmurs, we chose stenosis (arterial stenosis and aortic valve stenosis) as the disease of interest to study its hemodynamics and acoustics. The current project aims to achieve the following four objectives:

1.3.1 Objective 1: Improve current computational modeling and simulation methodology for hemoacoustics

The flow dynamics here is simulated through a ghost-cell method based sharp-interface immersed boundary solver [39]. This flow solver is developed in house, and has previously been successfully applied in the study of external flow problems such as thrust production in competitive swimming [40], optimal wing stroke in flapping flight [41]. However, this flow solver suffers from reduced efficiency when used to simulate internal flow problems, such as those commonly seen in the cardiovascular system. This inefficiency is attributed to the deteriorated performance of the geometric multigrid pressure Poisson solver and the significant computational overhead.
caused by the wasted grid points. Our first step is to upgrade this flow solver for efficient solution of internal flow problems.

1.3.2 Objective 2: Understand the role of shear wave in hemoacoustics

To understand the role of the shear wave in murmurs, we extend the previous 2D study by our group [23] to include shear modulus in the tissue model. A classic vector decomposition is employed to separate the murmurs into the compression wave and the shear wave, which help us to gain fundamental understanding of how the shear wave can affect the characteristics of the murmur signal and complicate the task of source localization.

1.3.3 Objective 3: Study the coupled hemodynamics-acoustics of the aortic stenosis

In this part, we focus on aortic stenosis as the disease condition of choice due to the fact that it is the most common valvular disease and is known to create a very distinct systolic murmur [42]. This problem is studied in a one-way coupled fashion. The flow past a modeled aortic stenosis is first studied through direct numerical simulation. Then, the flow information (i.e. wall pressure) is used as the input for the acoustic simulation to study the wave propagation inside a modeled thorax. Core
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Flow structures that are responsible for the murmur generation are identified, and connections between the source and the murmur signals are drawn.

1.3.4 Objective 4: Investigate the effect of valve motion on murmur generation

In reality, the aortic stenosis is caused by incomplete opening of the aortic valve. Thus, the dynamics of the valve leaflet interacting with the aortic jet may generate additional flow features and associated murmur signals. We have developed a novel, reduced degree-of-freedom (DOF) valve model, and we will incorporate this valve model into our blood flow model to investigate the effect of valve leaflet dynamics on the murmur generation.

1.3.5 Thesis Outline

The rest of the thesis is structured as follows. The improvement of the flow solver is described in Chapter 2. The hemoacoustic simulation and the role of the shear wave are discussed in Chapter 3. The detailed study the generation and propagation of the murmurs associated with the aortic stenosis is reported in Chapter 4 (hemodynamics) and Chapter 5 (acoustics). The effect of pulsatility and valve leaflets is investigated in Chapter 6. A summary of the current work is provided in Chapter 7.
Chapter 2

A Highly Scalable Sharp-Interface Immersed Boundary Method for Large-Scale Parallel Computers

2.1 Sharp-Interface Immersed Boundary Method

Immersed boundary (IB) methods enable simulations of flow past bodies with complex shapes as well as moving/deforming bodies on stationary Cartesian grids. IB methods avoid the computational cost and complexities associated with the use of body-conformal grids, which can be particularly severe for moving and/or deforming boundaries [43]. Moreover, IB method solvers can be parallelized relatively easily by employing a Cartesian domain decomposition topology [44]. Depending on how the immersed boundary is treated, the IB methods can roughly be separated into two categories: the sharp-interface IB methods and the diffused-interface IB methods [43]. The sharp-interface IB methods, the subject of the work here, refer to the methods
in which the boundary conditions are applied precisely at the interface without any ‘diffusion’, making it well-suited for high Reynolds number flows.

The flow solver employed in this study is a well-documented three-dimensional, viscous, incompressible Cartesian grid based solver (referred to as ViCar3D) developed in-house \[39\]. The governing equations solved in the solver are given by

\[ \frac{\partial u_i}{\partial x_i} = 0, \]
\[ \frac{\partial u_i}{\partial t} + \frac{\partial(u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_j} + \nu \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} \right). \] (2.1)

where \( i, j = 1, 2, 3 \), \( u_i \) are the velocity components, \( p \) is the pressure, and \( \rho \) and \( \nu \) are the fluid density and kinematic viscosity. The primitive variables \((u_i, p)\) are defined at the cell-center in the computational domain. To remedy this non-staggered arrangement, another set of velocities, \( U_i \), are calculated at the center of cell faces \[45\]. Only the face velocity component normal to the cell-face is calculated and stored. A variant of the fractional-step method proposed by Zang et al. \[45\] is used to advance the solution in time. The solution procedure consists of the following three steps:

\[ \frac{u_i^n - u_i^n}{\Delta t} = \frac{1}{2} \left( -\frac{\delta(U_j u_i)}{\delta x_j} + \nu \frac{\delta}{\delta x_j} \frac{\delta u_i}{\delta x_j} \right) + \frac{1}{2} \left( -\frac{\delta(U_j u_i)}{\delta x_j} + \nu \frac{\delta}{\delta x_j} \frac{\delta u_i}{\delta x_j} \right), \] (2.2)

\[ \frac{1}{\rho} \frac{\delta p^{n+1}}{\delta x_j} = -\frac{1}{\Delta t} \left( \frac{\delta U_i^*}{\delta x_i} \right), \] (2.3)

\[ u_i^{n+1} = u_i^* - \frac{\Delta t}{\rho} \left( \frac{\delta p^{n+1}}{\delta x_i} \right)_{cc}, \quad U_i^{n+1} = U_i^* - \frac{\Delta t}{\rho} \left( \frac{\delta p^{n+1}}{\delta x_i} \right)_{fc}. \] (2.4)

In the first step (Eq. 2.2), an intermediate velocity field \((u_i^*)\) is obtained through
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Figure 2.1: Schematic of the computation domain for a patient-specific coronary artery network.

solving the advection-diffusion equation using the flow field at time step $n$. Here, both the advection term and the diffusion term are treated implicitly with second-order Crank-Nicolson method to eliminate the stability requirement. In the second step (Eq. 2.3), a pressure Poisson equation (PPE) is solved with geometric multigrid method to obtain the pressure at time step $(n+1)$. In the final step (Eq. 2.4), two correction equations are used to update the cell center velocity and the face center velocity. In the above procedure, $\delta$ represents the central difference operator, and subscripts $cc$ and $fc$ indicate cell center and face center, respectively.

This fractional-step method is coupled with the sharp-interface IB method developed by Mittal et al. [39] to handle the complex geometries. Fig. 2.1 shows an
example of such application, in which a CT-derived patient-specific coronary artery network is immersed inside a Cartesian grid to study the flow rate distribution among the branches. The artery wall is represented by unstructured triangular elements. The vertices of each elements are referred to as body markers. The effect of the immersed boundary on the flow is included through a multi-dimensional ghost-cell method (GCM) [39]. The procedure is described here. As shown in Fig. 2.2, the Cartesian cells are first separated into solid cells and fluid cells based on the relative locations of the cell centers to the immersed surface. For those solid cells that have at least one fluid cell as a neighbor, we identify them as ghost cell (GC). From each ghost cell, a probe that is normal to the immersed boundary is drawn toward the fluid domain and intercepts the immersed boundary at the boundary intercept point.
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(BI). This probe is further extended into the fluid domain, and an image point (IP) is defined such that the boundary intercept point is located at the midway between the ghost cell and the image point. Then, a bi-linear interpolation (tri-linear in 3D problem) is employed to compute the value of a generic flow variable (e.g. $\phi$) at the image point:

$$\phi(x_1, x_2) = c_1 x_1 x_2 + c_2 x_1 + c_3 x_2 + c_4. \quad (2.5)$$

The four unknown coefficients are determined by the values at the four surrounding nodes as

$$\{C\} = [V]^{-1}\phi. \quad (2.6)$$

where $C$ is vector formed by coefficients $c_i$, and $[V]$ is the Vandermonde matrix built from the coordinate of the surrounding nodes. Once the coefficients are determined, the value at the image point can be expressed in terms of the value of surrounding nodes.

$$\phi_{IP} = \sum \beta_i \phi_i. \quad (2.7)$$

where $i$ is from 1 to 4 (for 2D) or 8 (for 3D) and represents the $i^{th}$ surrounding node for the image point, and $\beta_i$ are the interpolation weights. Both Dirichlet boundary condition and Neumann boundary condition can be imposed along the probe through
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the following equations,

\[ \phi_{GC} + \sum \beta_i \phi_i = 2\phi_{BI}, \] (Dirichlet); \hspace{1cm} (2.8)

\[ \phi_{GC} - \sum \beta_i \phi_i = -\Delta l \left( \frac{\delta \phi}{\delta n} \right)_{BI}, \] (Neumann). \hspace{1cm} (2.9)

where \( \Delta l \) is the probe length. A cut-cell based approach has also been implemented to improve the conservation around the immersed body \[46]\.

This GCM based sharp-interface IB flow solver has been successfully applied to many external flow problems, such as thrust production in competitive swimming \[40]\, optimal wing stroke in flapping flight \[41]\,. However, when applied to internal flow problems such as those commonly occur in the cardiovascular system (see Fig. 2.1), the current solver suffers from two challenges. The first challenge is the decreased efficiency of the geometric multigrid pressure Poisson solver, and the second challenge is the significant computational overhead incurred by the wasted grid points. These two challenges are addressed individually in the following sections.

2.2 BiCGSTAB Pressure Poisson Equation Solver

2.2.1 Background

Though the sharp-interface IB method usually has higher accuracy around the immersed body than the diffused-interface method, this sharp representation of the
boundary introduces extra difficulties when solving the pressure Poisson equation (PPE, Eq. 2.3). Obtaining the solution of the PPE is the most time-consuming step in any incompressible flow solver, and the problem is even more prominent in the sharp-interface IB method since the solver has to address the rapid change of parameters at the interface and the complicated stencils around the immersed boundary. Mittal et al. [39] developed a powerful geometric multigrid (GMG) solver that was tailored for such sharp-interface IB method. However, the performance of this GMG solver deteriorates for internal flows with narrow channels (e.g. Fig. 2.1). In such geometries, the grids coarsening, which is an essential step of GMG, introduces additional errors around the immersed boundary that slows down the convergence of this method. This problem becomes even more significant on parallel computers with large-partition counts where the GMG also suffers from extra communication-to-computation at the coarser grid levels. In this section, we describe the implementation of the biconjugate gradient stabilized method (BiCGSTAB) to the sharp-interface IB method. The parallel performance of the BiCGSTAB solver is compared to the GMG solver for a benchmark problem.

2.2.2 Biconjugate Gradient Stabilized Method for Ghost-Cell Method

In order to alleviate the problems associated with GMG method, we implement the BiCGSTAB method to solve the PPE and optimize it for performance on large-scale
parallel computers. BiCGSTAB method is a Krylov space method that is designed for solving general sparse matrix systems \cite{BiCM}. Unlike the GMG, it is not attached to the underline Cartesian grid, and this helps to avoid error accumulation around the immersed boundary. The BiCGSTAB used here employs a Jacobi preconditioner $K$ to solve the linear system $Ax = b$. It proceeds in the following way:

1. Provide initial guess $x_0$, \textit{e.g.}, using $x$ from last time step, and calculate the initial residual $r_0 = b - Ax_0$.

2. Choose $\hat{r}_0$ such that the inner product $(\hat{r}_0, r_0) \neq 0$.

3. $\rho_0 = \alpha = \omega_0 = 1.0$.

4. $\nu_0 = p_0 = 0$.

5. While $x_i$ is not accurate enough

   (a) $\rho_i = (\hat{r}_0, r_{i-1})$,

   (b) $\beta = (\rho_i/\rho_{i-1})(\alpha/\omega_{i-1})$,

   (c) $p_i = r_{i-1} + \beta(p_{i-1} - \omega_{i-1}\nu_{i-1})$,

   (d) $y = K^{-1}p_i$,

   (e) $\nu_i = Ay$,

   (f) $\alpha = \rho_i/(\hat{r}_0, \nu_i)$,

   (g) $s = r_{i-1} - \alpha\nu_i$, 

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(h) $z = K^{-1}s,$

(i) $t = Az,$

(j) $\omega_i = (K^{-1}t, K^{-1}s)/(K^{-1}t, K^{-1}t),$

(k) $x_i = x_{i-1} + \alpha y + \omega_i z,$

(l) $r_i = s - \omega_i t.$

The key to applying this BiCGSTAB to the sharp-interface IB method lies in the assembly of matrix $A$. Unlike GMG where the ghost-cell values can be updated explicitly in an iterative fashion, the matrix $A$ in BiCGSTAB is required to be kept the same during the solution process. Therefore, the equations for the ghost cells (i.e. Eqs. 2.8 and 2.9) have to be included in matrix $A$ directly. For a fluid cell, the second-order central difference is used to approximate the spatial derivative, which results in a 5-point (in 2D) or 7-point (in 3D) stencil. For a ghost cell, its value is governed by Eq. 2.8 or Eq. 2.9 depending on the boundary condition at the location, and coefficients $\beta_i$ are used to build the matrix. Since coefficients $\beta_i$ are solely determined by the coordinates of the surrounding points and the image point, they remain unchanged for stationary boundary problems. With all the ghost cell equations explicitly included in the matrix, strong coupling among the ghost cells and the fluid cells is taken into consideration automatically, and this leads to a faster convergence.
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Figure 2.3: (a) Schematic of the computational domain of the flow past a sphere array; (b) a sample $4 \times 2$ domain decomposition. The gray planes indicate the boundaries between parallel sub-domains.

2.2.3 Benchmark Study

The flow through a $2 \times 2 \times 2$ sphere array is chosen as the benchmark study (see Fig. 2.3(a)). These 8 spheres with diameter $D$ are equally-space in a $7D \times 7D \times 14D$ domain. The lower right corner sphere is centered at $(2.5D, 2.5D, 2.5D)$. Each sphere is represented by 4764 elements and contains 2462 body markers. The computational domain is discretized by a $256 \times 256 \times 512$ ($\sim$ 33 million points) mesh, which is uniformly distributed around the spheres, and stretches towards the outer boundaries. Each sphere is resolved by 40 grid points across the diameter. A uniform inflow velocity $U$ is enforced at plane $Z = 0$, and an outflow boundary condition is applied at the other boundaries. The Reynolds number calculated from the inflow velocity and the sphere diameter is 350. Each simulation is run with $dt = 0.02D/U$. A 2D
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Figure 2.4: Parallel performance of ViCar3D with BiCGSTAB Poisson solver. (a) Speed-up; (b) Efficiency.

Cartesian domain decomposition, along $x$ and $y$ directions, is applied for the MPI parallelization (see Fig. 2.3(b)).

In this study, a series of simulations are carried out on TACC-Stampede (Intel Xeon E5 processor) to test the scaling property of the flow solver with BiCGSTAB as the Poisson solver. Eight different decompositions, $4 \times 4$, $8 \times 4$, $8 \times 8$, $16 \times 8$, $16 \times 16$, $32 \times 16$, $32 \times 32$ and $32 \times 64$ are used to solve the benchmark problem for 1000 time steps. Fig. 2.4 summarizes the parallel performance of the upgraded solver. The wall time from the $4 \times 4$ case is chosen as the baseline to calculate the speed up and efficiency. As shown in Fig. 2.4(a), this solver demonstrates excellent strong scalability up to 1024 cores, and shows near-perfect scaling below 256 cores. The efficiency also maintains relatively well and reaches 90% up to 512 cores, and it is still around 60% when 1024 cores are used, which is acceptable for such large scale computations. It is noted the efficiency can go beyond 100% when less than 256 cores
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Figure 2.5: Average wall time comparison of the flow simulation using BiCGSTAB or GMG as Poisson solver.

are used. This is due to the fact that the change in the domain decomposition also affects the distribution of the body markers in each parallel sub-domain. The effect of this redistribution of body markers varies based on the problem. But in this case, it contributes positively to the efficiency of the computation.

While scalability is important, an even more critical metric is the improvement in the wall time. The same benchmark problem is also solved with different decompositions by the ViCar3D using GMG as the Poisson solver. Fig. 2.5 shows a direct comparison of the average wall time the flow solver takes to advance one time step when BiCGSTAB or GMG is used to solve the PPE. This clearly shows the advantage of BiCGSTAB, since, over a wide range of number of cores, the upgraded solver consistently out-performs the old one. On average, the flow solver using BiCGSTAB is eight times faster than the one with GMG.
2.3 A Graph-Partitioned Sharp-Interface Immersed Boundary Solver

2.3.1 Background

While the desirable features of IB methods, such as easy to parallelize, non-body conformal Cartesian mesh, have made them popular for simulations of external flows, their application to internal flows leads to some problems. Internal flows with geometrically complex boundaries are encountered in a variety of application areas, including but not limited to, physiology (e.g. blood flow in arteries and airflow in bronchial networks), chemical engineering (e.g. micromixing, bioreactors) and other configurations involving flow in curved or branching pipes and channels. IB simulations typically employ a single rectangular (or cuboidal in 3D) domain with a boundary non-conformal Cartesian grid covering this domain. A consequence of this is that only a subset of the total points of the grid lie in the fluid domain, while the rest fall outside in what we refer to here as the ‘solid’ domain. These grid points within the solid are in-principle associated with wasted computational cost. For external flows, the volume of the solid body is generally much smaller than the volume occupied by the flow and therefore, the proportion of these wasted solid points is small. In contrast, for internal flows, the points within the solid might account for a very large fraction (sometimes even the majority) of the total grid points, and represent a significant source of computational inefficiency for an IB method based solution.
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For example, consider the flow through the arterial network shown in Fig. 2.1. A uniform, isotropic grid with $650 \times 410 \times 468$ ($\sim 125$ million) points provides sufficient resolution (roughly $20 \times 20$ points across the narrowest branch) to resolve the details of the flow in the smallest artery of the network, but this leads to $98.22\%$ of the total points being in the solid.

For diffuse interface methods [48], there is no particular distinction between the fluid and solid points since the same governing flow equations are solved for all the points in the computational domain, and a suitably localized body force is used to represent the effect of the solid on the fluid. Thus, for diffuse interface methods there is no simple way to reduce the collateral expense associated with these solid points, and simulation of a flow such as in Fig. 2.1 would not be feasible using these methods. On the other hand, for sharp-interface immersed boundary methods [39], except for, in some cases, a thin layer of points immediately adjacent to the solid boundary, the grid points within the solid are ‘inactive’ in that they have no bearing on the computed flow field. Thus, there is a possibility of designing algorithms that reduce or eliminate the overhead cost associated with these inactive points and make simulations such as those in Fig. 2.1 tractable using IB methods.

Most sharp-interface IB method solvers impose a trivial equation at these inactive points (such as $(\hat{u}, p) = (0, 0)$) and assemble this equation with the flow equations for the fluid points into one large sparse system of coupled equations for the entire set of grid points. While these inactive grid points do not affect the computed flow field,
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Figure 2.6: (a) Example of a $2 \times 2$ ($x$ direction $\times y$ direction) 2D domain decomposition using Cartesian topology. The gray planes depict the boundary of the parallel sub-domains; the active grid points in each sub-domain is shown in different colors. (b) Number density of active grid points in 96 sub-domains. (c) Number density of active grid points in 384 sub-domains. (d) Number density of active grid points in 96 sub-domains with increased grid resolution.

They do have a significant impact on the solution procedure and the computational cost. This is firstly because the sparse matrix solvers usually carry out the same operations for the grid points inside the solid as they do for points within the fluid region. Thus, the inactive solid points have a non-trivial contribution to the total floating-point operation (FLOP) count as well as the memory requirement. The
situations further deteriorates when parallelization is taken into consideration. A Cartesian domain decomposition strategy is the obvious choice for parallelization (see Fig. 2.6(a)). However, even though each Cartesian sub-domain/process superficially contains nearly equal number of grid points, the number of active grid points can vary significantly depending on the sub-domain topology and the geometry of the immersed body. For complicated geometries, there might be cases where many sub-domains consist entirely of inactive grid points leading to large load imbalance.

In order to further examine this issue, we consider again the arterial network geometry in Fig. 2.6(a). The 125 million point grid is decomposed into 96 (12 × 8) Cartesian domains with equal number of total grid points, but this results in a wide distribution of active grid points per domain (see Fig. 2.6(b)). As a matter of fact, 47 of the 96 sub-domains (49%) do not contain any active grid points at all. We quantify the load imbalance by the parameter $\lambda = \left( \frac{L_{max}}{\overline{L}} - 1 \right) \times 100\%$, where $L_{max}$ and $\overline{L}$ are the maximum and mean load across the processes [49]. In the current context, the true computational load for each process is assumed to be proportional to the number of active grid points in this process. For the Cartesian decomposition in Fig. 2.6(b), the percent imbalance is 675%, indicating severe load imbalance. Moreover, as shown in Fig. 2.6(c), increasing the number of sub-domains to 384 (24 × 16) makes the problem worse, with 251 sub-domains (65%) now containing no active grid points and $\lambda = 1032\%$. This issue is also not alleviated by increasing the grid resolution as shown in Fig. 2.6(d). Lastly, even in many of the domains that contain active
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points, a significant proportion of boundary communication between adjacent sub-domains involves ‘useless’ communication to and from inactive points, thereby further increasing the computational overhead.

Users of IB methods are aware of this problem and have come up with various strategies to circumvent this issue. Anupindi et al. [50] developed a multiblock method and applied it to the flow inside a thoracic aortic aneurysm. Instead of using a single computational domain to cover the whole geometry, they employed a large number of smaller domains (blocks) to follow the overall shape of the geometry more closely. Each block was discretized using a uniform Cartesian mesh with the same resolution. However, the block sizes and locations had to be determined manually. Also, the number of inactive grid points is significantly reduced only when a relatively large number of blocks are used. In a different approach, Zelicourt et al. [51] discarded all of the inactive grid points by re-indexing the active grid points into a one-dimensional array, essentially converting the Cartesian mesh into an unstructured mesh. The advantage of this strategy was successfully demonstrated in a single processor implementation. However, this approach requires significant modification of any existing solver and furthermore, an efficient framework is still required to implement this method on large-scale parallel computing systems.

In this section, we present an easy-to-implement, graph-partitioning approach for sharp-interface IB methods that can solve internal flow problems with complex geometries more efficiently on large-scale parallel computers. Compared with other
methods, this method still operates on a single-domain Cartesian mesh and employs a well-established open-source tool to partition the active grid points for effective load balancing. Moreover, an effective precoarsening process is proposed to ensure that sufficient overlapping layers are available at the sub-domain interfaces to implement the discretization. The effectiveness of the method is demonstrated by simulating a small set of cases.

2.3.2 Graph-Partitioning Based Parallel Framework

The current approach can be viewed as a parallel framework to efficiently handle the underlying data structure for certain genre of immersed boundary methods. The proposed method is implemented with ViCar3D described in Section 2.1 and the BiCGSTAB solver introduced in Section 2.2 is used to solver the PPE. Though the solver is applied to 3D problems, the description in this section is mostly for a 2D configuration.

The key to solving internal flow problems more efficiently is to operate only on the active grid points, which, in the aforementioned sharp-interface IB method, are composed of the fluid points and the ghost points. The distribution of the active grid points closely shadows the shape of the immersed boundary, which may be highly irregular. To handle these irregular geometries, graph-partitioning with the open-source software package METIS \[52\] is adopted. METIS is widely used in the finite-element modeling community and is designed to deal with unstructured grids. It
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Figure 2.7: Vertex (solid circle) and the edges (dash lines) it forms with its neighbors (circles). (a) Inside the fluid region; (b) near the immersed boundary.

takes as input a graph constructed from the grid points and puts out partitions of these grid points that can be used in parallel computations. These partitions can be optimized to be well-balanced in total volume and have minimal communication interfaces. However, implementation of graph-partitioning within the context of the sharp-interface IB method requires some additional considerations, and these are described here.

The basic elements of a graph are the vertices and the edges. In this case, the vertices are the set of active points (i.e., the fluid and ghost points) while the edges represent connectivities between these active points. The existence of an edge between two vertices implies a coupling between these vertices. For a Cartesian mesh, the most common coupling is caused by the discretization of the derivatives. For instance, a second-order central scheme for the second derivatives in a two-dimensional grid couples the grid point with its neighbors along two axial directions. However, due
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Figure 2.8: (a) A zoom-in view at the interface between two partitions. Only two partitions are shown here and all the grid points are active grid points. The grid points inside the dash line are the first sub-domain overlapping layer of partition 2 from partition 1, and the grid points inside the dot line are the first sub-domain overlapping layer of partition 1 from partition 2. (b) Illustration of the coarsening-partitioning process when two sub-domain overlapping layers are required.

to the ghost-cell method depicted in Fig. 2.2 the coupling can also occur along the diagonal directions. In order to apply the ghost-cell method correctly, the coupling (or edges) along the diagonal direction should also be included when present (see Fig. 2.7).

Once the graph constructed from the active grid points is supplied, METIS generates a map where each vertex is assigned a partition number. Fig. 2.8(a) shows an example of partition in 2D. Active grid points with the same partition number are grouped into one sub-domain, which is assigned to one MPI process. Layers of vertices that overlap adjacent partitions are usually required to ensure the accuracy
and smoothness of the solution at the sub-domain interface. As shown in Fig. 2.8(a), a one-layer overlap between sub-domains is always well-defined via comparison of partition numbers of two edge-sharing vertices. However, multi-layer overlaps are required for reasons such as larger spatial discretization stencils [53], ghost-cell discretization (see above), and cell merging in cut-cell type methods [46, 54], but these may not always be correctly defined in the current situation. For example in Fig. 2.8(a), a two-cell thick sub-domain overlapping layers would be ill-defined for the highlighted point in partition-2, since a second overlap layer for this point would intrude back into partition-2. Although such points constitute a very small proportion of the total number of interface points, they lead to large numerical errors and even numerical instability, if not treated properly. More complicated situations will arise in 3D problems or when the number of required overlapping layers increases, and this significantly complicates the implementation process.

To solve this problem, we have devised a precoarsening process which is carried out before the partitioning step. Assuming that sub-domain overlapping layers of thickness $N_{ol}$ are required, the original grid is first coarsened by a factor of $N_{ol}$ in each direction; Fig. 2.8(b) shows an example of coarsening in 2D for $N_{ol} = 2$. Following this, the graph-partitioning is carried out on the coarsened mesh and the resulting partitioning is then simply applied to the original (uncoarsened) mesh. By following this procedure, ill-defined situations such as in Fig. 2.8(a) can be avoided, and well-defined and consistent sub-domain overlapping layers with sufficient thickness are
Figure 2.9: An example of a Cartesian bounding box enclosing the partition and one sub-domain overlapping layer. The red solid circles represent the active points in the current partition, and the black circles are the active grid points from surrounding partitions.

guaranteed for all interface points. The grid points inside the sub-domain overlapping layers are assembled into a vector, which is then communicated to the neighboring partitions.

The graph partitioning leads to complex shaped sub-domains, and the usual practice is to employ an unstructured grid topology to address the point inside the sub-domain \([55]\). In the current graph-partitioned solver we avoid the complexities arising from this by retaining a structured \((i, j, k)\) topology for each sub-domain. This is accomplished by defining a Cartesian bounding box that encloses each partition and its sub-domain overlapping layers (see Fig. 2.9 for example). The following pseudo code shows how the Cartesian bounding box is implemented with the graph topology.
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Here, \((n_x, n_y, n_z)\) is the size of the Cartesian bounding box. Two benefits are provided

\[
\begin{align*}
1: & \text{ for } i = 1 \text{ to } n_x \text{ do} \\
2: & \quad \text{ for } j = 1 \text{ to } n_y \text{ do} \\
3: & \quad \quad \text{ for } k = 1 \text{ to } n_z \text{ do} \\
4: & \quad \quad \quad \text{ if point } (i, j, k) \text{ does not belong to current partition then} \\
5: & \quad \quad \quad \quad \text{ Skip} \\
6: & \quad \quad \quad \text{ else} \\
7: & \quad \quad \quad \quad \text{ Computation on point } (i, j, k) \\
8: & \quad \quad \quad \text{ end if} \\
9: & \quad \text{ end for} \\
10: & \text{ end for} \\
11: & \text{ end for}
\end{align*}
\]

through this practice: first, matrix type data structure provides faster data access than the vector type in calculating multi-direction derivatives. Second, for legacy IB method solvers that employ Cartesian parallel topology, this practice greatly simplifies the transition from Cartesian to graph topology. However, these benefits come at the price of some extra memory usage, since ‘dummy’ points have to be added to fill the locations in the Cartesian bounding box that are not occupied by the active points associated with this sub-domain. Fortunately, METIS usually provides very high quality, compact partitions, and the extra memory usage is quite manageable in practice. Furthermore, since these dummy points carry no useful information, no computational operations are carried out for them.

Though the proposed method is implemented in the sharp-interface IB method developed by Mittal et al. [39], it is worth emphasizing that it can be effectively applied in any flavor of IB method that has a separation between the solid points and fluid points. The definition of the active grid points would vary depending on how
the boundary conditions at the immersed boundary are enforced. Moreover, due to
the irregular shape of the partitions, solvers designed for general sparse matrix sys-
tems (e.g. BiCGSTAB, GMRES) should be adopted. Other than this, the proposed
method should require very limited modifications of the solution procedure and code
structure for any existing IB method solver.

2.3.3 Benchmark Study

The accuracy and versatility of the original sharp-interface IB method have been
demonstrated previously in Refs. [39,46]. In this section, we focus on the performance
benefits the proposed framework provide for internal flow problems. The first test
is flow through a wavy channel, the baseline geometry and dimensions of which are
illustrated in Fig. 2.10(a). The wavy wall has a sinusoidal profile with wavelength
$2H$ and amplitude $0.5H$. The top and bottom walls have no phase shift and are
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separated to form a channel with height $H$. The channel size is half wavelength in the spanwise and five wavelengths in the streamwise direction. A uniform steady inflow $V_0$ is prescribed at the inlet, and a Neumann boundary condition is applied at the outlet. No slip and no penetration boundary condition is prescribed in the spanwise direction and on the top and bottom walls. The Reynolds number based on the channel height $H$ and inlet velocity $V_0$ is 400, and this Reynolds number leads to a complex, fully three-dimensional unsteady flow inside the channel. This type of channel is a good test for the current method since it naturally introduces large regions with inactive points. In addition, to further demonstrate the efficiency of the current solver in handling even larger proportion of inactive points, the entire channel is rotated with angle $\theta$ to introduce even more inactive points while maintaining the fluid volume and appropriate inflow velocity condition. As shown in Fig. 2.10(b), two additional cases with $\theta$ equals 5° and 10° are simulated, and the time-step for all the cases are $0.001H/V_0$. Thus, we generate a range of cases where the fluid domain and the flow field are the same but the sub-domain topology and the number of inactive points are quite different. Thus, these latter two factors are the only ones that can affect the comparative computational performance for these cases.

The details about the simulations are summarized in Table 2.1, where $N_x$, $N_y$ and $N_z$ represent the number of grid points in $x$, $y$ and $z$ directions, respectively. These three channels are solved using grids with the same resolution. It is apparent that as the angle increases, the total number of grid points as well as the number of inactive
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Table 2.1: Summary of the simulation details for the different wavy channel configurations.

<table>
<thead>
<tr>
<th>Case</th>
<th>Topology</th>
<th>No. of Partitions</th>
<th>((N_x, N_y, N_z))</th>
<th>Total Grid Points</th>
<th>Active Grid Points (Proportion of Total Grid Points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Graph</td>
<td>288</td>
<td>(640, 128, 80)</td>
<td>6,553,600</td>
<td>3,094,080 (47%)</td>
</tr>
<tr>
<td>2</td>
<td>Graph</td>
<td>288</td>
<td>(638, 176, 80)</td>
<td>8,983,040</td>
<td>3,084,560 (34%)</td>
</tr>
<tr>
<td>3</td>
<td>Graph</td>
<td>288</td>
<td>(630, 232, 80)</td>
<td>11,692,800</td>
<td>3,088,400 (26%)</td>
</tr>
<tr>
<td>4</td>
<td>Cartesian</td>
<td>36 \times 8</td>
<td>(630, 232, 80)</td>
<td>11,692,800</td>
<td>3,088,400 (26%)</td>
</tr>
</tbody>
</table>

Table 2.2: Summary of simulation performance for carrying out 10,000 time-steps for the different wavy channel configurations.

<table>
<thead>
<tr>
<th>Case</th>
<th>Wall Time Per Active Grid Point (seconds)</th>
<th>Communication Cost (%)</th>
<th>Total Allocated Memory (GB)</th>
<th>Percent Imbalance ((\lambda))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.61E-2</td>
<td>1.42%</td>
<td>252</td>
<td>6%</td>
</tr>
<tr>
<td>2</td>
<td>2.54E-2</td>
<td>1.48%</td>
<td>252</td>
<td>6%</td>
</tr>
<tr>
<td>3</td>
<td>2.56E-2</td>
<td>1.45%</td>
<td>252</td>
<td>6%</td>
</tr>
<tr>
<td>4</td>
<td>4.86E-2</td>
<td>8.28%</td>
<td>342</td>
<td>289%</td>
</tr>
</tbody>
</table>

grid points increase rapidly, but the number of active grid points is nearly the same. The active grid points are partitioned into 288 sub-domains using METIS, and the simulations are carried out on TACC-Stampede2 with 288 Intel®Skylake Cores. The flow generated in these channels is transitional in nature, and all results shown here are accumulated over 10,000 time steps after the simulations reach a stationary state. Fig. 2.11(a,b) plot the velocity magnitude and the fluctuation kinetic energy at the center plane of the baseline case. The complex, unsteady nature of the flow ensures that the solver performs non-trivial computations at each time-step, thereby making this comparison a reasonable test of the solver efficiency. Fig. 2.11(c,d) plot the flow field for \(\theta = 5^\circ\) and \(10^\circ\). As can be seen here, the flow physics are very similar among
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Figure 2.11: Compute flow on the spanwise center-plane of the channel using the graph parallel topology. (a) Velocity magnitude based on the mean flow and (b) fluctuation kinetic energy (FKE) for $\theta = 0^\circ$ channel. (c) Velocity magnitude based on the mean flow for the $\theta = 5^\circ$ channel. (d) Velocity magnitude based on the mean flow for $\theta = 10^\circ$.

The three cases and the flow field should therefore not be a factor in the performance comparison.

It is clear in Table 2.2 that the wall time per active grid point is quite similar among the first three cases, with the maximum difference being lower than 3% of the baseline case. This is a direct indicator that the inactive grid points have negligible effect on the speed of the simulation, indicating that they are effectively ‘invisible’ to the solver. The communication cost, which is measured as the percentage of total wall time that is spent on pure MPI communication (send/receive operations), is also maintained around 1.45% for all three cases. Moreover, the memory usage monitored through Intel®Vtune™ Amplifier also shows negligible differences between the various cases that employ the graph partitioning. This further demonstrates that the solver is very
efficient in ignoring the inactive grid points. The load is also well balanced when
graph topology is adopted, with only 6% imbalance for all three cases.

To enable a direct comparison between the graph parallel framework and the
Cartesian parallel framework, another case (Case 4 in Table 2.1) is simulation in
which the same channel configuration as Case 3 is adopted. But the original Carte-
sian domain decomposition is used to separate the computational domain into $36 \times 8$
(288) sub-domains. The performance is also summarized in Table 2.2. It is seen
that the allocated memory goes up by 35% compared to the baseline case. Further-
more, the wall time per active grid point goes up by 86%, and the communication
cost increases by 984%. The percent imbalance of the Cartesian topology is as high
as 289%, representing a highly imbalanced computing load. Thus, even for this rela-
tively simple, nominally 2D internal flow case, the current method provides significant
advantages.

It is worth noting that the graph partitioning here is performed as a pre-processing
procedure, and the partition information is supplied to flow solver as an input. Hence,
the computational cost of the graph partitioning procedure is not factored into the
overall computational cost. The partitioning process is however very fast, and the
wall time required for this partitioning is equivalent to $O(100)$ time-steps for the flow
solver.
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Figure 2.12: (a) Results of the graph partitioning for the coronary arteries with 96 partitions and 2 sub-domain overlapping layers. (b) Number density of active grid points in 96 sub-domains. (c) Number density of active grid points in 384 sub-domains. (d) Number density of active grid points in 96 sub-domains with increased grid resolution. The grid resolution, number of active points and number of domains in (b-d) are the same as those in Fig. 2.6(b-d).

2.3.4 Flow through an Arterial Network

One of the major applications for the upgraded solver is the modeling of cardiovascular flows. Fig 2.1 shows a section of the coronary artery reconstructed from the computed tomography (CT) scan of a patient. This arterial network shows several
small arteries branching from the same primary vessel. As noted earlier, such a geometry is virtually out of reach for all immersed boundary type methods and here, we use this configuration to demonstrate the feasibility of simulating such flows with the current graph-partitioned immersed boundary method.

In the current simulation, a constant pressure drop of 3mmHg (399.97Pa) is prescribed between the inlet and all the outlets. This is within the range of physiological conditions for the coronary artery and results in a Reynolds number of about 450 for the largest artery \[56\]. In order to provide reasonable resolution of the flow in the smallest arteries of this network (approximately \(20 \times 20\) grid across the cross-section of the smallest artery), the computational domain is discretized by 650, 410 and 468 grid points in \(x\), \(y\) and \(z\) directions, respectively. This leads to around 125 million grid points in total, which make this problem very challenging for typical IB methods that employ Cartesian parallel topology. However, only around 2.2 million (or 1.78\%)
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of the total grid points here are active, making this an excellent case to apply the proposed methodology.

Following the procedure described earlier, 96 partitions are created and 23,067 grid points are assigned to each partition on average (see Fig. 2.12(a)). From Fig. 2.12(b), it can be noted that the load is reasonably well-balanced among sub-domains with \( \lambda = 10\% \). Moreover, compared to the Cartesian topology shown in Fig. 2.6(c,d), the load balance is well maintained with either increased number of partitions or increased grid resolution as demonstrated in Fig. 2.12(c,d). During the simulation, an adaptive time-step is used to maintain a maximum CFL number of 0.75 while reducing the wall time needed to reach stationary state (which requires about 2500 time-steps). Fig. 2.13(a) shows the wall pressure distribution and a monotonic pressure drop is observed from the inlet to the outlet. The distribution of total velocity on the cross-section of all branches is plotted in Fig. 2.13(b), and we note that there are a number of small-scale features and sharp gradients in the contours of the narrowest branches, confirming the need for the relatively high grid resolution employed here.

2.4 Conclusion

In this chapter, the flow solver used in our study, \( i.e. \) a ghost-cell method based sharp-interface immersed boundary flow solver (ViCar3D) is described briefly. Despite its previous success in simulating external flow problems, the solver faces a couple of challenges when solving internal flows, such as flow inside an arterial network. The
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challenges come from two fronts. The first is the slow geometric multigrid Poisson equation solver, and the second is the non-trivial computational overhead associated with inactive grid points.

To tackle the first challenge, a BiCGSTAB pressure Poisson solver is incorporated into ViCar3D. The flow past a sphere array is used as the benchmark test to evaluate the performance of the upgraded flow solver. The new solver shows strong scalability and well-maintained efficiency over a wide range of number of cores (up to 1024 cores). A wall time comparison with the preexisting geometric multigrid (GMG) Poisson solver demonstrates the advantage of the BiCGSTAB algorithm.

For the second challenge, a graph-partitioning framework for efficient simulation of internal flow problems is proposed. This framework helps the solver to focus the computational resources on the active grid points while effectively ignore the inactive ones. The solver is subjected to a carefully designed benchmark test, which clearly shows that the computational cost, including wall-time and memory usage, only scales with the number of active grid points. The effectiveness of the method is also demonstrated by successfully simulating a flow which would effectively be out of reach of most immersed boundary methods: flow in a network of branching arteries on a grid that has a total of $O(10^8)$ points.
Chapter 3

A Computational Study of the Effect of Shear Wave in Murmur Propagation and Detection

3.1 Introduction

The abnormal narrowing of a blood vessel or a ‘stenosis’ can occur at a variety of locations in the vasculature including systemic, pulmonary and cerebral arteries. The stenoses, especially those occurring in arteries, usually leads to altered local flow patterns, which are known to create distinct murmurs (or bruits). In this chapter, computational hemoacoustics is used to study the role of the shear wave in murmurs generated by an arterial stenosis.

As mentioned in Section 1.2, murmurs propagate inside the thorax in form of elastic waves, which can be separated into the longitudinal component (compression wave, curl free) and the transverse component (shear wave, divergence free). These
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two components have drastically different properties (i.e. wave length, wave speed). Previous study by Seo & Mittal [23] focused on the propagation of the compression wave inside the tissue layer, and the present study can be viewed as the extension of that study. Here, a computational framework for simulating the generation and propagation of murmurs in biological tissues, as well as analyzing the individual roles of the compression wave and the shear wave is developed. The effect of shear wave is included in the current computational model, and the main objective of the study is the delineation of the effect of the compression and shear wave components on the propagation and detection of arterial murmurs. The problem is tackled with a one-way coupled hemodynamic-acoustic method. First, the flow in a modeled stenosed vessel is simulated. Subsequently, the pressure fluctuations from the flow serve as the input for the acoustic simulation. The medium is treated as a homogeneous linear elastic material, and contributions from the compression wave and the shear wave are separated through a classic decomposition. The accuracy of the computational method is verified by comparison against an analytically derived solution for a point source. Simulation results are used to explore the physics of murmur generation and propagation.
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Figure 3.1: Schematic of the constricted channel model and acoustic domain employed in the current study. The dimensions in the schematic are not to scale.

3.2 Numerical Methods

3.2.1 Model Configuration

As shown in Fig. 3.1, the stenosed artery is modeled as a two-dimensional constricted channel similar to the one employed in Ref. [23]. Even though this is not a physiological geometry, it captures the key features of a stenosed artery. The constriction is asymmetric, and has the following profile

\[ y = D - \frac{b}{2} \left[ 1 + \cos \left( \frac{\pi (x - x_0)}{D} \right) \right], \quad -D \leq x - x_0 \leq D, \]  

(3.1)

where \( D \) is the diameter of the modeled artery, \( b \) represents the severity of the stenosis, and \( x_0 \) is the center of the stenosis. Here, \( b \) and \( x_0 \) are set to 0.75\( D \) and 0\( D \).
CHAPTER 3. EFFECT OF SHEAR WAVE

flow inside the channel is driven by a pulsatile pressure drop between the inlet and outlet with the following profile

$$\Delta P = \rho_f U_{\text{max}}^2 \left[ A + B \sin (2\pi ft) \right],$$  \hspace{1cm} (3.2)

where $\rho_f$ is the density of the fluid, $U_{\text{max}}$ is the maximum centerline velocity at the inlet, and constants $A$ and $B$ are set to 0.75 and 1.5. The maximum and minimum flow rates resulting from this pressure profile are $0.55U_{\text{max}}D$ and $0.03U_{\text{max}}D$, respectively. The Strouhal number based on the diameter and the maximum centerline inlet velocity is $St = fD/U_{\text{max}} = 0.024$. For typical values in the human aorta where $D = 2.0cm$ and $U_{\text{max}} = 0.98m/s$, this Strouhal number corresponds to a heart rate of 60 bpm. The Reynolds number in this study is $Re = U_{\text{max}}D/\nu_f = 2000$, where $\nu_f$ is the kinematic viscosity of the blood. It is worth noting that it is the intensity of the stenosis-induced jet that has the dominant effect on the sound generation, rather than the profile of the pressure drop [23].

The murmurs generated by the abnormal blood flow propagate through an isotropic, homogeneous tissue layer. Even though the current computational framework can in-principle include the blood vessel wall, this feature is not included here so as to maintain the simplicity of the model. The exclusion of the blood vessel wall is not expected to change the results significantly, since the wavelength of the most energetic components of the murmurs is much longer than the wall thickness. The density
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and compression wave speed of the blood and the tissue are obtained from Ref. [57], and these values are set at 1.05g/cm$^3$ and 1500m/s, and 1.20g/cm$^3$ and 1720m/s, respectively. The shear modulus ($G$) to bulk modulus ($K$) ratio of the tissue layer is set to $2 \times 10^{-4}$, and this is based on the widely-used tissue-mimicking silicone gel, Ecoflex-10. The sound signals from various locations on the epidermal surface are monitored and analyzed.

3.2.2 Computational Hemodynamics

The current study focuses on modeling the flow in a stenosed artery, and the blood in these large vessels can be approximated as a Newtonian fluid [58], which is governed by the incompressible Navier-Stokes equation.

\[
\frac{\partial U_i}{\partial x_i} = 0, \\
\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{1}{\rho_f} \frac{\partial P}{\partial x_j} + \nu_f \frac{\partial}{\partial x_j} \left( \frac{\partial U_i}{\partial x_j} \right),
\]

where $U_i$ is the flow velocity vector, and $P$ is the pressure. The sharp-interface immersed boundary method described in Section 2.1 is used to treat the complex geometry. The fluid region is discretized by a 768 × 128 nonuniform grid. The grids are clustered around the stenosis with minimum spacing $\Delta x = 0.01D$ and are stretched toward the inlet and outlet. More details about the flow simulation can be found in Ref. [23].
3.2.3 Computational Hemoacoustics

Arterial murmurs or murmurs that are generated by the blood flow propagate through the tissue layer and are detected on the epidermal surface. The sound generation inside the blood flow can be described by the linearized perturbed compressible equations (LPCE) shown below \[59\],

\[
\begin{align*}
\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_i} (u_j U_j) + \frac{1}{\rho_f} \frac{\partial p}{\partial x_i} &= 0, \\
\frac{\partial p}{\partial t} + U_j \frac{\partial p}{\partial x_j} + \rho_f c_f^2 \frac{\partial u_j}{\partial x_j} + u_j \frac{\partial P}{\partial x_j} &= -\frac{D P}{D t},
\end{align*}
\]

(3.4)

where \(u_i\) is the acoustic velocity perturbation vector, \(p\) is the acoustic pressure perturbation, \(c_f\) is the speed of sound in the fluid region, and \(D/Dt\) represents the material derivative. The incompressible Navier-Stokes/LPCE method is a two-step, one-way coupled approach suitable for fluid induced sound at low Mach numbers and more details can be found in Ref. \[19\].

In the current study, the propagation of the flow induced sound in the tissue layer is governed by the following linear elastic wave equation:

\[
\begin{align*}
\frac{\partial u_i}{\partial t} + \frac{1}{\rho_s} \frac{\partial p_{ij}}{\partial x_j} &= 0, \\
\frac{\partial p_{ij}}{\partial t} + \lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) &= 0,
\end{align*}
\]

(3.5)

where \(u_i\) is the acoustic velocity vector in the tissue, \(p_{ij}\) is the stress tensor, \(\rho_s\) is the structural density, \(\delta_{ij}\) is the Kronecker delta, and \(\lambda\) and \(\mu\) are the first and second
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Lame constants of the material, respectively.

To solve Eqs. 3.4 and 3.5 efficiently, these two equation-sets are combined into
the following single set of equations

\[
\frac{\partial u_i}{\partial t} + \frac{1}{\rho(x)} \frac{\partial p_{ij}}{\partial x_j} + H(x) \left( u_k \frac{\partial U_k}{\partial x_i} + U_k \frac{\partial u_k}{\partial x_i} \right) = 0,
\]

\[
\frac{\partial p_{ij}}{\partial t} + \lambda(x) \frac{\partial u_k}{\partial x_k} \delta_{ij} + \mu(x) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) + H(x) \left( u_k \frac{\partial P}{\partial x_k} + U_k \frac{\partial p_{ij}}{\partial x_k} + \frac{\partial P}{\partial t} \right) \delta_{ij} = 0,
\]

(3.6)

As can be seen here, the inhomogeneity of the domain is accounted for by the space-
dependent material properties \((\rho(x), \lambda(x), \mu(x))\) where \(\lambda = \rho f c^2_f, \mu = 0\) in
the blood region, as well as the Heaviside function \(H(x)\), which equals 1 inside the
blood region and equals 0 in the tissue layer. Eq. 3.6 is discretized by a sixth-
order compact finite difference scheme \[53\], and advanced in time by a four-stage
Runge-Kutta method. The flow Mach number \(M = U/c\), where \(c\) is the speed of
sound, is set to 0.01 in the current study, instead of the physiological value, which
is \(O(10^{-3})\). This 10-fold increment is employed to ameliorate the stringent time-step
size constraint imposed by the large gap between the velocity scales of the flow and
the sound. The same approach is adopted in other studies \[23, 60\] as well and has
been shown to have no significant effect on the detected murmurs. The stability
constraint is more stringent for the acoustic simulations and in order to maintain
overall stability of the computational method, the incompressible flow simulation
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time-step is divided into 16 sub-steps for the acoustic simulation. The intermediate values of flow variables between two flow time-steps are obtained through a second-order Lagrangian interpolation [19].

A uniform Cartesian grid is used to discretize the acoustic domain with a grid spacing of 0.02\(D\). The shear wave length is about 0.14\(D\) at \(St = 10.0\), which is resolved by around 7 points. This resolution is guided by our previous work [23] and grid refinement studies indicate that this grid provides sufficient resolution to accurately resolve the quantities of interest. The accuracy of the computation is also established in an analytic study, which will be described in Section 3.3.2. A bi-linear interpolation is used to interpolate the results of the flow simulation onto the acoustic grid. A zero-stress boundary condition is imposed at the epidermal surface, while the non-reflecting energy transfer and annihilating (ETA) boundary condition [61] is applied on the other boundaries.

3.2.4 Elastic Wave Decomposition

One objective of the current study is to investigate the relative contributions of the compression and shear wave components to the detected murmurs and here we make use of a decomposition of the elastic wave equation that can be derived analytically [62]. The linear elastic wave equation [3.5] can be written as

\[
\rho_s \frac{\partial^2 v_i}{\partial t^2} = (\lambda + 2\mu) \frac{\partial}{\partial x_i} \left( \frac{\partial v_j}{\partial x_j} \right) - \mu \epsilon_{ijk} \frac{\partial}{\partial x_j} \left( \epsilon_{klm} \frac{\partial v_m}{\partial x_l} \right), \tag{3.7}
\]
where $v_i$ is the displacement vector, $\epsilon_{ijk}$ is the alternating tensor. We note that the first term on the right-hand side is curl free and therefore only responsible for the compression component of the acceleration, while the second term, which is divergence free, results in the shear component of the acceleration. We further define the following variable:

$$\Phi = (\lambda + 2\mu) \frac{\partial v_j}{\partial x_j}. \quad (3.8)$$

Noting that $\Phi$ is related to the first invariant of the stress tensor $-p_{ij} = \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$, it can be given the following more concrete form in two-dimensional problems

$$\Phi = -\frac{1}{2} \left( \frac{\lambda + 2\mu}{\lambda + \mu} \right) p_{kk}. \quad (3.9)$$

The compression and shear components of the acceleration vector can then be expressed as

$$a_{c,i} = \frac{1}{\rho_s} \frac{\partial \Phi}{\partial x_i}, \quad a_{s,i} = \frac{1}{\rho_s} \left( -\frac{\partial p_{ij}}{\partial x_j} - \frac{\partial \Phi}{\partial x_i} \right), \quad (3.10)$$

respectively. Thus, the two components of acceleration can be obtained via a post-processing of the deformation and stress distributions in the tissue layer. Note that conventional stethoscopes sense the normal acceleration of the skin surface and we therefore focus primarily on this quantity.
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3.3 Results and Discussion

To ensure that the results are presented consistently, results from both the flow simulation and the acoustic simulations are non-dimensionalized by the same characteristic parameters: velocity scale, $U_{max}$; length scale, $D$; time scale, $D/U_{max}$; and pressure/stress scale, $\rho U_{max}^2$.
Figure 3.3: (a) Time histories and (b) the corresponding spectra of the nondimensionalized time-derivative of the pressure on the upper lumen at three different downstream locations.
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3.3.1 Hemodynamics

Fig. 3.2 shows snapshots of the vorticity and pressure field in the modeled artery. As the flow starts to accelerate through the stenosis, a vortex street consisting of fairly evenly spaced counter-rotating vortices is formed and this is convected into the post-stenotic region. As the flow decelerates, the shear layers in the post-stenotic region become unstable and complex patterns of vortex mergers can be observed. The pressure distribution in the post-stenotic region is driven by the vortices with the centers of these vortices coinciding with the regions of low pressure. It has been shown in Ref. [23] that arterial murmurs are well correlated with the time-derivative of the integrated pressure force on the arterial wall, instead of the pressure fluctuation.

Figure 3.4: Spatial distribution of the spectral energy of the nondimensionalized pressure time-derivative along the upper lumen.
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in the blood flow region. Thus, the time histories and the corresponding spectra of the time-derivative of the pressure on the upper lumen at different downstream locations are plotted in Fig. 3.3. The lower limit of the $St$ in the spectra corresponds to the fundamental frequency of the pulsatile flow, $i.e., St = 0.024$. The general trend of the spectra shows initial decline after the fundamental frequency, but more energy is generated in the high frequency region due to the stenosis-induced pressure fluctuation. The strongest pressure fluctuation is observed around $St = 0.9$. Based on the spectra, the intensity of the time-derivative of the pressure is calculated and plotted along the upper lumen, as shown in Fig. 3.4. This plot clearly shows that the source of the murmurs is located around $4D$ downstream of the stenosis.

3.3.2 Verification of Acoustic Modeling Approach

The computational model for the acoustics as well as the method for computing the compression and shear waves associated with arterial murmurs presented here are new, and the fidelity of this method needs to be verified. This is particularly important since the relatively short wavelength of the shear wave increases the resolution requirement. It is therefore desirable to establish that the current method accurately resolves the propagation of all the relevant waves in the tissue layer. In order to accomplish this verification in the most comprehensive manner possible, we employ the exact solution associated with a point source in the infinite domain and then incorporate the time history of the actual sound source obtained from the above
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flow simulation. This allows us to examine the accuracy of the computational method for the entire range of relevant frequencies.

The governing equation for wave generation by a point source is as follows:

$$\rho_s \frac{\partial^2 v_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left[ \lambda \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right] = s_i(t) \delta(\vec{x}),$$  \hspace{1cm} (3.11)

where $v_i$ is the displacement vector, $s_i(t)$ is the acceleration source vector, and $\delta(\vec{x})$ is the Dirac delta function used to localize the source. The exact solution for a free-space propagation can be obtained in the frequency domain with the help of the Green’s function $G_{ij}(\vec{x}, \omega)$

$$\langle v_i \rangle(\vec{x}, \omega) = G_{ij}(\vec{x}, \omega) \langle s_i \rangle(\omega),$$  \hspace{1cm} (3.12)

where $\langle \cdot \rangle$ represents the Fourier transform of the function, and $\omega$ is the angular frequency. The Green’s function for the 2D problem has the following expression

$$G_{ij}(\vec{x}, \omega) = \frac{i}{4(\lambda + 2\mu)} \left\{ \frac{1}{k_p r} H_1^{(1)}(k_p r) \delta_{ij} - \frac{x_i x_j}{r^2} H_2^{(1)}(k_p r) \right\}$$

$$+ \frac{i}{4\mu} \left\{ \frac{1}{k_s r} H_0^{(1)}(k_s r) - \frac{1}{k_s r} H_1^{(1)}(k_s r) \right\} \delta_{ij} + \frac{x_i x_j}{r^2} H_2^{(1)}(k_s r),$$  \hspace{1cm} (3.13)

where $H^{(1)}$ is the Hankel function of the first kind, and $k_p, k_s, r$ are defined as

$$k_p = \omega/c_p, \quad k_s = \omega/c_s, \quad r = |\vec{x}|.$$  \hspace{1cm} (3.14)

In the above equations, $c_p$ and $c_s$ are the compression and shear wave speeds, re-
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Figure 3.5: Schematic of the computational domain with an acceleration point source located at $(0D, 0D)$ and two monitor points at $(2D, 0D)$ and $(0D, 2D)$. 

respectively. It is also noted that the first term of the Green’s function represents the compression wave, while the second term is solely related to the shear wave. Finally, the acceleration at any location is obtained as $\langle a_i \rangle(\bar{x}, \omega) = -\omega^2 G_{ij}(\bar{x}, \omega) \langle s_j \rangle(\omega)$.

As shown in Fig. 3.5, the acceleration point source given by $s_i = (0, s_2(t))$ is placed at the origin, $(0D, 0D)$, and we monitor the response at two points located at $(2D, 0D)$ and $(0D, 2D)$. The signal at one location in the flow simulation (4D downstream of the stenosis on the upper lumen wall) is used as the point source. Its spectrum is shown in Fig. 3.3(b). The shear to bulk modulus ratio here is also set to $2 \times 10^{-4}$, and a uniform grid with the same spacing, $0.02D$, is employed for the spatial discretization of the domain. At this resolution, the shear wave at the highest frequency, $St = 10$, is resolved by 7 points per wave length. Consistent with the
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Figure 3.6: Spectra of the nondimensionalized vertical acceleration signal collected at monitor point (a) \((2D, 0D)\) and (b) \((0D, 2D)\).

The exact solution in free-space represented by Eq. 3.12, the ETA boundary condition is applied at all boundaries to allow for the transmission of the waves.

Fig. 3.6 plots the spectra of the vertical acceleration signal at locations (a) and (b). Plotted in the same figure along with the numerical solutions are the exact
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solutions obtained from the Green’s function. The numerical solutions match well with the exact solutions even at $St = 10$, which suggests that 7 points per wave length provides adequate resolution of the acoustic wave. However, there are some noticeable differences between the numerical solutions and the exact solutions for low frequencies around $St \sim 10^{-1}$. These discrepancies can be attributed to the inherent limitations of the ETA boundary treatment for low frequency band [61] and not to any resolution issues. Fortunately, the frequencies of murmurs are usually orders of magnitude higher than the fundamental frequency, i.e., the heart rate [21]. Thus, the simulations still possess high fidelity in the frequency range of interest here.

3.3.3 Characteristics of Murmur Propagation and Auscultation Signal

With the murmur propagation modeling and simulation verified, we turn to examining the characteristics of the propagation of the murmurs. Fig. 3.7 shows the computed instantaneous vertical acceleration field when the intensity of the hydrodynamic source ($DP/Dt$) reaches its maximum. The compression component and the shear component are obtained through the aforementioned decomposition. As shown in the figure, the wave fronts of the compression component align parallel to the arterial lumen and the high intensity region is located immediately downstream of the stenosis. On the other hand, the shear component is transmitted radially away from a region localized in the vicinity of the stenosis. The contours clearly indicate
that the energy of the shear component propagates through the tissue in an oblique angle to the lumen, and its effect on the surface signal will be felt more at locations that are significantly downstream of the stenosis. This phenomenon is related to the characteristics of the shear wave. Since the fluid cannot sustain shear movements, shear wave can only be created at the interface of the blood region and tissue layer. The pressure force exerted by the blood on the interface acts as a point source aligned
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in the vertical direction. Thus, the generated shear wave will mainly propagate in the
downstream direction. Its region of influence will grow as the wave travels, forming
the oblique wave pattern shown in Fig. 3.7(b).

Despite the different wave patterns, both plots in Fig. 3.7 suggest that the main
source of the murmurs is located in the post-stenotic region, where the wall pressure
fluctuation is the strongest. This observation is consistent with the results of the flow
simulation. Since one issue of interest here is to localize the source of the murmurs
using auscultation, the following analyses focus on the acoustic signal measured on
the epidermal surface after the stenosis.

To study the effect of shear modulus ($G$) on wave propagation, a separate simu-
lation in which the shear modulus is set to zero, is also conducted. Fig. 3.8(a) shows
the comparison of the surface acceleration at $4.5D$ downstream of the stenosis from
simulations with and without the shear modulus. This specific location is chosen
because it has the highest signal intensity, which will be shown later. Assuming the
influence of the fundamental frequency is negligible above its tenth harmonic, the
analyses focus on the high frequency range ($St \in [0.24, 10]$) which carries most of
the energy of the murmurs. Interestingly, the presence of shear wave propagation
mechanisms has little effect on the vertical acceleration spectrum at this location.
To further investigate the effect of the shear modulus, the decomposition described
before is applied to separate the shear component from the compression component
and the result is shown in Fig. 3.8(b). When $St$ is below 0.9, the compression wave
and the shear wave share similar amplitudes, but the phase difference between them results in a total signal with a much lower amplitude. When $St$ is above 0.9, the shear wave energy decreases sharply, leaving the compression wave as the dominant
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Figure 3.9: (a) Comparison of nondimensionalized vertical acceleration signal between the simulations with and without shear modulus on the epidermal surface at $x = 15D$ and (b) spectra of the components of the nondimensionalized vertical acceleration from the simulation with shear modulus at the same location.

It is noted that the compression wave spectrum in Fig. 3.8(b) is different from the signal spectrum for $G = 0$ in Fig. 3.8(a) in the low frequency range. This signifies the fact that the shear and compression wave mechanisms are not linearly
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Figure 3.10: Spectral energy of the nondimensionalized vertical acceleration along the epidermal surface for simulations with and without the shear wave mechanism for tissue thickness of $9D$.

additive. Instead, the inclusion of the shear modulus induces complex interaction between the compression wave and the shear wave at the surface, which can only be captured by including both effects simultaneously. The importance of including the shear modulus into these murmur propagation models is further demonstrated by the signal at $x/D = 15D$ in Fig. 3.9. As shown in Fig. 3.9(a), the influence of the shear wave is more apparent at this location. Significant differences still exist when $St > 1.0$ in Fig. 3.9(a), while the corresponding decomposition in Fig. 3.9(b) shows clear compression dominance in this region.

Motivated by the notion of using quantitative auscultation to localize the sound source, we examine the spatial distribution of the surface signal, and the band-limited spectral energy of the total vertical acceleration, which is calculated from the spectrum using $f^2 = \sum_{St=0.24}^{10} \langle f \rangle^2$, is plotted along the epidermal surface in Fig. 3.10.
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As shown here, the intensities from both cases overlap very well up to $8D$ after the stenosis, indicating a trivial contribution from the shear component in this region. However, the case with shear modulus produces a stronger surface signal further downstream, due to the increasing contribution from the shear wave. In Fig. 3.10, both simulations with and without the shear modulus put the source of the murmurs around $4.5D$ after the stenosis, which is in good agreement with the actual source location from the flow simulation. It is worth noting that the prediction here does not contradict with the observation in Ref. [23], where the peak is located between $5D$ and $6D$, since the latter is predicted by the vertical velocity fluctuation on the epidermal surface.

The simulation results (Figs. 3.7 and 3.10) show that the effect of the shear wave on the auscultation signal is felt further downstream from the stenosis. This implies the thickness of the tissue between the lumen and the epidermal surface should have a differential effect on the contribution of the compression and shear wave. The thickness of the intervening tissue may vary significantly based on individual anatomy (body size, body-mass index) as well as the artery of interest. For instance, peripheral arteries such as the carotid and femoral lie closer to surface whereas other vessels commonly implicated in arterial occlusive diseases such as the coronary and iliac arteries, as well as the aorta, lie deeper in the body.

To explore the effect of tissue thickness on the murmur propagation and detection, another set of simulations with and without the shear modulus are carried out, but
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Figure 3.11: Spectral energy of the nondimensionalized vertical acceleration along the epidermal surface for simulations with and without the shear-wave mechanism for tissue thickness of $6D$.

with the thickness of the tissue layer reduced from $9D$ to $6D$. Fig. 3.11 shows the spectral energy distribution along the epidermal surface for both cases. A single peak at $4.5D$ after the stenosis is observed when the shear modulus is not included, while two peaks are observed when the shear modulus is included. For this latter case, the first peak is again at $4.5D$ after the stenosis, which is in line with the peak generated in the previous case. The other peak, which is at $10.5D$ downstream of the stenosis, is related to the contribution from the shear wave. Based on the wave patterns in Fig. 3.7, we can see that the decrease of thickness has little effect on the propagation of the compression wave, and the maximum signal intensity is observed at the same location. On the other hand, reducing the thickness of tissue layer also reduces the distance between the stenosis and the location where the shear waves with high intensity interact with the surface. Hence, the contribution from the shear...
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wave comes into effect earlier, creating a second peak slightly downstream of the first peak. This second peak also has an intensity level that is comparable to the first peak and would therefore appear as an equally strong auscultatory signal. The appearance of this second peak could therefore complicate the task of source localization.

3.4 Conclusion

A coupled hemodynamic-acoustic simulation method and a wave-decomposition based analysis approach are developed to study the generation and propagation of arterial murmurs. A key feature of the model is that it includes compression as well as shear wave contributions to the murmurs. The model and wave-decomposition method is successfully verified against a canonical problem solution, and is then used to study the effect of shear waves on the propagation and detection of murmurs generated by a modeled stenosis. The decomposition reveals that the compression wave propagates perpendicular to the arterial lumen, while the shear wave is mainly transmitted in an oblique direction through the tissue. As a result, the shear wave contributes more significantly to the surface signal further downstream of the stenosis. For a relatively thick tissue layer between the lumen and tissue surface, the shear wave has a limited effect on the auscultatory signal. For this case, the peak location of the surface signal corresponds reasonably well to the source location. However, a reduction in tissue thickness amplifies the contribution from the shear wave, creating a second peak in the auscultatory signal slightly downstream of the first peak.
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This ‘false’ peak could confound the task of source localization and this issue will be explored in future simulations. Besides the thickness of the tissue, there are other factors that can affect the generation and propagation of the murmurs, such as curvature of the artery and heterogeneity of the intervening tissue. The current method can be extended to study these issues in the future. Even though it is difficult to separate compression and shear components from the surface signal in the clinic with available stethoscopes, the present computational framework can potentially facilitate the development of more sophisticated auscultation tools and signal processing algorithms. For example, the simulation results can be used to fine tune the source localization algorithm to take account of potential ‘false’ peaks and generate more accurate information for diagnosis. Such analysis could also be ‘inverted’ to obtain information about the tissue properties, and in doing so, provide a noninvasive means for tissue biopsy. Finally, it is noted that while the focus of this chapter is on arterial murmurs, the current methodology is equally applicable to heart murmurs where the source location lies within the cardiac chambers.
Chapter 4

Analysis of Hemodynamics for the
Murmur Generation in a Simple Model of
Aortic Stenosis

4.1 Introduction

Previous 2D study uses a canonical geometry to study arterial stenosis in general. In this chapter, we focus on aortic stenosis as the disease condition of choice due to the fact that it is the most common valvular disease and is known to create a very distinct ejection murmur [42,64,65].

Post-stenotic flow in general has been studied extensively through both experiments [21,66,80] and numerical simulations [22,81,93]. The majority of the studies cited here modeled the blood vessel of interest as a straight circular tube, and both axisymmetric and asymmetric stenoses were studied. Steady as well as pulsatile flow conditions were used in these studies, with steady flow studies usually serving as the
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prequel to pulsatile flow studies. In the context of aortic stenosis or more generally, arterial stenosis, researchers have mainly attempted to find 1) under what conditions the post-stenotic flow becomes turbulent, 2) the generation mechanism(s) of the murmurs, 3) the source location of the murmurs, and 4) the connection between the severity of the stenosis and the signal characteristics. An overview of these results is presented in the next few paragraphs. In the context of this chapter, the severity of the stenosis is denoted by the percentage of area that is occluded, and the Reynolds number \((Re)\) is defined using the diameter \((D)\) and mean cross-sectional velocity of the unconstricted (healthy) part of the circular tube.

It is well established that laminar flow is a valid assumption in the majority of the healthy (normal) human vascular system \([94]\). However, due to the high flow rate through the aortic valve, turbulent flow can sometimes be observed in the aorta of health subjects during peak systole as shown in the in-vivo measurements by Stein & Sabbah \([95]\). This study also found that turbulent flow could be detected in the ascending aorta of all the subjects with an aortic valvular disease during most of the systole. Ahmed & Giddens \([75]\) conducted in-vitro experiments to investigate the post-stenotic flow inside a straight tube with steady inflow at moderate Reynolds numbers (500-2000). Their study showed that for a stenosis with 50% area reduction, the post valvular flow was dominated by discrete-frequency vortex shedding when the Reynolds number was lower than 1000, while both periodic vortex shedding and turbulence contributed significantly to the post-valvular flow fluctuations at higher
Reynolds number. For the case with area reduction of 75%, the turbulence could be observed for Reynolds number as low as 1000. Sherwin & Blackburn [86] used linear stability analysis to investigate the turbulent transition in a similar set-up and the critical Reynolds number predicted for the 75% stenosis was 722. To model the onset of the turbulence, the Reynolds-averaged Navier-Stokes (RANS) equation approach [85], large eddy simulation (LES) [22,84], and direction numerical simulation (DNS) [22,86,88,89] have been used.

While the post-stenotic flow in the aorta is turbulent, the sound directly generated by the aortic jet (quadruple jet noise) is significantly weaker than the sound generated by the pressure fluctuations at the vessel wall (surface dipole) due to the relatively low flow Mach number [96]. Clark [69] reached a similar conclusion, based on which, he postulated that the murmurs were mainly generated from the wall pressure fluctuations caused by the onset of turbulence. Similarly, based on their experimental observation that murmur signal spectra collected from subjects with carotid and femoral stenoses shared strikingly similarity to the spectra of turbulent wall pressure fluctuations from a rigid tube, Lees & Dewey [6] concluded that the turbulent wall pressure fluctuations were responsible for the murmur generation. On the other hand, Tobin & Chang [21] measured the spectra of wall pressure fluctuations generated by a steady flow in a rigid tube with different axisymmetric constrictions at physiologically relevant Reynolds numbers (500-4000). Unlike Lees & Dewey [6], the spectra they obtained didn’t exactly match that of the fully developed turbulent pipe flow, and
the location with maximum intensity of wall pressure fluctuation was determined by
the reattachment of the jet shear layer. Ahmed & Giddens [74, 75] showed that, for
the steady jet flow after a 75% stenosis with $Re = 2000$, the centerline energy spec-
trum showed discrete-frequency vortex shedding behavior shortly after the stenosis
($1.5D$ downstream) and the flow demonstrated both vortex shedding and turbulent
characteristics between $1.5D$ and $2.5D$, while the reattachment location was around
$2.8D$. The flow displayed clear characteristics associated with turbulence further
downstream of the stenosis. Clearly, the final murmur signal will include contribu-
tions from wall pressure fluctuations caused by both shear layer vortex shedding and
turbulence, and although it is practically impossible to separate these contributions,
the consensus is that the murmurs are generated from the pressure fluctuations on
the vessel wall regardless of the precise source of these pressure fluctuations.

With this understanding of the murmur generation mechanism, the source loca-
tion of the murmurs is usually identified as the location with maximum wall pressure
fluctuation. Identifying the murmur source location has great clinical significance,
since this can help diagnose the underlying condition. Physicians have however long
been aware that the location of murmur source and the location of disease do not nec-
essarily match. For example, to determine if there are ejection murmurs caused by
the aortic stenosis, doctors will place the stethoscope at the second right intercostal
space, which is slightly downstream of the aortic valve [97]. Tobin & Chang [21] found
that for a steady flow through stenoses with 75%, 85% and 90% area reduction in
a straight tube, the spatial distribution of the intensity of wall pressure fluctuation shared very similar behaviors at Reynolds number around 3000. The intensity slowly increases after the stenosis, reaches the maximum between $1.5D - 2.5D$ downstream of the stenosis, and decreases sharply afterwards. They also noticed that the source location was just upstream from the flow reattachment location. Lu et al. \cite{76} reported the source location to be $2.1D$ downstream of the stenosis for a 89% stenosis, and in Ahmed & Giddens \cite{74}, the source location for a 75% stenosis at $Re = 2000$ was about 2.8D downstream. It seems that after a certain critical Reynolds number, the maximum wall pressure fluctuation location only varies slightly over a wide range of Reynolds number and the severity of stenosis. However, the reattachment behavior is slightly complex in low Reynolds number regime. Ahmed & Giddens \cite{74} demonstrated that, for the case with 75% stenosis, the shear layers reattached to the wall at $4D$ distal to the stenosis at $Re = 500$, but the reattachment location was between $5D$ and $6D$ when $Re = 1000$. If the Reynolds number was further increased to 2000, the reattachment occurred roughly $2.8D$ after the stenosis. This is due to the fact that as the Reynolds number passes the critical value, the onset of transition to turbulence of the post-stenotic jet will change the reattachment location. The transition moves closer to the stenosis as the Reynolds number further increases, and so does the location of reattachment \cite{74}.

While localization of the murmur source is one way to utilize the auscultation signals, researchers have also focused on non-invasively determining the degree of
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stenosis through the characteristics of the murmur signals. As previously mentioned, periodic vortex shedding plays an important role in the generation of the murmurs, so this shedding frequency is studied extensively. The first such study dates back to 1930 [66], in which orifices were placed inside a straight rigid tube to model stenoses. For the case with 75% area constriction, the steady volume rate was modulated in order to vary the Reynolds number from 222 to 1020. This study found that Strouhal numbers of the vortex shedding was about 0.60 irrespective of the flow rate. It is worth noting that, since this shear layer vortex shedding is governed by the jet instability, the Reynolds number and Strouhal number here are non-dimensionalized by the diameter and mean velocity at the orifice. Other experimental studies [69, 72, 75] also reported vortex shedding Strouhal number around 0.60 at a higher Reynolds numbers as well as different degrees of stenosis. As a matter of fact, the Strouhal number of the vortex rings formed from a free circular jet is around 0.63 [98]. This is a clear evidence that before the shear layer reaches the wall, the confined jet is governed by the same instability mechanism as the free jet. It is also noted that the Strouhal number of a two-dimensional jet reduces to 0.43 [98], which hints that the shape of the jet is very critical in determining the shedding frequency.

From the above discussion, it appears that we already have a basic understanding of the hemodynamics of post-stenotic flows that generate murmurs. However, the knowledge derived from previous studies does not necessarily apply to the case of aortic stenosis. First of all, the aorta has a high degree of curvature in the post-
valvular region which is expected to affect the post-stenotic jet and it is therefore difficult to extrapolate our understanding from previous studies with straight tube models. In fact, it is known that wall curvature introduces secondary flows which should affect the wall pressure fluctuation in complex ways. Another issue is that the post-stenotic jet is unlikely to remain circular due to its interaction with the curved wall and this could affect the jet characteristics. There are several studies [83, 93, 99] that explored the flow in curved pipes. However, because these earlier studies have either used the assumption of laminar flow or did not employ the geometric configuration that represents an aortic stenosis, they provide limited understanding of the post-stenotic hemodynamics of an aortic stenosis. To explore these issues, we use computational modeling and simulation to study the flow downstream of aortic stenoses with the aorta being modeled as a curved pipe. This model although simple, incorporates many of the key features of a realistic aortic stenosis.

4.2 Model Configuration

4.2.1 Geometry and Numerical Methods

The geometry employed in this study is shown in Fig. 4.1(a). The aorta is modeled as a pipe with a 180° turn, and the unconstricted part of the aorta shares a uniform diameter $D$. An axisymmetric, smooth constriction is placed $1D$ ($L_i = 1D$) downstream of the inlet of the modeled aorta as the stenosis, and its profile is given in
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Figure 4.1: (a) Schematic of the modeled aorta with an axisymmetric 75% stenosis. The dash line represents the geometric centerline of the modeled aorta. (b) Frontal plane view of the mesh used in the current study. For the sake of clarity, every fourth mesh point in each direction is plotted here.

Appendix A. This shape represents the incomplete opening of the aortic valves due to stenosis, and the same profile is used in Seo et al. [100]. Similarly contoured occlusions have also been used in other studies to represent stenoses [72, 74, 83, 86, 88]. The area stenosis ratio ($AS$), which represents the percentage of area that is occluded, is used to denote the severity/degree of stenosis, and can be calculated by $1 - (D_j/D)^2$, where $D_j$ is the minimal diameter of the stenosis. In the current study, three different degrees of stenosis, $AS = 50\%$, $62.5\%$ and $75\%$, are considered, which represent mild to severe aortic stenoses. The cross-sectional averaged velocity at the location with the minimum diameter (jet velocity) can be derived from the mass conservation as $V_j = V_{in}(D/D_j)^2 = V_{in}/(1 - AS)$, where $V_{in}$ is the mean inlet velocity. The geometric centerline of the modeled aorta lies in the $y-z$ plane, and has a radius
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of $R_c = 2D$. Human aortas exhibit a certain degree of tortuosity, but we neglect this here to focus on the effect of the curvature. The vessel wall is assumed to be rigid, i.e., the translational motion and compliance of the wall are not considered. This is justified by Jin, Oshinski & Giddens [101], where they showed that the wall shear stress predicted from the rigid wall model and the deformable model agreed reasonably well. Moreover, the cross-sectional velocity distribution calculated from the rigid-wall patient-specific models correlated well with the corresponding MRI data [101].

The blood in the large arteries are usually treated as a Newtonian fluid [58] and in keeping with this, flow inside the modeled aorta is governed by the incompressible Navier-Stokes equations. The resting human heart rate is $O(1)$ Hz, while the murmur frequency is usually $O(100)$ Hz. Thus, the pulsatility of the flow is considered a slow variation compared to the murmur signal [21], and the blood flow in the current model is assumed to be driven by a uniform steady inflow ($V_{in}$) at the inlet. Peiffer et al. [102] demonstrated through their computational study of flow inside rabbit aortas that parabolic or skewed inflow profiles produced very similar results to the uniform inflow. The Reynolds number, which is defined as $Re = V_{in}D/\nu$, is set to 2000 in all the simulations, and it is close to the mean value in human aorta [103] and facilitates comparison with experiments that have used a similar value [72,74,75]. This Reynolds number is located in the transitional flow regime, and direct numerical simulation (DNS) is used to resolve relevant spatial and temporal scales of the flow. The Dean’s
number \((De)\) based on the provided \(R_c\) and \(Re\) is \(De = 4\sqrt{D/RcRe} = 5656.85\), which is well-within the typical physiological value \([103]\).

The complicated geometry is handled by a sharp-interface immersed boundary method described in Chapter 2. Fig. 4.1(b) shows the frontal plane view of the mesh employed in the simulation. The number of mesh points in \(x\), \(y\) and \(z\) direction are 128, 384 and 370, which results in a mesh with around 18 million points. This grid has been subjected to an extensive grid refinements study (see Appendix B.1). The minimum grid spacing is \(7.8 \times 10^{-3}D\), and the time-step is fixed at \(1 \times 10^{-3}D/V_{in}\). The maximum CFL number in these simulations is about 0.82. Simulations are carried out on the TACC-Stampede supercomputers with 768 cores, and it takes about 82 hours of wall-time to complete 3.5 flow-through-time for the \(AS = 75\%\) case. Here, one flow-through-time is defined as the length of the geometric centerline of the modeled aorta divided by \(V_{in}\).

It is useful to enumerate the key limitations of the current work. First of all, the steady flow assumption is adopted here, but the physiological flow is pulsatile with a complex time varying profile. The effect of the flow acceleration and deceleration on the dynamics of the jet is not included here. However, pulsatility introduces additional parameters into the problem which would significantly expand the scope of the simulations. Our expectation is that the current steady inflow model will serve as a baseline for exploration into the effect of pulsatility, which will be shown in Chapter 6. The second limitation is that the current model does not include the
aortic-valve leaflets. The opening and closing of these leaflets affect the dynamics of the jet and its interaction with the aortic lumen. The effect of the valve leaflets will also be discussed briefly through a straight tube model in Chapter 6. Last but not least, the human aorta typically has additional geometric complexities that are not included here. For example, the non-planar geometry of the aorta could induce an asymmetric flow with a significant swirl inside the aortic arch [104], and such effects are excluded in the current study.

Figure 4.2: (a) Schematics of the modeled aorta with 75% stenosis. The cutout shows the cross section of the aorta, and $r$, $\phi$ and $s$ represent the radial, azimuthal and streamwise directions respectively. (b) Monitor points inside the computational domain. The red squares locate on the anterior/posterior surface of the aorta, and the black circles locate at the geometric centerline of the aorta.
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4.2.2 Data Presentation

In the simulations shown in this chapter, monitor points are placed at different locations in the computational domain to record the temporal history for subsequent analysis. There are two sets of monitor points, and they are located on the anterior surface (squares) and the geometric centerline (circles) of the modeled aorta, as shown in Fig. 4.2. Monitor points are usually referred to through an angle $\theta$ measured from the starting location of the aortic arch (see Fig. 4.2(b)).

As stated before, the simulations are conducted for 3.5 flow-through-times and statistics are accumulated over the last 2.5 flow-through-times so as to exclude the transients. For a general flow quantity $f$, the mean over the averaging time $T$ is computed as

$$\bar{f}(x, y, z) = \frac{1}{T} \int_{t_0}^{t_0+T} f(x, y, z, t) dt,$$

where $t_0$ represents the initial time of the averaging. Accordingly, the fluctuation of the flow quantity is defined as

$$f'(x, y, z, t) = f(x, y, z, t) - \bar{f}(x, y, z).$$

The Root-Mean-Square (RMS) is computed by

$$f_{rms}(x, y, z) = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} [f'(x, y, z, t)]^2 dt}.$$
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Please note that the RMS here is defined on the fluctuation quantity.

Unless otherwise stated, the results presented in this chapter are non-dimensionalized by the following characteristic parameters: velocity scale, $V_j$; length scale, $D_j$; time scale, $D_j/V_j$; and pressure/stress scale, $\frac{1}{2}\rho V_j^2$. Particularly, the Strouhal number here is defined as $St = fD_j/V_j$.

4.3 Results and Discussion

4.3.1 Vortex Dynamics

Fig. 4.3 shows the snapshots of the azimuthal vorticity on the frontal plane of the three cases, $AS = 50\%$, $62.5\%$, and $75\%$, after the simulations reach stationary state. These plots demonstrate the general behaviors of the post-stenotic flow. The jet that is formed at the stenosis shares many characteristics among the three cases simulated here. As the jet advances into the aortic arch, the outer part of the jet impinges at the outer wall, while the inner part of the jet propagates further downstream and eventually starts shedding vortices, forming two separate recirculation areas denoted in the figures by arrow A and B. The inner part of the shear layer starts to exhibit discrete vortex shedding at roughly the same location of about $1.2D$ (arrow C) after the stenosis for all three cases. However, due to the different diameters of the jet, the location of jet impingement on the wall varies slightly between the three cases. Furthermore, after the outer portion of the jet hits the wall, a thin boundary layer is
Figure 4.3: Instantaneous non-dimensionalized azimuthal vorticity distribution on the frontal plane for (a) $AS = 50\%$, (b) $AS = 62.5\%$, and (c) $AS = 75\%$. All the plots share the same contour levels as shown in (c). Arrow A: small recirculation zone; arrow B: large recirculation zone; arrow C: starting location of the periodic shear layer vortex shedding.
formed near the outer wall. This boundary layer shields the outer wall from sensing the vortex shedding occurring inside the modeled aorta in case $AS = 50\%$ and $62.5\%$ due to the relative weak jet intensity. On the other hand, the vortices shed from the inner portion of the jet can be observed to penetrate the near-wall boundary layer in the aortic arch region for the $75\%$ case due to more intense jet as well as the shorter distance between the onset of shedding and the outer wall. Lastly, compared with the other two cases, due to the lower jet velocity, the flow in $50\%$ stenosis shows fewer small vortical structures in the aortic arch and the descending aorta.

Flow through a curved pipe is known to create secondary flows [105]. Here, we use the cross-sectional vorticity distribution at $\theta = 35^\circ$ of $AS = 62.5\%$ to examine some key features of the secondary flows generated here. Fig. 4.2(a) establishes the terminology for this analysis and Fig. 4.4(a) and 4.4(b) show the mean streamwise ($\overline{\omega}_s$) and azimuthal ($\overline{\omega}_\phi$) vorticity at the aforementioned plane. The mean streamwise vortical structures exhibit bilateral symmetry, with the flow near anterior wall rotating in the counterclockwise direction and the flow near the posterior wall rotating clockwise. These vortical structures can be roughly divided into three groups: the primary vortices, the secondary vortical regions and the near-wall vortical regions. Among the three vortical structures on the same side of the plane of symmetry, the primary vortex and the secondary vortical region share the same sign, while the near-wall vortical region has the opposite sign. Lee et al. [106] conducted numerical simulation of the flow in a similar curved pipe, but with no stenosis. At $Re = 500$,
Figure 4.4: (a) Mean streamwise vorticity at $\theta = 35^\circ$ of $AS = 62.5\%$. The vectors show the direction of the in-plane motion of the fluid elements. (b) Mean azimuthal vorticity at $\theta = 35^\circ$ of $AS = 62.5\%$. (c) Mean $z$ vorticity component at $\theta = 35^\circ$ of $AS = 62.5\%$. (d) Evolution of mean $y$ vorticity component at different $z$-planes in the ascending aorta of $AS = 62.5\%$. All the plots share the same contour levels shown in (d).
they also observed three groups of vortical structures. However, unlike the current flow, the primary vortex (Dean vortex) and the secondary vortical region in Lee et al. [106] have different signs. This hints at the different origins of the secondary vortical region in these two cases. In the traditional flow inside a smooth curved pipe, the secondary vortical region is formed from the near-wall vortical structure peeling off the wall. Hence, it has the same sign as the near-wall vortical region, but has a sign opposite to the primary vortex. In contrast, as will be shown later, the secondary vortical region is associated with the shear layer surrounding the jet in the current flow. Since the streamwise vorticity is the vector projection of the $z$-component and $y$-component of the vorticity onto the streamwise direction, we plot both components in Figs. 4.4(c) and 4.4(d) to help understand the origins of the primary vortices and secondary vortical regions in the current study. The mean $z$ vorticity component in Fig. 4.4(c) captures the primary vortices, which can be identified as the so-called Dean vortices [105, 106]. This is further confirmed by examining the vector field in Fig. 4.4(a), wherein the primary vortices overlap with regions with strong in-plane swirling motions. It is worth noting that the Dean vortices can be established quickly as shown in experimental studies by Bulusu et al. [107] where a physiological pulsatile inflow condition was adopted.

Fig. 4.4(d) shows the spatial evolution of the shear layer surrounding the core of the jet via contours of $\overline{w_y}$ on different $z$ planes. As can be seen, the inner part of the shear layer maintains its shape even after the outer part vanishes due to the jet
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![Figure 4.5: Non-dimensionalized mean streamwise vorticity of the three cases plotted at different angle, $\theta$, to illustrate its spatial evolution.](image)

impingement. This remaining shear layer generates $y$ vorticity on both sides of the plane of symmetry that have the same sign as the primary vortices. We note that since the inner portion of the jet flows mainly in the $z$ direction immediately after the stenosis (see Fig. 4.3), plotting the cross-sectional azimuthal vorticity will not reveal the true origin of the secondary vortical region, as demonstrated in Fig. 4.4(b).

With the key structures of the streamwise vorticity identified, we examine how these features evolve with downstream distance. Fig. 4.5 shows the mean streamwise vorticity at different angle $\theta$ for all three models. The region of vorticity deficit near the outer wall represents the core of the jet, and its area decreases as the stenosis becomes more severe. The aforementioned three groups of vortical structures can be clearly identified when $\theta \leq 45^\circ$. Nevertheless, as shown in Fig. 4.3, the inner
shear layer quickly breaks up as the jet advances downstream, introducing increased stochasticity in the flow behavior. Hence, the secondary vortical regions are quick to dissipate in all three cases after $55^\circ$, while the primary vortices persist further downstream for about $10^\circ$. Eventually, all the large vortical structures will disintegrate into small vortices in the descending aorta. In case $AS = 75\%$, the primary vortices break up significantly earlier than the other two due to the strong jet intensity.

In summary, the post-stenotic flow in this model of an aorta with an aortic stenosis is dominated by the jet created at the stenosis as well as the secondary flow induced by the curvature. The shear layer formed around the jet is the source of the secondary vortical regions and it separates the core of the jet from the secondary flow that generates the primary/Dean vortices. In previous experimental studies, where the aorta is modeled as a straight tube, the flow distal to the stenosis is mainly governed by the jet dynamics, and no secondary flow is reported $[72,74,75]$. The post-stenotic flow in a straight pipe is highly axisymmetric and the reattachment location is determined by the growth of the shear layer surrounding the jet $[108]$. However, the inclusion of curvature breaks the axisymmetry of the flow, and the reattachment of the outer part of the shear layer is clearly due to the geometric confinement, while the reattachment of the inner part of the jet cannot be easily defined.
Figure 4.6: Vortex structures corresponding to the mean flow visualized through isosurface of $\lambda_2$ and colored by the mean non-dimensional pressure (a) $AS = 50\%$, (b) $AS = 62.5\%$ and (c) $AS = 75\%$. (d) Decomposition of the vortex structures of $AS = 75\%$; left: shell structure; right: two finger-like structures.
4.3.2 Vortex Structures and Pressure

Since the current work is primarily motivated by auscultation-based diagnosis of aortic stenoses, we focus here on the characteristics of the pressure field in the post-stenotic flow and its correlation to the vortical structures. Figs. 4.6(a-c) show the vortical structures corresponding to the mean flow visualized by the isosurface of $\lambda_2$ in the three cases and these structures are colored by the mean pressure. The vortical structures can be roughly separated into two groups: a shell structure on the top and two finger-like structures beneath it (see Fig. 4.6(d)). From the pressure contours it can be seen that the shell structure is mainly correlated with high pressure, while the finger-like structures, which are the Dean vortices, are correlated with low pressure. The high pressure in the shell structure is caused by the high pressure carried inside the core of the jet, which results from the conversion of dynamic to static pressure due to the impact of the jet on the outer wall. These two vortical structures are well separated in the ascending aorta, but merge further downstream as the large vortical structures start to disintegrate due to the transition to turbulence (see Figs. 4.6(b) and 4.6(c)). However, in the case $AS = 50\%$, there seems to be no merging of these two structures, and the flow shows clear re-laminarization in the descending aorta, and the vortical structures are stretched and extended to the outlet of the modeled aorta.

Fig. 4.7(a) shows the cross-sectional distribution of the mean pressure at $\theta = 35^\circ$ of $AS = 62.5\%$ and we can see that the low pressure cores locate near the inner wall of
Figure 4.7: (a) Mean pressure distribution of case $AS = 62.5\%$ at $\theta = 35^\circ$. (b) The contour line of the pressure shown in (a) overlapped with mean streamwise vorticity.

the aorta, while high pressure region is localized in the jet region near the outer wall.

In Fig. 4.7(b), this pressure distribution is overlapped with the streamwise vorticity at the same location, and it is readily noticed that the two low pressure cores coincide well with the Dean vortices, which is consistent with Fig. 4.6. We also note that the secondary vortical regions mark the boundary between the low pressure region and the high pressure region.

Fig. 4.8 shows an array of cross-sectional mean pressure distribution at different angles of all three cases. Constant high pressure is observed in the jet region (near outer wall). As the severity of stenosis increases, the initial diameter of the jet decreases, and the size of the high pressure core decreases accordingly. This is especially obvious at the initial stage of the jet ($\theta < 45^\circ$). When compared with the streamwise vorticity in Fig. 4.5, Fig. 4.8 shows that the two low pressure cores are well-correlated with primary vortices in regions where the Dean flows are well developed ($35^\circ \leq \theta \leq 65^\circ$). Meanwhile, in the same region, the two low pressure
cores are moving towards the outer wall as the angle increases and also come closer to the anterior and posterior surfaces. For the 62.5% and 75% stenosis, the core size of the jet is smaller and the low pressure cores are able to migrate to the centerline of the aorta, whereas they remain confined to the lower half for the 50% case. This observation can also be verified by the locations of the finger-like vortical structures in Fig. 4.6.

4.3.3 Transition and Turbulence

As stated earlier, the flow studied here lies in the transitional region and in this section, we describe the transitional and turbulent characteristics of the flow. Fig. 4.9(a) shows the turbulent kinetic energy (TKE) distribution at $\theta = 60^\circ$ for the case...
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\[ \frac{1}{2} \frac{\bar{\rho} D_j}{V_j^2} \]

\[ \bar{\omega}_s D_j/V_j \]

\[ \bar{\omega}_\phi D_j/V_j \]

**Figure 4.9:** (a) TKE distribution of case \( AS = 62.5\% \) at \( \theta = 60^\circ \). (b) The contour line of the TKE shown in (a) overlapped with mean streamwise vorticity. (c) The contour line of the TKE shown in (a) overlapped with mean azimuthal vorticity.

\( AS = 62.5\% \). The high TKE region forms a bridge-like shape with two ends locating near the anterior and posterior surfaces of the aorta. In Figs. 4.9(b) and 4.9(c), the contour lines of the TKE distribution are overlapped with the mean streamwise (\( \bar{\omega}_s \)) and azimuthal (\( \bar{\omega}_\phi \)) vorticity components at the same location. As shown in Fig. 4.9(b), the Dean vortices partially overlap with the two ends of the bridge-like TKE distribution, indicating the contribution from the stochastic fluctuations within the secondary flows to the TKE. Moreover, the secondary flows wrap around the jet and force it into the shape of a crescent, as can be seen in Fig. 4.9(c). It is also noted that the majority of the bridge-shape structure is well-correlated with the strong azimuthal vorticity, and this azimuthal vorticity is related to the inner portion of the jet, where the shear layer vortex shedding and the subsequent vortex break-up mainly happens. Based on previous analysis of the vortex dynamics (see Figs. 4.3(b), 4.6(b)), this specific angle is where the periodic vortex shedding transitions into a more stochastic flow behavior, which accounts for the high TKE.
Fig. 4.10 shows the spatial evolution of the TKE for $AS = 50\%$, $62.5\%$, and $75\%$. At the initial stage of the jet, the flow is still predominantly laminar, and the plots show low TKE concentration for $\theta \leq 35^\circ$. From Fig. 4.3, we can see that, compared with the other two cases, the vortex shedding in $50\%$ case is considerably less intense. Thus, the TKE for $AS = 50\%$ is not noticeable until $\theta = 55^\circ$, and the majority of the TKE comes from the region of the Dean vortices. On the other hand, for cases $AS = 62.5\%$ and $75\%$, the intensity of the TKE is already quite noticeable at $\theta = 45^\circ$, where the jet is transitioning from periodic vortex shedding into more stochastic flow, and contributions from both the secondary vortical regions and the Dean vortices are visible. Similar to what we observed in Fig. 4.8, due to the size of the jet, the high TKE region moves closer to the outer wall and the anterior/posterior surface as the
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severity of the stenosis increases. It is worth mentioning that the TKE in Fig. 4.10 is non-dimensionalized by the mean jet velocity, $V_j$. Should it be non-dimensionalized by the inlet velocity, $V_{in}$, the TKE intensity in $AS = 75\%$ will appear much stronger than the other two under the same contour level.

Ahmed & Giddens [74, 75] studied flow past 50\% and 75\% stenosis at $Re = 2000$, and, similar to the current study, they also found that the turbulence is always preceded by the discrete-frequency vortex shedding. However, since the vessel is modeled as a straight tube in their study, the turbulent energy comes purely from the break-up of the jet shear layer. In the curved pipe model, the secondary flow induced by the curvature also contributes significantly to the total energy of turbulence. This is especially true for the 50\% stenosis, where the turbulent kinetic energy resulting from the shear layer break-up is significantly lower than that from the secondary flow.

### 4.3.4 Surface Force Analysis

As mentioned in the introduction, wall pressure fluctuations are known to be responsible for the generation of the murmurs and in this section, we focus on the characteristics of forces generated by the flow near the wall of the aorta.

Previous experimental studies of flows in straight tubes concluded that the maximum pressure fluctuation occurs slightly upstream of the reattachment position. Moreover, the reattachment is determined by the growth of the shear layer, and its position is relatively insensitive to the Reynolds number and the severity once the
flow enters the regime where the shear layer growth is dominated by the transition to turbulence. However, the inclusion of the curvature complicates the behavior, since the reattachment location is difficult to define and the secondary flows potentially introduce an additional source of fluctuation.

Fig. 4.11 and Fig. 4.12 plot the distribution of Root-Mean Square (RMS) of wall pressure and wall shear stress (WSS). The pressure fluctuations are expected to gen-
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enerate both compression (longitudinal) and shear (transverse) waves inside the thorax, while WSS fluctuations will primarily generate shear waves \[20, 100\]. Interestingly, the region in the vicinity of the stenosis and the location of jet impingement on the wall are correlated with low magnitude of pressure fluctuations as well as low WSS fluctuations. The strong surface force fluctuations are found to occur in the small recirculation region immediately after the stenosis as well as on the anterior and posterior surfaces of the ascending aorta. The surface force fluctuations on the anterior aortic lumen are of particular importance, since this surface faces the chest, where heart murmurs are typically detected.

It is readily noticed that the intensity of wall pressure fluctuation has very different distributions in this region for the three cases. For \( AS = 50\% \), a single high intensity region is observed on the anterior surface around \( \theta = 70^\circ \). This location is well-correlated with strong TKE cores in Fig. 4.10, which also occur in this general region. This TKE is mainly caused by the strong secondary flow and it is therefore reasonable to deduce that this strong wall pressure fluctuation is also related to the stochastic fluctuations inside the Dean vortices. On the other hand, the region of strong wall pressure fluctuations for \( AS = 62.5\% \), 75\% spans a large area on the anterior (and posterior) wall of the modeled aorta. In the \( AS = 75\% \) case, the anterior and posterior regions of high wall pressure fluctuation are joined together to form a single region that spans a large portion of the aortic arch. These behaviors can also be traced back to the TKE distribution. First of all, from Fig. 4.10 the
two ends of the TKE bridge lies very close to the anterior/posterior surface, and they show strong intensity over a wide angle ($55^\circ \leq \theta \leq 75^\circ$). This causes the large hot-spot on the anterior surface. Secondly, the break-up of the jet shear layer occurs closer to the outer wall as the severity of the stenosis increases, which also introduces significant pressure fluctuations near the outer wall in the last two cases. It is readily noticed that the TKE reaches its maximum around $65^\circ$ for the last two cases, and this explains why the source location for the $AS = 50\%$ case appears to be further downstream.

Unlike the RMS of wall pressure, the surface distribution of the WSS fluctuations is more consistent among the three cases. Apart from the hot-spot caused by the small recirculation zone immediately after the stenosis (arrow A in Fig. 4.3), the maximum intensity of WSS fluctuation is located on the anterior and posterior surfaces of the ascending aorta, and is closely associated with the location of the Dean vortices.
The RMS of pressure and WSS are extracted along the anterior monitor points and plotted in Fig. 4.13. It clearly shows that the peak magnitude of the WSS fluctuation is about 5% of that of the pressure fluctuation. Hence, the pressure fluctuation plays a dominant role in the murmur generation. From Fig. 4.13(a), the locations of the pressure fluctuation, i.e. the murmur source locations of AS = 50%, 62.5%, 75%, are found to be at $\theta = 70^\circ, 60^\circ, 55^\circ$, respectively. It is noted that even though the source locations span over 15°, they actually lie within 0.6D of each other. This relative insensitivity of the murmur source to the stenotic severity is a reflection of the dominant role of the aortic curvature on the generation of pressure fluctuations.

### 4.3.5 Spectral Analysis

One of the major motivations of this study is to correlate the characteristics of the murmurs, which are generated by the lumenal pressure fluctuations, with the degree of stenosis. Since the severity of the stenosis has a direct impact on the signal strength, researchers previously examined the scaling of the maximum wall pressure fluctuation, with the intention to find a universal scaling law that could relate the severity to the fluctuation intensity [21, 22, 71]. However, this is not an ideal metric in clinical practice since it requires careful calibration of the sensors and the measured signal strength is subject to various extraneous factors such as sensor contact condition, sensitivity and placement, as well as the thoracic anatomy of the patient. On the other hand, the frequency of the murmur signal can be measured with
greater confidence as long as it is within the operating range of the sensor. In this section, we will focus on examining the frequency characteristics of the post-stenotic flow in the modeled aorta.

Fig. 4.14(a) shows the spectra corresponding to the streamwise velocity perturbation along the geometric centerline of the modeled aorta of 75% case. We can see that at the exit of the stenosis ($\theta = 0^\circ$), the flow is still laminar and has very low fluctuation intensity. As the jet propagates further downstream, the shear layer of the jet exhibits periodic vortex shedding, and the spectrum at $\theta = 20^\circ$ shows a peak around $St = 0.93$. The reason the peak is not very sharp is that the signal is collected in the core region of the jet instead of the shear layer. This shedding frequency is higher than that of the circular jet reported by other researchers. For instance,
Beavers & Wilson [98] studied the natural shedding frequency of the free circular jet and observed a shedding frequency around 0.63 over a wide range of Reynolds numbers (500 – 3000). Similar numbers are also reported by other studies of post-stenotic jet in a confined straight tube [66, 72, 75]. The reason for a similar shedding frequency for both free and confined circular jet is that these post-stenotic flows are still dominated by the dynamics of the jet. The shear layer surrounding the core of the jet is less affected by the wall immediately after the stenosis, and its stability is still governed by the intrinsic mechanism of the shear layer instability. However, in the current study, even though the jet still maintains a circular shape right after the stenosis, the outer part of the jet impinges the outer wall before the shedding behavior occurs. The resulting shear layer no longer maintains a circular shape and hence, the characteristic shedding frequency from straight tubes are not expected to predict the shedding frequency in this case.

Beyond the angle of 40°, the discrete shedding frequency is no longer visible in the spectrum, and the flow starts to transition to turbulence. At 75°, the spectrum corresponding to the velocity fluctuation clearly demonstrates turbulent flow scaling as shown in Fig. 4.14(b). Two other spectra, corresponding to the signals at different locations of 50% and 62.5% cases, are also plotted in Fig. 4.14(b). Both of them show reasonable agreement with the classic scalings [110, 111], verifying the existence of turbulence in such flows. It is worth emphasizing that signals plotted here are collected along the geometric centerline of the aorta. But, it is clearly shown in Fig.
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\[ \frac{p'}{\left( \frac{1}{2} \rho V^2 \right)} \]

Figure 4.15: Temporal variation of non-dimensionalized pressure fluctuation at \( \theta = 35^\circ \) of \( AS = 75\% \). The y-axis is set to \([-0.2, 0.2]\) to facilitate comparison.

4.10 that the locations selected here are not necessarily where turbulence develops first or is the strongest. This can also potentially explain the imperfect scaling of the 50\% case at the high frequency range, since the TKE shows low intensity along the centerline there.

Pressure is of great interest in this study as it is responsible for the generation of the murmurs. In the following discussion, we will first explain the results for 75\% stenosis in detail, and then present the results from the other two cases in comparison to the 75\% case. Fig. 4.15 shows the temporal history of the pressure fluctuations at \( \theta = 35^\circ \) of 75\% stenosis. Even though no clear periodicity can be observed in the signal from the centerline, it will be shown later that there is periodic vortex shedding happening at this location. Fig. 4.16(a) illustrates how the spectrum of the pressure
fluctuation evolves along the centerline. The flow is relatively quiescent coming out of the stenosis, and weak pressure fluctuations are observed at $\theta = 0^\circ$. As the jet propagates downstream, the shear layer becomes unsteady and discrete-frequency vortex shedding with a Strouhal number of 0.93 appears, as shown by the spectrum at $\theta = 20^\circ$. This characteristic frequency is consistent with the value obtained previously from the velocity spectrum. A clear peak at $St = 0.93$ can be observed in the pressure spectrum from $20^\circ$ to $35^\circ$, indicating a region where the vortex shedding is dominant. As the flow moves downstream, it transitions to turbulence and the aforementioned peak is no longer visible. The spectra of the centerline pressure fluctuation of 50% and 62.5% cases demonstrate similar spatial evolution. Particularly, the discrete vortex shedding frequency, when non-dimensionalized by the jet diameter and jet velocity, is around 0.93 for all three cases, as shown in Fig. 4.16(b). This specific frequency can
be observed from $20^\circ$ to $55^\circ$ for $AS = 50\%$, and from $25^\circ$ to $40^\circ$ for $AS = 62.5\%$. It is noted that the vortex shedding develops around the same angle for all three cases. However, as the jet intensity increases with the severity of the stenosis, the location of the transition to turbulence moves upstream, leading to an early disappearance of the corresponding peak in the spectrum. Nevertheless, as will be shown later, the absence of this peak does not indicate a lack of contribution from the periodic vortex shedding to the flow in this region.

The surface pressure fluctuations are directly relevant to the generation of murmurs, and in Fig. 4.17(a), we plot the spectra of the pressure fluctuation on the anterior surface at the same angles as in Fig. 4.16(a). It is immediately apparent that there is no clear peak at $\theta = 20^\circ$, but, interestingly, the slope of the spectra at
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Table 4.1: Details of the least squares based linear curve fitting. $\tilde{f}_{bc}$ is the observed break frequency and $f_{bc}$ is the break frequency calculated from the linear regression.

<table>
<thead>
<tr>
<th>Figure</th>
<th>AS</th>
<th>Angle</th>
<th>$f_{bc}$</th>
<th>$f_{bc}$</th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.17(b)</td>
<td>75%</td>
<td>40°</td>
<td>0.93</td>
<td>0.92</td>
<td>$E_p = 10^{-2.2590} St^{0.2396}$</td>
<td>$E_p = 10^{-2.3743} St^{-3.0000}$</td>
</tr>
<tr>
<td>4.18(a)</td>
<td>50%</td>
<td>70°</td>
<td>0.93</td>
<td>0.97</td>
<td>$E_p = 10^{-1.8183} St^{0.1481}$</td>
<td>$E_p = 10^{-1.8692} St^{-3.6764}$</td>
</tr>
<tr>
<td>4.18(b)</td>
<td>62.5%</td>
<td>60°</td>
<td>0.93</td>
<td>0.92</td>
<td>$E_p = 10^{-1.7210} St^{0.2625}$</td>
<td>$E_p = 10^{-1.8342} St^{-2.8570}$</td>
</tr>
<tr>
<td>4.18(c)</td>
<td>75%</td>
<td>55°</td>
<td>0.93</td>
<td>1.02</td>
<td>$E_p = 10^{-1.9565} St^{0.0689}$</td>
<td>$E_p = 10^{-1.9308} St^{-2.6387}$</td>
</tr>
</tbody>
</table>

$\theta = 40^\circ$ seems to change significantly across $St = 0.93$. This change of slope has been observed in previous experimental [21] and numerical [23] studies. The frequency at which the slope changes, usually called the ‘break frequency’ or ‘corner frequency’, has been identified as an important metric in electronic auscultation based determination of the degree of stenosis [7].

Using a piecewise linear regression, we have obtained the best fit lines of each side of the observed break frequency, $\tilde{f}_{bc}$, as shown in Fig. 4.17(b). Here, the intersection of these two linear regressions gives the break frequency ($f_{bc}$) and the difference between $f_{bc}$ and $\tilde{f}_{bc}$ indicates the quality of the approximation. From Table 4.1, we can see that, for Fig. 4.17(b), the observed break frequency, which is also the shedding frequency of the shear layer, is sufficiently close to $f_{bc}$. This means that even though no distinct peak from vortex shedding is present in the spectrum, the break frequency shows a significant contribution from the discrete vortex shedding to the pressure fluctuation.

It is also useful to study the break frequency in the surface region identified as the primary source for the murmurs. Figs. 4.18(a) to 4.18(c) plot the spectra of pressure fluctuation at the source location identified in Fig. 4.13(a) along with the linear
Figure 4.18: Spectra of the pressure fluctuation at the source locations identified in figure 4.13(a) plotted with the corresponding linear curve fittings. Details of the linear curve fitting are provided in Table 4.1. The vertical line denotes at \( St = 0.93 \).
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curve fit, and $f_{bc}$ are determined to be sufficiently close to 0.93 for $AS = 50\%$ and 62.5\%, indicating the significant contribution from the shear layer vortex shedding. On the other hand, $f_{bc}$ deviates slightly but noticeable away from this frequency for $AS = 75\%$ and this could be due to several reasons. First of all, the smaller jet size means that the vortex shedding occurs closer to the geometric centerline of the modeled aorta, and away from the anterior surface, making the frequency harder to transmit to the surface. Moreover, the strong jet is also likely to induce stronger turbulence, and increase the relative contribution of stochastic turbulent fluctuation to the wall pressure fluctuation.

Combining the results in Table 4.1 and Fig. 4.10 provides additional insights into the mechanism for murmurs generation. Previous analysis shows that the location of strong wall pressure fluctuations is spatially well-correlated with the location of intense turbulent fluctuations. However, the study of break frequency reveals that the total pressure fluctuations also contain contribution from the shear layer vortex shedding from upstream. In clinical practice, a 75\% area reduction is at the threshold between a mild and a severe stenosis. Severe stenosis causes noticeable symptoms in the patient and are therefore easy to detect. Mild stenosis on the other hand, are asymptomatic and thus harder to detect. The spectra analysis shows that, for mild stenosis, the discrete-frequency shear layer shedding frequency is also the break frequency of the murmur source, and can potentially be sensed on the epidermal surface. Thus, auscultation may be a promising tool for detecting and screening of
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early-stage aortic stenosis.

4.4 Conclusion

Direct numerical simulations of steady flow in a modeled stenosed aorta are conducted. Three different degrees of stenosis, $AS = 50\%, 62.5\%$ and $75\%$, are studied and the Reynolds number based on the inlet velocity and diameter is fixed at 2000. Compared with the previous experimental studies [67, 69, 74, 75] and numerical studies [22, 84, 86, 88, 89], the major difference here is that the aortic arch is included as a curved pipe with a $180^\circ$ turn. The inclusion of this curvature induces strong secondary flows including Dean vortices in the aortic jet, which originate in the ascending area and persist all the way into the aortic arch.

The current study is particularly focused on the implication of the flow dynamics on the generation and detection of the associated heart murmurs. The murmur source is identified as the location on the anterior aortic lumen with maximum wall pressure fluctuations. A key finding of the current study is that in all the cases studied here, this source is spatially about $2.1D$ downstream from the aortic valve area. Thus, the sound source is not co-located with the underlying pathology, and this has implication for the detection of this disease via automated source localization. The source location is also found to be relatively insensitive to the severity of the stenosis. This is because, unlike stenoses in straight tubes, where the source location is determined by the natural instability of the shear layers, the murmur source here
includes significant contributions not only from the intrinsic instability of the aortic jet but more importantly, from the unsteadiness associated with the secondary flows induced by the aortic curvature.

Inspection of the spectra of the pressure fluctuations reveals that the post-stenotic jet in all the cases exhibits a distinct frequency due to the shear layer vortex formation and this characteristic shedding frequency, when non-dimensionalized by the jet diameter and jet velocity, is around 0.93 for all three cases. This frequency is also identifiable in the wall pressure spectra at the source location in the form of a break frequency for all three cases.

The identifications of the source location and the spectral characteristics are particularly relevant for automated auscultation-based diagnosis. First, the lack of dependence between the source location and the severity of the stenosis hints that source localization algorithms can potentially be used to distinguish aortic stenosis from other heart conditions that generate systolic murmurs such as tricuspid insufficiency, the murmurs of which are expected to be best detected around the lower left sternal border[97]. Second, the break frequency provides a universal scaling for cases with different degrees of stenosis. However, it is important to emphasize that this frequency is measured at the source location, while the final murmur signal is measured on the skin surface. The effect the propagation of the elastic waves through the thorax is investigated in the next chapter.
Chapter 5

Computational Modeling and Analysis of Murmurs Generated by Modeled Aortic Stenoses

5.1 Introduction

In this chapter, we focus on the propagation of heart murmurs generated by the aortic stenosis studied in Chapter 4. Compared with studies by other researchers [21, 28, 31, 35, 37], here, the generation and propagation of murmurs associated with the aortic stenosis are studied in a coupled fashion using numerical simulations from first principles. First, hemodynamic simulations are performed to study the post-stenotic flows. The key flow patterns, source locations as well as the characteristic frequencies of the source are identified in the previous chapter. Then, the flow information is used as the input in the acoustic simulations to study the propagation of the elastic waves inside the modeled thorax, source localization and the characteristics of the
resulting murmurs measured on the epidermal surface. Another important feature of the current study is the inclusion of shear wave propagation. The study in Chapter 3 already demonstrates through a classic vector decomposition that the shear wave has a significant impact on the spectra of the murmurs and acoustic energy distribution on the epidermal surface.

This chapter is organized as follows. First, the model employed in the study is introduced in Section 5.2. The results from the acoustic simulations are presented in Sections 5.3.1 and 5.3.2. Then, a discussion is provided in Section 5.3.3 with the help of free-space Green’s function. Finally, some concluding remarks as well as the implications of the current study to the cardiac auscultation are presented in Section 5.4.

5.2 Model Configuration

Fig. 5.1 shows the geometry adopted in the current study. The thorax is modeled as an elliptic cylinder in this study, as shown in Fig. 5.1(a), and dimensions of the geometry are provided in Fig. 5.1(b). The modeled aorta used in Chapter 4 is embedded inside this thorax model to serve as the source of the murmurs. In reality, the human thorax is highly inhomogeneous and consists of bones, lungs, muscles and other tissues. However, to simplify the problem, the thorax is treated as a homogeneous, linear material here, and the viscoelastic behavior of the tissue is described by the Kelvin-Voigt model. The resulting governing equations for elastic
Figure 5.1: (a) Schematic of the modeled thorax; (b) dimensions of the modeled thorax. The aorta has the same geometry and dimension as in Fig. 4.1.
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wave propagation are

\[ \rho_s \frac{\partial u_i}{\partial t} + \frac{\partial p_{ij}}{\partial x_j} = \eta \frac{\partial}{\partial x_j} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \]  

(5.1)

\[ \frac{\partial p_{ij}}{\partial t} + \lambda \frac{\partial u_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = 0, \]  

(5.2)

where the indices \( i \) and \( j \) range from 1 to 3. \( u_i \) and \( p_{ij} \) are the acoustic velocity vector and the stress tensor in the thorax. \( \rho_s, \eta, \lambda, \) and \( \mu \) are the density, viscosity, first and second Lame constants of the material, respectively. \( \delta_{ij} \) is the Kronecker delta function, and it has value 1 when \( i = j \) and value 0 when \( i \neq j \). The same material model is also employed in earlier studies [20,100].

Previous studies of the fluid dynamics and acoustics associated with heart murmurs [22,24,36,69,96] have concluded that the generation of the murmurs are closely associated with the pressure force on the wall of the blood vessel. To incorporate the wall pressure from a more realistic model, this hemoacoustic problem is solved through a one-way coupled approach [100]. First, the flow inside the modeled aorta with stenosis is simulated with a sharp-interface immersed boundary flow solver with improved mass conservation [39,46,112]. Since this study aims to delineate the connections between the abnormal fluid behaviors and the murmur signals, a uniform steady inflow \( (V_{in}) \) is prescribed at the inlet of the aorta to avoid the extra complications and parameters introduced by the pulsatility. During the hemodynamic simulations, the temporal history of the wall pressure is recorded and is subsequently
applied in the acoustic simulations as the boundary condition at the vessel-tissue interface.

To reveal the effect of the severity on the characteristics of murmurs, the results from three cases \( (AS = 50\%, 62.5\% \text{ and } 75\%) \) in Chapter 4 are used to generate murmurs in the current acoustic study. These three cases represent stenoses ranging from mild to severe. In the hemodynamic simulations, the inlet velocity \( (V_{in}) \) is set at \( 0.25 m/s \), and the diameter \( (D) \) at the unconstricted part of the aorta is \( 2.5 cm \). Along with a kinematic viscosity \( (\nu) \) of \( 3.125 \times 10^{-6} m^2/s \), these parameters result in a Reynolds number \( (Re = V_{in}D/\nu) \) of 2000, which is within the physiological range [103] and matches the Reynolds number employed in other experimental studies [72,74,75]. More details about the hemodynamic simulations can be found in Chapter 4.

In the hemoacoustic simulations, the material properties of the thorax are modeled after a widely-used tissue mimicking material, EcoFlex-10. The density of the material is \( 1040 kg/m^3 \). The shear wave speed \( (c_s) \) and compression wave speed \( (c_p) \) are \( 4.2/s \) and \( 1000m/s \), respectively, and the viscosity of the material is set to be \( 14 Pa\cdot s \). It is noted that the compression wave Mach number \( (V_{in}/c_p) \) of the problem is \( 2.5 \times 10^{-4} \). This large separation of velocity scales imposes a very strict CFL condition based time-step size constraint on the hemoacoustic simulations. In order to simulate the problem with reasonable computational cost, the compression wave speed in this study is reduced by one order of magnitude to \( 100m/s \) while the shear wave speed is kept the same. The effect of this practice is rigorously tested in Ref. [100] and it
shows little effect on the wave propagation since the wave length of the compression wave with decreased speed is still much longer than the characteristic length of the thorax at the frequency range of interest \((f < 1000Hz)\). This 10-fold reduction of the compression wave speed is also adopted previously \([20, 23]\), and has been shown to have no significant effect on the final signal.

The acoustic simulations are conducted with the high-order, immersed boundary based, finite-difference solver developed in Ref. \([100]\). The computational domain in the acoustic simulations is discretized by a uniform, isotropic grid with 190, 360 and 220 (~15 million) grid points along the minor axis, major axis and height directions, respectively. The grid spacing is about 0.027\(D\), and a shear wave at 400\(Hz\) is resolved by around 16 points/wavelength with this grid spacing, which is well above the 8 points/wavelength recommended in Ref. \([100]\) for this high-order solver. The ETA (Energy Transfer and Annihilating) non-reflecting boundary condition \([61]\) is adopted at the top and bottom of the elliptical cylinder and the buffer zone is set to be 20 grid points on each end. A traction-free boundary condition is applied on the side of the modeled thorax to model the air-tissue interface. It takes 96 hours of wall time to obtain 1.2s of acoustic signals on TACC-Stampedede1 supercomputer using 600 cores. More details regarding the numerical methods employed in the hemoacoustic simulations as well as validation against experiments can be found in Ref. \([100]\).
5.3 Results and Discussion

In this section, the acoustic simulations for three cases with different degrees of stenosis (50%, 62.5% and 75%) are presented. Results from the 75% case are presented in detail first, and results from the other two cases are subsequently discussed in comparison to the 75% case. To make sure that the results are presented consistently, the following characteristic scales are used to non-dimensionalize the results in this chapter unless otherwise stated: velocity scale, \( c_p \); length scale, \( D \); time scale, \( D/c_p \); pressure/stress scale, \( \rho_s c_p^2 \). The Strouhal number is defined as \( St = fD/c_p \). Moreover, since the problem is symmetric with regard to the frontal plane, ensemble average of results from the anterior and posterior surface is taken when possible.

5.3.1 Murmur Propagation for 75% Stenosis Case

Fig. 5.2 demonstrates the wave pattern inside the tissue through contours of the instantaneous velocity. It clearly shows the generation of elastic waves at the interface of the aorta and the tissue, as well as the propagation of these waves inside the tissue. On the frontal plane, a distinct wave pattern is observed between 45° and 90° (the angle is measured from the end of the stenosis toward the ascending aorta as shown in Fig. 4.2(b)). However, in clinical practice, doctors can only measure signals on the epidermal surface with the help of a stethoscope or other sensors. In order to mimic this in the simulations, four surface monitor points are placed across the epidermal surface of the modeled thorax (see Fig. 5.2), and the signals at these locations are
Figure 5.2: Instantaneous wave pattern inside the modeled thorax with 75% stenosis demonstrated by the velocity contour. Frontal plane plots the velocity along the major axis \((y)\) direction and the sagittal plane plots the velocity along the minor axis \((x)\) direction.
The temporal histories of the wall-normal velocity perturbations are plotted in Fig. 5.3, and the wall-normal velocity is defined as $u_n = u_x n_x + u_y n_y$, where $(n_x, n_y)$ is the unit surface norm. The signal at location A has visibly stronger perturbations as it lies almost directly above the actual source location.

Spectra of the wall-normal acceleration calculated from these temporal signals are plotted in Fig. 5.4. Apart from the location D, the spectra from the other three locations share very similar trend. The amplitude first increases gradually as the frequency increases up to $St \approx 5 \times 10^{-3}$. The spectra then exhibit a short plateau.
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Figure 5.4: Spectra of the wall-normal acceleration at the four surface locations shown in Fig. 5.2 for 75% stenosis. The Strouhal number is defined as $St = fD/c_p$.

before they drop sharply after $St \approx 2 \times 10^{-2}$. The acoustic break frequency can therefore be identified around $St = 2 \times 10^{-2}$ for this case. It should be emphasized that the break frequency defined here is associated with the murmur signal instead of the wall pressure fluctuations on the vessel wall.

Based on the trend of the murmur spectra, the frequency range is separated into the three bands indicated in Fig. 5.4. These three frequency bands are:

I : $f \in (10, 20] \text{Hz};$ or $St \in (2.5 \times 10^{-3}, 5 \times 10^{-3}]$;

II : $f \in (20, 80] \text{Hz};$ or $St \in (5 \times 10^{-3}, 2 \times 10^{-2}]$;

III : $f \in (80, 400] \text{Hz};$ or $St \in (2 \times 10^{-2}, 10^{-1}]$;

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Figure 5.5: Two-dimensional projection of the spectral energy of wall-normal acceleration on the anterior/posterior surface over the entire frequency band (I+II+III) for 75% case. Please note that range of the contour is adjusted to best reflect the signal intensity in each case and the outline of the modeled aorta is included for clarity. (a) With shear modulus; (b) without shear modulus.

The low frequency band starts from $10\text{Hz}$ assuming that the energy above the tenth harmonics of the cardiac cycle is not heavily influenced by whether the inflow is steady or pulsatile. One intuitive way to predict the source location of the murmurs is to inspect the intensity distribution of the surface signals. The underlying rationale is that for such a simple geometry and material model, the epidermal surface directly above the source location is expected to receive the strongest signal. The intensity of the signal in the frequency band $(f_1, f_2]$ can be quantified by the band-limited spectral
energy, which is defined as \( a_n^2 = \sum_{f_1}^{f_2} |\langle a_n(f) \rangle|^2 \), where \( \langle \cdot \rangle \) represents the Fourier transform. As the monitor points are placed across the entire epidermal surface, we are able to calculate the band-limited spectral energy over the entire anterior/posterior surface and project the three-dimensional surface into a two-dimensional plane as shown in Fig. 5.5. The outline of the modeled aorta is also included here for clarity. The energy peak on these plots not only indicates the location on the epidermal surface with strongest acoustic signals but also serves as the prediction of source location on the modeled aorta.

Fig. 5.5(a) shows the energy distribution over the entire frequency band, and two peaks with similar magnitudes are observed. One is located above the ascending aorta region and overlaps with the correct source location, while the other one is near the outlet of the modeled aorta and is a ‘false’ peak. Similar behaviors are observed in the previous two-dimensional study [20] where a ‘false’ peak is observed due to the propagation of the shear wave. Based on the previous study [20,23], compression wave is convenient for predicting the source location due to its long wave length and better directionality. On the other hand, the shear wave can cause incorrect prediction due to the orthogonality between its propagation direction and the oscillation direction of the local material element. Similar conclusions can also be drawn from the current three-dimensional case. As shown in Fig. 5.5(b), the surface energy contours show a single peak in the vicinity of the actual source location when shear wave propagation is not considered in the simulation. Moreover, in Fig. 5.6(a), the spectra energy
Figure 5.6: Two-dimensional projection of the spectral energy of wall-normal acceleration on the anterior/posterior surface over different frequency bands for 75% case. The range of the contour is adjusted to best reflect the signal intensity in each frequency range and the outline of the modeled aorta is included for clarity. (a) frequency band I; (b) frequency band II; (c) frequency band III.
distribution in the low frequency band (I) is plotted, and the peak predicts the source location reasonably well. The energy at the ‘false’ source location becomes stronger as the frequency increases (Figs. 5.6(b,c)). This deterioration of prediction accuracy is related to the alternating dominance of shear wave and compression wave at different frequency bands [20][100]. At the low frequency band, the compression wave has strength comparable to the shear wave, and its long wavelength provides better directionality for source localization. On the other hand, the shear wave dominates the middle frequency band, leading to false peaks, while the short wavelength of high frequency waves might make it more susceptible to the wall effect and ineffective in source localization.

5.3.2 Effects of the Severity of Stenosis

The focus of this study is on two particular aspects of auscultation: the first one is regarding the localization of the murmur source from surface signals, and the second is the connections between the source spectra and the murmur spectra. In this section, the results of the three different degrees of stenosis, 50%, 62.5% and 75% are presented in order to explore these two aspects.

Apart from the aortic stenosis, several other heart conditions are known to generate systolic murmurs, such as obstructive hypertrophic cardiomyopathy, mitral valve regurgitation, pulmonary valve stenosis and tricuspid insufficiency. However, the source of the murmur usually localizes in the vicinity of the underlying pathology,
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Figure 5.7: Two-dimensional projection of the spectral energy of wall-normal acceleration on the anterior/posterior surface over the low frequency band I. A black diamond symbol is plotted in the same spatial location in Fig. 5.6(a) and Fig. 5.7 to facilitate comparison. (a) 50%; (b) 62.5%.

a fact that clinicians know and exploit in their assessment. For example, murmurs originating from the mitral valve are heard best near the cardiac apex; murmurs generated by the pulmonary valve disease are usually best heard in the second and third left interspaces close to the sternum; and murmurs arising from the tricuspid valve are detected best around the lower left sternal border [97].

According to our previous study of the hemodynamics of this configuration, the source locations for all three severities localize downstream of the stenosis and are
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within 0.6D of each other (Figs. 4.11 and 4.13(a)). This suggests that the source location of the aortic stenosis is relatively insensitive to its severity and, within the context of automated auscultation, this could potentially be used to determine whether a systolic murmur is caused by an aortic stenosis. It is already shown in Fig. 5.6(a) that the low-frequency-band spectral energy distribution on the epidermal surface has predicted a source location that is sufficiently close to the actual location for 75% case.

Fig. 5.7(a) and Fig. 5.7(b) plot the same energy distribution for 50% and 62.5% case. These figures show that the energy peak directly above the modeled aorta predict a similar source location for different severities, and these locations lie in close proximity to the actual source locations. It is noted that there is also a local maximum located on the left lateral surface on the thorax for the 50% stenosis case. This peak is generated due to the strong pressure fluctuations inside small recirculation area (arrow A in Fig. 4.3). However, this peak can be easily dismissed as a source location since it is not spatially correlated with the modeled aorta. Also, whether this peak is still visible in a physiologically accurate thorax models is questionable due to the presence of the lungs.

Another important aspect of auscultation is the spectral characteristics of the murmurs. As mentioned before, researchers have long believed that the break frequency of the murmurs carries information about severity of the stenosis [6, 7]. In the current study, there is no clear preferred location to study the frequency since the break frequency can be observed over a wide spatial range (see Fig. 5.4). In
order to make the study consistent, signals measured at the location of the black diamond symbols in Fig. 5.6(a), Fig. 5.7(a) and Fig. 5.7(b) are used to calculate the spectra shown in Fig. 5.8. These three spectra share similar trends: the amplitude first increases as the frequency increases, and they level off afterwards, and also the amplitude decreases sharply when the frequency is increased further. The acoustic break frequency is generally around $St = 2 \times 10^{-2}$ for the three cases, but decreases slightly as the severity decreases.

It is very critical to understand the difference between the signal spectra from the flow and the acoustic studies. In Fig. 5.9, the same murmur spectra are renormalized by the jet velocity and jet diameter. Compared with source spectra plotted in Fig.
Figure 5.9: Spectra of the wall-normal acceleration on the epidermal surface of the modeled aorta renormalized using jet velocity ($V_j$) and jet diameter ($D_j$). The signals are collected at the same spatial location indicated by the black diamond symbol. Here, the vertical dash line represents $St_j = 0.93$, which is the hemodynamic break frequency.
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4.18 it can be seen that the propagation inside the tissue alters the source spectra significantly. The low frequency band energy is much lower in the murmur signal while the sharp slope change around the break frequency is more clearly defined in the murmur spectra than the source spectra. Moreover, there is no clear connection between the hemodynamic break frequency and the acoustic break frequency. This is especially true for the 50% and 62.5% case, as $St_j = 0.93$ lies in the plateau of the murmur spectra. These discrepancies between the flow and acoustic spectra clearly reveal the importance of understanding the propagation of the murmurs.

5.3.3 Analysis using Green’s Function

The previous section presents the results from the hemoacoustic simulations. In this section, motivated by the studies of Royston et al. [34] and Seo et al. [100], we try to gain a better understanding of the results from a more theoretical perspective with the help of the free-space Green’s function. Compared to the vector decomposition proposed in Chapter 3, the free-space Green’s function can provide insight into the shear wave and compression wave behavior at a significantly lower computational cost. The free-space Green’s function for the current homogeneous, linear viscoelastic material is summarized in the Appendix C. With the help of this function, the murmur spectrum at the skin surface is approximated using the pressure fluctuations on the aortic lumen wall (i.e. the ‘forward’ problem) in Section 5.3.3.1. In Section 5.3.3.2 we explore the possibility of localizing the murmur source from the acoustic signals.
5.3.3.1 Forward Problem

In the forward problem, the pressure recorded on the wall of the modeled aorta is used to evaluate the signal at the epidermal surface of the thorax. Instead of finding the Green’s function for the current problem, *i.e.* a point source inside an elliptical cylinder, the free-space Green’s function is used to approximate the solution. The same practice is adopted in Refs. [34,100]. Even though using the free-space Green’s function doesn’t take into consideration of the shear/compression wave conversion at the surface, it still provides reasonable agreement with the analytical solution [34] and the numerical solution [100].
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The set-up of the problem is illustrated in Fig. 5.10. The surface force in Eq. C.5 is replaced with a distribution of 37 point sources on the lumen surface (black dots in Fig. 5.10) which are spaced 5° away from each other on both the anterior surface and the posterior surface of the aorta. The temporal histories of the pressure on these points are recorded and serve as the point sources in calculating the resulting acoustic signal in a given point in space through the following equation

\[
\frac{\langle a_n \rangle (\vec{x}, \omega)}{\Delta A} = 2 \sum_{j=1}^{J} (i\omega)^2 N_p G_{pq} (\vec{x} - \vec{X}_j, \omega) M_q (\vec{X}_j) (P(\vec{X}_j, \omega)).
\]

Here, \( J \) is the total number of point sources, and it equals 74 (from both anterior and posterior surface) in this study. \( \vec{X}_j \) is the coordinate of the \( j^{th} \) point source, and \( \vec{x} \) is the coordinate of the target point on the epidermal surface of the thorax, which is chosen to correspond to the location of the black diamond symbol in Fig. 5.6(a). \( \vec{M}(\vec{X}) \) and \( \vec{N}(\vec{x}) \) are the unit normal vector at the locations of the point source and the target point, respectively. Finally, \( P(\vec{X}) \) is the hydrodynamic pressure force at point source \( \vec{X} \). The indices \( p \) and \( q \) range from 1 to 3 and \( \Delta A \) is the surface area occupied by each point source. The multiplicative factor of two preceding the summation is a crude approximation of the wave reflection at the epidermal surface based on the suggestion in Ref. [34].

Fig. 5.11 shows the results for all three cases, and the spectra demonstrate significant similarity to the results from the hemoacoustic simulations in Fig. 5.8. Moreover,
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Figure 5.11: Spectra of the wall-normal acceleration calculated from the free-space Green’s function at the same location as in Fig. 5.8.

Figure 5.12: Spectra calculated from individual point source at different angle $\theta$ for 75% case.
the acoustic break frequencies based on the approximations match those from the numerical simulations reasonably well, and they show the same slight decrease with decrease in severity. The above observation clearly shows that 74 point sources are sufficient to capture the characteristics of the spectra of the wall-normal acceleration. This also indicates that, at least in this case, the geometry of the modeled thorax is not the determining factor for the shape of the spectra and the break frequency in particular. Fig. 5.11 is the result of all the point sources superimposed at the target location, while Fig. 5.12 shows the contribution from individual point source at different angle $\theta$ for 75% case. It is apparent that the spectra generated from single point source does not exhibit the aforementioned shape and break frequency, which echoes the finding of Fredberg [24] that it is the superimposition of murmurs generated at different locations of the modeled aorta that determines the shape of the final spectrum. Thus, using a single point source, even the one at the dominant source location, does not provide a reasonable representation of the overall signal in experimental or numerical studies.

5.3.3.2 Inverse Problem

Since the free-space Green’s function performs relatively well in terms of predicting the shape of the spectra of the murmur signals, it is possible to use it to develop other source localization methods. Fig. 5.13 illustrates the process of the source localization, i.e. using the surface signal collected from the hemoacoustic simulations
Figure 5.13: Illustration for the source localization problem using free-space Green’s function. The green patch is the plane where source location is evaluated, and it is discretized into a $5 \times 5$ grid. The red circles on the skin surface represent the $4 \times 4$ sensor array.

to locate the source of the murmurs. During the hemoacoustic simulations, wall-normal accelerations are recorded on the epidermal surface at different locations.

Assuming there are $K$ monitor points in total, we have

$$
\langle a_n \rangle (\vec{x}_k, \omega) = \sum_{j=1}^{J} (i\omega)^2 N_p(\vec{x}_k) G_{pq}(\vec{x}_k - \vec{X}_j, \omega) M_q(\vec{X}_j) \Delta A(P)(\vec{X}_j, \omega).
$$

(5.4)

All the parameters have the same meanings as before, except that now $\langle a_n \rangle$ is known, and $\langle P \rangle$ is unknown. The distribution of the pressure will reveal the source location.

Employing the following short-hand $f_k$, $f_j$ and $f_{kj}$ for $f(\vec{x}_k)$, $f(\vec{X}_j)$ and $f(\vec{x}_k - \vec{X}_j)$ respectively. We have

$$
\langle a_n \rangle_k(\omega) = G_{kj}(\omega) \langle P \rangle_j(\omega).
$$

(5.5)
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\( \mathcal{G} \) is a matrix of size \( K \times J \), where \( K \) is the number of target points while \( J \) is the number of point sources. Generally, these two number are not necessarily equal, and hence the pressure distribution at angular frequency \( \omega \) can be expressed as

\[
\langle P \rangle_j(\omega) = [\mathcal{G}_{kj}(\omega)]^\perp \langle a_n \rangle_k(\omega).
\] (5.6)

where \( \perp \) represents Moore-Penrose pseudo-inverse [113].

During the hemoacoustic simulations of 75\% case, 16 evenly spaced monitor points (mimicking an array of stethoscopes [15]) are placed on the epidermal surface of the modeled aorta with the spacing in \( y \) direction being \( 1.5D \) and the spacing in \( z \) direction being \( 1.3D \). The potential source location resides in the frontal plane (marked as green in Fig. 5.13) and the plane is evenly discretized by 25 mesh points with spacing \( 0.625D \) in each direction. It is noted that Eq. (5.6) is frequency dependent, and thus, for each angular frequency \( \omega \), a system of equations that can be solved to obtain the amplitude of the pressure. We have already observed from the study in Section 5.3.2 that the signal intensity of frequency band I is spatially well correlated with the murmur source. Therefore, the wall-normal acceleration at the lower frequency band is used to calculate the band-limited energy distribution of the pressure by

\[
P^2 = \sum_{\omega \in I} \{[\mathcal{G}_{kj}(\omega)]^\perp \langle a_n \rangle_k(\omega) \}^2.
\] (5.7)

The result is plotted in Fig. 5.14. It can be seen that this prediction does a reasonable
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Figure 5.14: Source localization for 75% case on the frontal plane. The black diamond symbol represents the same spatial location in Fig. 5.6(a).

job of placing the expected source location downstream of the modeled aorta and in the vicinity of the actual source location.

5.4 Conclusion

In this study, a high-order immersed boundary method based acoustic solver is used to simulate the propagation of murmurs generated by a modeled aorta with an aortic stenosis. The wall pressure on the modeled aorta is applied as the boundary condition in the hemoacoustic simulations. Three cases with stenoses ranging from mild to severe (50%, 62.5% and 75%) are investigated in order to gain better understanding of the murmur propagation. The source locations predicted by the acoustic signal intensity on the skin surface and the free-space Green’s function are in good agreement with the actual locations determined in the hemodynamic studies. The fact that this source location is relatively insensitive to the severity of the stenosis
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makes it a potential diagnostic criterion to differentiate aortic stenosis from other heart conditions that generate systolic murmurs. The spectral analysis of the wall normal acceleration at the skin surface shows a break frequency for all three cases study here. This break frequency is a consequence of the superimposition of murmurs generated from wall pressure fluctuation at different locations on the modeled aorta. It is important to note that this break frequency generally does not equal to the underlying jet shear layer shedding frequency, i.e. the hemodynamic break frequency.

Last but not least, current study has several limitations. First, by assuming a steady inflow into the aorta we might be missing effects associated with the pulsatile nature of the aortic jet. Second, the modeled thorax is highly simplified in terms of geometry and material properties, although geometric and anatomical complexities of the thorax can be included in the current method [100].
Chapter 6

Effect of Pulsatility and Aortic Valve Motion on the Hemodynamics of Aortic Stenosis

6.1 Introduction

Our previous study of aortic stenosis adopts the steady flow assumption to focus on investigating the effect of curvature on the post-stenotic jet. However, the blood flow coming into the aorta has a complex pulsatile profile due to the pumping action of the left ventricle. A cardiac cycle can be divided into two major phases: the systole, during which the blood flow is pumped out of the left ventricle into the aorta; and the diastole, when the flow rate across the aortic valve reduces to around zero, and the blood is filled into the left ventricle. Thus, the murmurs associated with aortic stenosis are only expected to be generated during the systole when there is flow through the aortic valve. Also, the acceleration and deceleration of the flow is also
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projected to generate different hemodynamics than the steady case.

Furthermore, in the previous study of hemodynamics across aortic stenosis, the stenosis is modeled as a smooth constriction. Similar configurations have been used in many experiments \([74, 75, 77]\) and numerical simulations \([88, 89, 91]\). This assumption ignores the fact that an aortic stenosis is created by the incomplete opening of the aortic valve. The pulsatile blood flow drives the closing and opening of the valve leaflets. When the valve is open, it resembles an orifice but with a dynamically changing area. As a result, complex flow patterns are expected to occur in the post-valvular region.

In this chapter, we investigate the effect of the pulsatility and the valve on the hemodynamics of the aortic stenosis.

6.2 Computational Modeling of Pulsatile Flow through a Modeled Aortic Stenosis

6.2.1 Model Configuration

6.2.1.1 Geometry and Numerical Methods

The modeled aorta used in Chapter 4 is also employed in the current study (see Fig. 4.2(a)). The severity of the stenosis is quantified by the area stenosis ratio (AS), which represents the percentage of area that is occluded. Three different severities
of stenosis, $AS=50\%$, 62.5\% and 75\%, are investigated, and they represent mild to borderline severe stenoses. Following the assumptions made in Chapter 4, the tortuosity of the aorta and the compliance of the vessel wall are neglected in the current study to focus on investigating the effect of curvature and pulsatility.

The blood in the modeled aorta is treated as a Newtonian fluid and is governed by the incompressible Navier-Stokes equations $^5$. As shown in Fig. 6.1, a pulsatile flow is prescribed at the inlet, and its profile is provided in Appendix D. The duration of the cardiac cycle ($T$) is assumed to be 0.77s, which equals 78bpm, and the systole phase ($T_s$) lasts 0.33s. In order to make a fair comparison with the steady flow study in Chapter 4, the inlet flow velocity averaged over the systolic phase $\overline{V_{in}} = \frac{1}{T_s} \int_{0}^{T_s} V_{in} dt$ is equal to the steady inlet velocity, which is 0.25$m/s$. The stroke volume can be calculated as $SV = \frac{\pi D^2}{4} \int_{0}^{T_s} V_{in} dt = 40.5ml$, with $D$ set to 2.5cm. The mean Reynolds
number during the systole is \( Re = \frac{V_{in}D}{\nu} = 2000 \), and the peak Reynolds number during the cardiac cycle is \( Re_{max} = \frac{V_{max}D}{\nu} = 3637 \), where \( \nu \) is the dynamic viscosity of the blood and \( V_{max} \) is the peak inlet velocity. The Dean number based on the mean Reynolds number is 5657 (see Chapter 4) and the Womersley number is \( Wo = \frac{D}{2} \sqrt{\frac{2\pi}{\nu T}} = 20.2 \). A Dirichlet pressure boundary condition that mimics the physiological condition is prescribed at the outlet of the modeled aorta, and the pressure profile (see Fig. 6.2) is provided through the after-load model described in Appendix D.

Most of the parameters, such as the diameter, Dean number and Womersley number, used in the study are within range typical for aortic flows [103, 114, 115]. However, the stroke volume is lower than the common value measured in patients with aortic stenosis [116].

This relatively complex geometry is treated with the sharp-interface immersed boundary method described in Chapter 2. The whole geometry shown in Fig. 4.2(a)
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Figure 6.3: Monitor points inside the computational domain. The red squares locate on the anterior/posterior surface of the aorta, and the black circles locate at the 1/4 and 3/4 of the diameter connecting the anterior and posterior monitor points.

is immersed in a Cartesian mesh with a $128 \times 384 \times 369$ (18 million grid points) mesh. This grid is subjected to a grid convergence study (see Appendix B.2), and is demonstrated to be able to well resolve all the cases. A semi-adaptive time step size is used to reduce the computational cost. A small time-step size $1.923 \times 10^{-3} D/V_{in}$ is employed to resolve the systole and part of the diastole, i.e. $t \in [0T, 0.55T]$, while a coarse time step size $5.76924 \times 10^{-3} D/V_{in}$ is used to solve the rest of the diastole. Simulations are conducted on the TACC-Stampede supercomputers with 812 cores, and it takes around 13 hours of wall-time to compute one cardiac cycle for the 75% stenosis. Seven cardiac cycles are computed for each severity, and results of the last 6 cycles are collected and analyzed.
6.2.2 Data Presentation

As shown in Fig. 6.3, monitor points are placed at different locations of the modeled aorta to record temporal signals for later analysis. There are two major groups of monitor points. The first group locates on the anterior/posterior surface of the aorta, and is represented by the red squares. The second group, which is marked by the black circles, lies on the 1/4 and 3/4 of the diameters connecting the anterior and posterior monitor points, and is referred to as ‘quarter’ points. Both groups of monitor points are evenly spaced across the ascending aorta, aortic arch and descending aorta, and are referenced through an angle \( \theta \), which is measured from the end of the stenosis towards the ascending aorta.

For a general flow quantity \( f \), the phase average (or mean) is

\[
\bar{f}(x, y, z, t) = \frac{1}{M} \sum_{n=0}^{M-1} f(x, y, z, t + nT),
\]

(6.1)

where \( M \) is the number of cardiac cycles, \( T \) is the period and \( t \in [0, T] \). The turbulent fluctuations can be defined accordingly as

\[
f'' = f - \bar{f}.
\]

(6.2)

The intensity of the fluctuation is quantified by the root-mean-square (RMS), which
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is calculated by

\[
frms(x, y, z, t) = \sqrt{\frac{1}{M} \sum_{n=0}^{M} [f''(x, y, z, t + nT)]^2}.
\] (6.3)

The other important average is the temporal mean over the systolic phase

\[
\bar{f} = \frac{1}{T_s} \int_0^{T_s} f dt.
\] (6.4)

The aforementioned \( V_{in} \) is defined in this way. Another velocity scale, the systolic mean jet velocity \( V_j \), can also be defined, and it is related to systolic mean inlet velocity by \( V_j = V_{in}/(1 - AS) \).

To ensure the results are presented consistently between the steady and pulsatile flow models, the following characteristic scales are used to normalize some of the results in this section unless otherwise stated: velocity scale \( V_j \), length scale \( D_j \), time scale \( D_j/V_j \) and force scale \( 1/2\rho V_j^2 \). Therefore, the Strouhal number is defined as \( St = fD_j/V_j \). These characteristic scales are essentially the same as the steady flow case in Chapter 4, which will facilitate comparison. Also, due to the bilateral symmetry of the modeled aorta, ensemble average of the results from anterior and posterior portions of the fluid domain are presented when available.
6.2.3 Results and Discussion

6.2.3.1 Post-Stenotic Jet Dynamics

The stenosis acts as a nozzle in an otherwise smooth curved pipe. This nozzle creates a strong post-stenotic jet during systole, as shown in Fig. 6.4. In this figure, the instantaneous azimuthal vorticity fields at the four phases indicated in Fig. 6.1 are plotted for the three stenoses studied here. Phase I is in the early systole while the flow is still accelerating; phase II represents peak systole when the flow rate reaches maximum; phase III is in the deceleration phase of the systole; and phase IV indicates the end of systole and beginning of diastole. In phase I, the post-stenotic jet starts...
to develop, and a vortex ring is seen formed immediately after the stenosis for $AS=62.5\%$ and $75\%$. At this early stage, this vortex ring is relatively small, and the effect of the lumen wall is not prominent. This is clearly shown by the cutout in the $75\%$ case, in which the vortex ring is round and horizontal. As the flow rate continues to increase, the outer part of the vortex ring impinges on the outer wall, and the inner part moves downstream freely, as illustrated in phase II. While the outer part of the vortex ring in $50\%$ and $62.5\%$ cases still remains laminar, chaotic flow behaviors can be observed near the impingement location for the $75\%$ case. During the acceleration phase, the inner part of the vortex ring is seen to grow steadily for all three cases. The ring structures for the $50\%$ and $62.5\%$ are still connected to the jet shear layer, whereas the ring starts to separate from the rest of jet for $75\%$ case and short span of vortex shedding behavior can be observed at Arrow C. As soon as the flow enters the deceleration phase, both the inner part and outer part of the jet start to break up into small vortical structures. As the jet moves downstream, a small recirculation zone (marked by Arrow A) and a large recirculation zone (marked by Arrow B) are formed in the ascending aorta. The range the small vortical structures can reach in the modeled aorta is driven by the jet intensity, and can be determined through the plots at phase IV. As can be seen here, the small vortical structures are mostly confined to the ascending aorta for the $50\%$ case, whereas they can reach the aortic arch and the descending aorta for the other two cases.
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Figure 6.5: Phase averaged (mean) streamwise vorticity of (a) $AS=50\%$ at $\theta = 15^\circ$, (b) $AS=62.5\%$ at $\theta = 20^\circ$ and (b) $AS=75\%$ at $\theta = 35^\circ$ overlapped with the vectors showing the in-pane fluid motion. All the flow fields are shown at phase II.

6.2.3.2 Secondary Flow and Turbulence

Smooth curved pipes are known to induce strong secondary flows \cite{105, 117} and for steady inflow condition, the secondary flow is dominated by the so-called Dean vortices. However, more complicated structures can be observed when the inflow is pulsatile \cite{107, 117}. The secondary flow structure is expected to become more complex when the stenosis is introduced. In this section, we will focus on identifying the primary secondary flow structures.

Fig. 6.5 shows the terminology for location. We first inspect the flow field of $AS=62.5\%$ in detail, and the other two cases are presented in comparison to the $AS=62.5\%$ case. Fig. 6.6(a) shows the spatial evolution of the mean streamwise vortical structures of $AS=62.5\%$ at different phases. At peak systole (phase II), the large vortical structures are still intact, and the dominant secondary flow is associated with the Dean vortices that form in the ascending aorta around $20^\circ$. This is confirmed by Fig. 6.5 where the main vortices are superposed on the strong in-plane swirling...
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Figure 6.6: Cross-sectional distribution of (a) mean streamwise vorticity, (b) TKE and (c) mean streamwise velocity for AS=62.5% at different angle $\theta$ of phases II, III and IV.
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fluid motion. During phase III, the large coherent vortical structures can no longer be observed, and the small vortices are distributed all over most of the ascending aorta, except in the core region of the jet for $\theta < 20^\circ$. These small vortices decay rapidly at the end of the systole.

Fig. 6.6(b) plots the cross-sectional distribution of the turbulent kinetic energy (TKE) for $AS=62.5\%$. The turbulence is extremely weak during phase II, confirming that the flow is primarily laminar at peak systole. On the other hand, strong turbulence can be observed during phase III. Overall, the region with high TKE is well correlated with the distribution of small vortical structures in Fig. 6.6(a). The mean streamwise velocity are shown in Fig. 6.6(c). During phase III, a strong jet (forward flow) can be seen at the center of the cross section at $\theta = 10^\circ$, and a small backward flow region locates near the inner wall of the aorta. As the jet propagates downstream, the jet moves closer to the outer wall and forms a crescent shape around $40^\circ$. A backward flow region can be consistently observed near the inner wall, forming the large recirculation zone highlighted in Fig. 6.4. The region between the jet and the large recirculation zone includes the inner part of the jet shear layer and potentially secondary flows, and this region also has strong TKE.

The mean streamwise vorticity and TKE for $AS=50\%$ are shown in Fig. 6.7 and at peak systole, the flow features are quite similar to those for the $AS=62.5\%$ case. The flow is mainly laminar and low TKE is seen across the entire aorta at this phase. Nevertheless, the streamwise vortices are much smaller due to the large core
Figure 6.7: Cross-sectional distribution of (a) mean streamwise vorticity and (b) TKE for $AS=50\%$ at different angle $\theta$ of phases II and III.

region of the jet. During phase III, small vortical structures spread widely in the cross section until $\theta = 40\degree$, and the locations with strong TKE also match the small vortical structures quite well.

The flow field for the 75\% is quite different from the other two cases (see Fig. 6.8). First, the vortex ring formed by the jet is observed to break up as early as peak systole. The mean streamwise vorticity clearly shows that the stochastic flow behaviors exist near the outer wall at $\theta < 20\degree$ at phase II. This early breakup is caused by the higher jet velocity in the 75\% case, which makes the vortex ring travel downstream rapidly. This enables the vortex ring to impinge on the aortic wall at an
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Figure 6.8: Cross-sectional distribution of (a) mean streamwise vorticity and (b) TKE for $AS=75\%$ at different angle $\theta$ of phases II and III.
earlier phase than the other two cases and to have longer time for the disintegration to develop. Moreover, the Dean vortices have irregular shapes and small vortical structures can be seen at this phase. Inspection of the TKE distribution at the peak systole reveals that there is strong turbulence fluctuations around the jet shortly after the stenosis, and the high TKE region lies near the outer wall and inside the small recirculation zone. During phase III, no large vortical structures can be observed, and high TKE at $\theta \leq 30^\circ$ is clustered near the inner boundary of the jet which faces the inner wall of the modeled aorta.

To summarize, the main secondary flows for all three cases are still the Dean vortices, which are most visible during acceleration phase. However, unlike the steady case, the strong secondary flows during this phase are usually not associated with strong turbulent fluctuations. On the other hand, high TKE is commonly observed during the deceleration phase of the systole. The major source of the turbulent fluctuations is the disintegration of the inner part of the jet shear layer, which is caused by the deceleration of the flow instead of the instability mechanism governs the steady jet case. Of the four phases studied here, the strongest TKE occurs during phase III. The extent to which this high TKE is observed increases as the severity of stenosis increases: from $\theta < 40^\circ$ in $AS=50\%$ to $\theta < 60^\circ$ in $AS=62.5\%$ and to well beyond $80^\circ$ in $AS=75\%$. 

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Figure 6.9: (a) Pressure signal measured on the anterior surface of AS=75% at \( \theta = 40^\circ \). (b) Time derivative of the pressure signal at the same location. A bandpass filter with a passband between \( 10Hz \) and \( 1000Hz \) is applied to exclude noise and the effect of the pulsatile frequency.
6.2.3.3 Surface Pressure Fluctuations

It has been shown in previous studies that abnormal wall pressure fluctuations are responsible for the generation of heart murmurs [23]. More precisely, the time derivative of the wall pressure is designated as the source term in the governing equation of murmur generation. Investigation of the murmur source carries two significant purposes: first, the identification of the source location provides valuable information for diagnosis [97], as the source location of murmurs usually lies in the vicinity of the site of the disease. Second, a deeper understanding of the signal characteristics of the source can help to better predict the murmurs that are measured at the skin surface. The source location is studied in this section, and the signal characteristics are explored in the next section. Through our previous investigation of steady flow in Chapter 4, the source of the heart murmurs is found not co-located with the stenosis. Rather, it is about $2.1D$ downstream of the stenosis, and this location is almost the same for the three severities (50%, 62.5% and 75%). However, the inclusion of pulsatility is expected to complicate and perhaps, modify the situation.

One basic assumption adopted here is that the pressure signals on the anterior surface of the modeled aorta is responsible for the murmur generation, since these locations directly faces the anterior surface of the thorax, where the murmurs signals are usually measured. Therefore, the analysis mainly focuses on the pressure signal measured from the monitor points on the anterior/posterior surface (marked by red squares in Fig. 6.3). An example of the pressure signal in plotted in Fig. 6.9(a), which
is measured on the anterior surface of \( AS=75\% \) at \( \theta = 40^\circ \). As can be seen here, the pressure primarily follows the trend of the after load shown in Fig. 6.2 except at the deceleration phase, when small perturbations are clearly shown. As stated previously, the time-derivative of the pressure is the actual source term in the governing equation. Hence, the time-derivative of the pressure at the same location is plotted in Fig. 6.9(b). A bandpass filter with an allowed band between 10\( Hz \) and 1000\( Hz \) is applied. Inspection of the spectrum shows that the signal above 1000\( Hz \) can be considered noise, and the influence of the fundamental frequency (heart rate) is assumed to be negligible above 10\( Hz \). Fig. 6.9(b) clearly demonstrates strong fluctuations during the deceleration phase of the systole (marked by arrow A), and they are considered source of the murmurs. The timing of these fluctuations is consistent with the same perturbations in Fig. 6.9(a). A distinct pulse is also observed at the beginning of each cycle (marked by arrow B). This pulse is caused by the sudden change of flow rate and is not considered as murmur source, since it is expected to exist even if there is no stenosis.

The intensity of the source can be represented by \((dp/dt)^2\). After squaring the time derivative of pressure, an ensemble average can be computed to obtain the average signal strength over one cardiac cycle. Since a group of monitor points are placed on the anterior points, the average signal strength can be calculated along the anterior surface at each monitor location. The results for the three cases studied here are summarized in Fig. 6.10, and these heat maps show the spatial-temporal
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Figure 6.10: Temporal-spatial distribution of \((dp/dt)^2\) on the anterior surface of the modeled aorta with (a) 50\%, (b) 62.5\% and (c) 75\% stenoses. The unit of the contours plot is \([Pa^2/s^2]\) and the white dash line indicates the flow rate.
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distribution of \((dp/dt)^2\). The high intensity spots in these plots indicate potential source locations. It is readily noticed that, unlike the steady flow case, the murmurs here are not generated from one single static location. Rather, the source location travels downstream with time for all three cases. The initial source locations for the 62.5% and 75% stenoses start around 20°, but the source location for the 50% stenosis starts much closer to the stenosis. This is due to the fact that larger jet diameter for the 50% case reduces the distance between the jet and the anterior wall, and the perturbation in the flow can be sensed by the wall shortly after the stenosis. Moreover, the source region for the 75% case spreads across the ascending aorta, reaching as far as 60°, while the intensity of the source is well maintained only up to approximately 40° and 25° for 62.5% and 50% stenoses respectively. This positive correlation between the severity of the stenosis and the extent of the source is consistent with Fig. 6.4, where the small vortical structures with higher intensity reach further downstream. Besides the location of the source, the timing is also an important aspect. For 50% case, the high intensity is first generated around \(0.2T\), and continues to approximately \(0.27T\). For the 62.5% case, the source is first seen after the peak systole around \(0.18T\), and continues to about \(0.25T\). For 75% case, the source actually starts slightly before the peak systole, which is \(0.15T\), and extends to approximately \(0.24T\). These results clearly demonstrate that the more severe the stenosis, the earlier the murmurs are generated. The murmurs for less severe stenoses (\(e.g.\) 50% and 62.5%) are expected to occur during the deceleration phase of the
systole, while for the severe stenosis the murmurs could potentially be heard before peak systole. Above all, the overall timing of the murmurs is the deceleration phase of the systole.

The above observations can be connected to the post-stenotic flow features, especially the turbulence. At phase III \((t = 0.25T)\), the distribution of strong TKE is well correlated with the extent of the source locations for all three cases. Moreover, angles at which strong TKE are observed near the anterior/posterior surface are also those with strong sources. For example, as shown in Fig. 6.6(b), the TKE distribution is most intense near the anterior/posterior wall around 30° at phase III, and this is also the location where the strong time variance of pressure is seen in Fig. 6.10(b). Also, there is strong TKE at 20° of phase II for 75% stenosis, and this is temporally and spatially well correlated to the timing and location of the source in Fig. 6.10(c). Therefore, it is safe to conclude that the high pressure fluctuation at the wall is caused by the near wall chaotic flow behaviors. The same conclusion is reached in the steady flow study, but the precise origin(s) of these chaotic flows are different. In the steady flow, it is concluded that both the instability of the jet and the secondary flow contributes to the turbulence. Nevertheless, in the current pulsatile flow, the secondary flows are mainly formed in the acceleration phase of the systole, and are laminar in nature. Also, the duration of the jet is too short for any inherent instability to develop. Instead, the turbulent fluctuations are mainly caused by the instability caused by the deceleration.
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Last but not least, it is worth emphasizing that dimensional values are used in Fig. 6.10. The contour levels clearly indicate that the intensity of the source increases significantly as the severity increases.

6.2.3.4 Spectral Analysis

In addition to the source location, the spectra of the source signal also potentially contain valuable diagnostic information. In the steady flow study, the spectra of pressure contain a characteristic frequency, where the slope of the spectra changes significantly. This frequency is usually called break frequency and is caused by the periodic vortex shedding from the jet shear layer. It can be detected in the pressure signal measured inside the jet and on the anterior surface. More importantly, when non-dimensionalized by the jet velocity $V_j$ and jet diameter $D_j$, the break frequencies from the three severities (50%, 62.5% and 75%) collapse to a single Strouhal number of 0.93. In this section, we examine how the pulsatility affects the spectral characteristics of the flow signal.

Fig. 6.11 plots the spectra of TKE along the quarter points. As can be seen from Figs. 6.6-6.8, these monitor points lie right inside the jet shear layer, and are ideal to examine the existence of the break frequency or the turbulence. Fig. 6.11(a) shows the TKE spectra at selected angles of 75% stenosis. The vertical line denotes $St = 0.93$. This figure clearly shows the lack of any identifiable peak across the entire ascending aorta and the spectra show very similar trend in this region. The
spectra of TKE at selected angles from all three cases are shown in Fig. 6.11(b). These locations are inside the source region of these three stenoses. It is evident that turbulence exists in the ascending aorta for all three cases. Moreover, the spectra of \( dp/dt \) along the quarter points are compiled in Fig. 6.12. The vertical lines in these three plots are corresponding to \( St = 0.93 \) in each case. It is readily noticed that this specific frequency cannot be identified as break frequency in either 50% stenosis or 62.5% stenosis. However, shallow peaks are observed in the close vicinity of this frequency from 0° to 20° in the 75% case. It is potentially due to the fact that the stronger intensity/higher speed enables the jet to develop natural instability during the systolic phase, while for the less severe cases, the flow remains stable and no shedding happens. This can be confirmed by Fig. 6.4, which shows signs of shedding in the 75% case at the peak systole. It should be noted that this shedding frequency
Figure 6.12: Spectra of time derivative of pressure along the quarter points at selected angles for (a) 50%, (b) 62.5% and (c) 75% stenoses. The vertical lines represent $St = 0.93$ in each case.
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Figure 6.13: (a) Spectra of time derivative of pressure along the anterior surface at selected angles for 75% stenosis. (b) The composition of the pressure signal at 40° for 75% stenosis.

is only seen up to 20°, which is on the lower boundary of the source range identified in Fig. 6.10(c). Also, these shallow peaks do not exactly locate at $St = 0.93$. The small discrepancy is likely caused by the use of velocity scale $\bar{V}_j = 0.25m/s$, which might be slightly different from the velocity at which the shedding happens.

As emphasized before, it is the pressure on the lumen wall of the aorta that generates the murmurs. Here, we focus on the time derivative of the pressure on the anterior surface, since this surface faces the anterior of the thorax. The spectra of $dp/dt$ along the ascending aorta in the 75% case are shown in Fig. 6.13(a). The shallow peak near $St = 0.93$ can no longer been seen here. Apart from $\theta = 0°$, the spectra at other angles share similar trends. The energy first decreases sharply until around 10Hz, and after a brief increase, the energy drops again after 100Hz. Furthermore, the signal intensity also changes with the angle. The signal is quite
weak at $0^\circ$, and the intensity increases downstream. After reaching the peak around 40°, the signal intensity decreases as the angle increases. The details of signal at 40° is examined more closely in Fig. 6.13(b), in which different components (phase average and turbulence) of the total signal are plotted along with the after load. As can be seen here, the signal energy at low frequency ($f < 10 Hz$) mainly comes from the after load, while at high frequency range ($f > 100 Hz$), the turbulence plays the dominant role. This observation justifies the previous assumption that the signal is heavily influenced by the fundamental frequency when $f < 10 Hz$. In the middle frequency band ($10 Hz < f < 100 Hz$), the signal strength increases, and the total signal mostly overlaps with the phase average. This region represents the periodic component of the post-stenotic jet. This part of the fluctuation should still be considered as part of the murmur source since it would not exist in the absence of the stenosis. Fig. 6.14:

The composition of the pressure signal at (a) $20^\circ$ for 50% stenosis and (b) $30^\circ$ for 62.5% stenosis.
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6.14 plots the source spectra at 20° and 30° of 50% and 62.5% stenoses respectively. These locations are also the angles where the maximum signal intensity is observed. Similar to the 75% case, the low frequency band is dominated by the after load, while the turbulence is responsible for the mid-to-high frequency range. The influence of the periodic component of the post-stenotic jet on the source spectra is much weaker for 50% and 62.5% stenoses when $f > 10Hz$.

6.2.4 Conclusion

In this section, we have reported on our investigation of the effect of pulsatility on the hemodynamics across the aortic stenoses with 50%, 62.5% and 75% area blockage. Pulsatility is found to introduce extra complexity to the post-stenotic flow as compared to the steady flow case but the dominant secondary flow structures are still the Dean vortices, which are best observed before the flow starts to decelerate. The post-stenotic jet break-up mainly during the deceleration phase, and this disintegration of the jet shear layer is main source of the turbulent fluctuations inside the ascending aorta. Due to the pulsatile nature of the inflow, the source location, which is identified as the location on the anterior/posterior surface with strongest pressure fluctuations, travels downstream as the jet advances towards the descending aorta. For the three cases studied here, the murmurs are expected to be generated during systole around $0.2T$ in the ascending aorta. The precise source location shows well-defined dependence on the severity of the stenosis. For the 75% case, the source
location travels from $\theta = 20^\circ$ to around $\theta = 60^\circ$; for 62.5% case, the source location is identified between $\theta = 20^\circ$ and $40^\circ$; and the source location for the 50% case is bounded between $\theta = 15^\circ$ and $30^\circ$.

Further inspection of the pressure signal around source location reveals that no clear break frequency can be identified due to the lack of the periodic vortex shedding in the jet shear layer. The energy in the high frequency band is mainly contributed by the turbulent fluctuations in all three cases.

### 6.3 Computational Modeling of Aortic Stenosis with a Simple Aortic Valve Model

#### 6.3.1 Model Configuration

##### 6.3.1.1 Geometry and Numerical Methods

The geometric model employed in this study consists of the aorta and the aortic valve, as shown in Fig. 6.15. The aorta is modeled as a straight tube that can be divided into three sections: an inflow duct, the aortic root and the ascending aorta. The diameter of the inflow duct ($D_o$) is set to $2.5cm$, which is within the physiological range for both healthy subjects and patients with stenosis \[^{114}\]. The dimensions of the aortic root are shown in Fig. 6.15 and they are based on the generic model proposed by Reul \textit{et al.} \[^{118}\].
Figure 6.15: Schematic of the modeled aorta with a simple valve model. The gray surface represents the lumen wall, and the red surface is the aortic valve model. The valve in the lower right figure is fully open, and it is fully closed in the other two figures.
Figure 6.16: Inlet velocity profile.

Figure 6.17: After load profile employed in this study.
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The blood inside the aorta is considered Newtonian and is governed by the Navier-Stokes equation [58]. The flow inside the modeled aorta is driven by a pulsatile inflow. The inlet velocity profile is plotted in Fig. 6.16 and its expression can be found in Appendix D. This pulsatile inflow generates a stroke volume of 70ml. The systolic phase lasts 0.33s, and the duration of one cardiac cycle (T) is 0.9375s which is equivalent of 64bpm. The maximum Reynolds number ($Re_{max} = \frac{V_{in, max} D_o}{\nu}$) is around 6000, where $\nu$ is the kinematic viscosity of the blood. The Womersley number ($Wo = \frac{D}{2 \sqrt{\frac{2\pi}{T}}}$) is around 17.73. This peak Reynolds number and the Womersley number are within the typical range for aortic flows [115]. To determine the absolute pressure inside the aorta, the after-load model proposed by Watanabe et al. [119] is adopted in the current study. The formula is given in Appendix D and the pressure profile in Fig. 6.17 is prescribed at the outlet of the modeled aorta.

The complex geometry is treated by the sharp-interface immersed boundary (IB) method described in Chapter 2. A Cartesian grid of $154 \times 160 \times 256$ (~6.3million) resolution is used to discretize the computational domain. The grid is distributed uniformly in the cross section, and is clustered around the valve region in the streamwise direction. This results in around 128 grid points across the diameter of the aorta. To ensure that sufficient resolution is provided to resolve the flow, a grid convergence study is conducted and is summarized in Appendix B.3. It demonstrates that this grid is capable of adequately resolving all the cases studied here. In order to reduce computational cost, a semi-adaptive time step size is used in all of the
simulations. For each cardiac cycle, a small time-step size \(1.25 \times 10^{-4} s\) is used to resolve \(t \in [0 T, 0.46 T]\). This time period includes the systole and part of diastole, which is the most dynamic phase of the cardiac cycle. The rest of the diastolic phase is resolved by a coarser time-step \((3.75 \times 10^{-4} s)\). Simulations are carried out on the MARCC supercomputers with 288 cores, and it takes around 26 hours of wall-time to compute one cardiac cycle for the most time-consuming case. Seven cardiac cycles are computed for each case, and data from the last 6 cycles are collected and analyzed.

### 6.3.1.2 Reduced Degree-of-Freedom Valve Model

Many efforts have been expended into developing complicated and realistic valve models \([115]\). Here we propose a simplified valve model with reduced degrees-of-freedom (RDOF) and couple with fluid-structure interaction (FSI) to examine first-
order’ valve effects. The aortic valve employed here is a simplified model of the Trifecta aortic valve (St. Jude Medical Inc., Minneapolis), and similar valve model is used by de Tullio & Pascazio [120]. As shown in Fig. 6.18(a), the aortic valve contains three individual leaflets. Each leaflet is a shell structure represented by triangular elements (see Fig. 6.18(b)). The equation of motion of each triangular element can be described as:

\[ \alpha \frac{\partial \hat{v}_v}{\partial t} = \Delta P \hat{n} - K (\hat{d}_v - \hat{d}_{v,0}). \]  

(6.5)

Here, \( \alpha \) is a constant related to mass, which includes both the areal mass and the added mass; \( K \) is a stiffness constant; \( \hat{v}_v \) and \( \hat{d}_v \) are the velocity and displacement of the element; \( \Delta P \) is the pressure difference across the element; and \( \hat{n} \) is the norm of the element. The first term on the right hand side of the equation represents the pressure force from the flow, while the second term models the elastic recoil forces on the leaflet. It should be noted that \( \alpha \) and \( K \) are model parameters and need to be determined.

In the proposed RDOF FSI model, instead of solving for each triangle element in a leaflet, each leaflet is treated collectively so that it opens/closes following a prescribed mode. Here, this mode is defined by the vector \( \tilde{b} \) that connects each element at the fully open position to its fully closed position (see Fig. 6.18(b)). Hence, the motion
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of an element can be described by the following prescription

\[ b(x) = x_{\text{open}} - x_{\text{close}}, \quad (6.6) \]

\[ \vec{v}_v = a(t) \cdot \vec{b}(x), \quad (6.7) \]

\[ \vec{a}_v = c(t) \cdot \vec{b}(x). \quad (6.8) \]

The coefficients \( a(t) \) and \( c(t) \) are determined by the total force acting on each leaflet. Substituting the above expressions into Eq. 6.5 and integrate over the entire leaflet, we obtain the following:

\[ \frac{da}{dt} = \frac{\int \Delta P dS}{\int \alpha \vec{b} \cdot \vec{n} dS} - \frac{K}{\alpha} (c - c_0), \quad (6.9) \]

\[ \frac{dc}{dt} = a. \quad (6.10) \]

where \( c_0 \) is the initial condition for \( c \). By imposing the mode of the valve, the partial differential equation describing the motion of each element is reduced to an ordinary differential equation that describes the collective motion of the elements in each leaflet, and can be solved easily. It is worth emphasizing that the solution procedure is carried out separately on each individual leaflets, and contact between the leaflets is prohibited by appropriately prescribing the fully closed position. Eqs. 6.9 and 6.10 are explicitly coupled with the Navier-Stokes equation to impose the interaction of the valve with the flow.

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Figure 6.19: Inlet velocity profile used to determine stiffness coefficient $K$.

Figure 6.20: Normalized projected valve area (PVA) during one cardiac cycle.
Figure 6.21: (a) Instantaneous snapshots of the valve leaflets at the time instances highlighted in Fig. 6.20. The valve fully opens at phase II. (b) Leaflet motion during one cardiac cycle. The selected cross section is indicated by the dash line in (a).
As stated before, the parameters $\alpha$ and $K$ are model parameters that need to be determined beforehand. Based on our tests, changing $\alpha$ over a wide range has little effect on the valve motion. Hence, we set $\alpha = 40\,kg/m^2$, and vary $K$ to get a realistic opening/closing behavior. To achieve this, we simulate the problem that is also studied by de Tullio & Pascazio [120], in which the valve is treated as a non-linear anisotropic material described by a spring-network model. The inlet velocity is plotted in Fig. 6.19. This velocity profile is the same as the one used by de Tullio & Pascazio [120], except that the negative flow rate near the end of the systole is excluded here to avoid artificial regurgitation. Fig. 6.20 shows the normalized project valve area (PVA) over one cardiac cycle for the current model with $K = 10600\,Pa/m$. It also includes the result from de Tullio & Pascazio [120]. As can be seen here the RDOF valve model shows a rapid opening behavior similar to the model in Ref. [120], while the closing phase is not as smooth around 0.2s potentially due to the artificial stiffness introduced by the reduced-order motion. Despite these discrepancies, this figure clearly demonstrates that this simplified valve model can capture the first-order behavior of the more complicated valve model.

Snapshots of the valve at different time instances are shown in Fig. 6.21(a). The phases at which these snapshots are taken are highlighted in Fig. 6.20. The valve is rapidly opening at phase I and compared with the snapshot in de Tullio & Pascazio [120] at a similar phase, the current valve profile is smoother and lacks small variations near the tip of the leaflets. Phase II and III correspond to the fully open
position and early closing phases respectively. The envelop of the valve motion is shown in Fig. 6.21(b), and we can see that the leaflet follows the same mode shape during the closing and opening phase due to the adoption of same trajectory \( \vec{b} \) in both phases.

Based on the above comparison, it is concluded that this simplified valve model is expected to generate a flow field that is reasonably close to the valve model which uses more complicate constitutive relations. Moreover, the incomplete opening of the valve, \textit{i.e.} aortic stenosis, can be easily introduced by increasing the parameter \( K \).

### 6.3.2 Results and Discussion

In this section, results from three cases (a healthy case and two stenosed cases) are presented. To facilitate the clarity of the discussion, the data reduction described in Section 6.2.2 is adopted here.

#### 6.3.2.1 Severity of Aortic Stenosis

Aortic stenoses are the results of the incomplete opening of the aortic valve. One of the parameters that measures the severity of the stenosis is the area stenosis ratio \( (AS) \), which is defined as the percentage of area that is blocked by the valve during systole. It is related to the PVA through \( AS = 1 – \frac{\text{PVA}_{\text{max}}}{\text{PVA}_0} \), where \( \text{PVA}_{\text{max}} \) is the maximum opening area of the diseased valve and \( \text{PVA}_0 \) is the maximum opening area of the healthy valve. In the current RDOF valve model, the incomplete opening
Figure 6.22: (a) Normalized projected valve area (PVA) during one cardiac cycle for the healthy case and two stenosed cases (AS=31% and 47%). The parameters $K$ for the three cases are 10600 Pa/m, 15900 Pa/m and 42400 Pa/m, respectively. (b) Valve profiles at the fully open position for the three cases. The dash line represents the $z - x$ cross section and the dash-dot line indicates the $z - y$ cross section.
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of the valve can be achieved by increasing the stiffness parameter $K$. Depending on whether the same $K$ is used for all three leaflets, rotationally symmetric or asymmetric aortic stenosis can be created easily. Only rotationally symmetric stenosis is considered in the current study, \textit{i.e.} all three leaflets have the same stiffness constant. Three values of $K$ ($10600\text{Pa/m}$, $15900\text{Pa/m}$ and $42400\text{Pa/m}$) are used here, and they lead to the healthy case, and two stenosed cases with $AS$ of $31\%$ and $47\%$, respectively. The valve motion is summarized in Fig. 6.22(a). It is worth emphasizing that PVA is normalized by the maximum PVA of the healthy case, \textit{i.e.} $PVA_0$. The inlet velocity profile is also included in the figure for reference. From this figure we can see that the valve in all three cases opens rapidly, and reaches maximum opening before peak systole. Though the leaflets in the healthy case remain fully open for approximately $0.06T$, the stenosed cases immediately start closing after reaching maximum PVA. Moreover, the healthy valve is not fully closed until around $0.4T$, while the other two cases close at approximately $0.35T$, right after the flow rate reduces to zero. The valve remains closed during the diastole in all three cases. The valve profiles at the maximum opening position for the three cases are shown in Fig. 6.22(b). When fully opens, the valve in the healthy case creates a circular orifice and has minimum blockage of the flow, while the valve in the stenosed cases creates an orifice that is more triangular-shaped.
Figure 6.23: Instantaneous vortical structures visualized by the Q criterion. The isosurface of $Q=10^5 \text{s}^{-2}$, $3 \times 10^5 \text{s}^{-2}$ and $5 \times 10^5 \text{s}^{-2}$ are plotted for Healthy, 31% and 47% cases respectively. The isosurface is colored by the streamwise velocity. The phases selected here are indicated in Fig. 6.22(a).
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6.3.2.2 Post-Valvular Hemodynamics

The inclusion of the valve model and pulsatility is expected to further increase the complexity of the post-valvular hemodynamics. Fig. 6.23 shows the instantaneous vortical structures for the three cases studied here through the Q-criterion. Results from selected phases are plotted here to demonstrate the effect of the pulsatility. When the flow is accelerating (phase I), the valves in all three cases have already reached fully open position. Due to the orifice created by the valve, a jet is formed inside the aorta. For the healthy case and 31% stenosis, the jet is mainly laminar at this phase and a well-defined vortex ring is clearly visible in the aorta. On the other hand, due to the strong intensity of the jet created by the narrower orifice, the flow after the 47% stenosis already shows chaotic behavior and breakup of the vortical structures. At phase II, the inflow reaches maximum flow rate. While the valve in the healthy case still remains fully open, it starts to close in the stenosed cases. The decreased orifice area along with the increased jet intensity are potentially responsible for the breakup of the jet for the 31% case at phase II, while the vortical structures in the healthy case are still coherent and well-defined. The post-valvular jet created by the healthy valve does not break-up until the flow starts to decelerate (phase III), and the disintegration of the vortical structures is not as intense as the stenosed cases. Moreover, the extent of the jet penetration is also affected by the jet intensity, as the small vortical structures can reach further downstream when the stenosis becomes more severe. These small vortical structures continuously dissolve
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during the deceleration phase of the systole as shown in phase IV and V.

Fig. 6.24 summarizes the phase-averaged (mean) flow field for \( AS = 47\% \). Due to the existence of the aortic root, the flow field is not axisymmetric, and the flow fields in both the \( z - x \) plane and \( z - y \) plane are shown here. As can be seen in the phase-averaged streamwise velocity field (Fig. 6.24(a)), the flow is symmetric in the \( z - y \) plane but tilts toward the left side (towards the sinus) on the \( z - x \) plane. Negative velocities can be seen surrounding the core of the jet in both planes, indicating recirculation zones. The mean vorticity in Fig. 6.24(b) clearly shows the behavior of the shear layer around the jet. The shear layer already shows indications of breakup at phase I, which is consistent with the observation of instantaneous flow field in Fig. 6.23, and the reattachment location is estimated to be around \( s = 0.7D_o \) into the aorta based on the vorticity in \( z - y \) plane. Here, \( s \) is the distance into the aorta measured from the end of the aortic root. As the jet reaches peak flow rate, the reattachment location moves slightly past \( 1.0D_o \). Even though the reattachment location continues to move downstream for a short period of time during the deceleration phase, it eventually recedes toward the aortic root due to the closing valve and the decreasing jet intensity. It is noted that due to the non-symmetric flow field, the reattachment locations estimated based on the vorticity in \( z - x \) plane are slightly different from the \( z - y \) plane. However, the general trend remains the same. Fig. 6.24(c) plots the turbulent kinetic energy (TKE) distributions at different planes. The TKE shows that the turbulent fluctuations already exist before peak systole due to the early
Figure 6.24: Flow fields of AS=47% at selected phases. (a) Phase-averaged streamwise velocity, (b) phase-averaged vorticity, (c) TKE. The top row of each figure shows the y – z plane, and the bottom row shows the x – z plane (see Fig. 6.22(b)). The selected phases are highlighted in Fig. 6.22(a).
Figure 6.24: (cont.) Flow fields of AS=47% at selected phases. (a) Phase-averaged streamwise velocity, (b) phase-averaged vorticity, (c) TKE. The top row of each figure shows the y – z plane, and the bottom row shows the x – z plane (see Fig. 6.22(b)). The selected phases are highlighted in Fig. 6.22(a).
breakup of the jet and reach maximum intensity in the deceleration phase. Also, the location with high TKE is spatially well-correlated with the small vortical structures resulted from the breakup of the jet shear layer shown in Fig. 6.24(b).

The mean flow field for the healthy case and 31% stenosis are shown in Figs. 6.26 and 6.25. The results from the 31% case share great similarities to the 47% case. A strong jet is seen forming at the orifice and recirculation zones are observed surrounding the core of the jet. Nevertheless, the jet intensity is weaker in this less severe case, and the flow remains laminar during the acceleration phase as indicated by the coherent vortical structures at phase I. Furthermore, compared with the 47% case, the reattachment points are closer to the aortic root at the same phase. The majority of the small vortical structures are observed after $s = 1.0D_o$ during the deceleration phase. Also, the TKE distribution spatially overlaps these small vortical structures created from the jet shear layer. As expected, the healthy valve produces the weakest jet among the three cases. The flow remains laminar during most of the systole, and the majority of the jet breakup occurs during late systole (phase IV). It is noted that the same contour level is used for the TKE in Figs. 6.24, 6.26 and 6.25. These two figures clearly show the increasing of turbulent intensity as the severity increases.
Figure 6.25: Flow fields of AS=31% at selected phases. In each figure, the top row is the phase-averaged streamwise velocity; the middle row is the phase-averaged vorticity; and the bottom row is the TKE. The selected phases are highlighted in Fig. 6.22(a).
Figure 6.26: Flow fields of the healthy case at selected phases. In each figure, the top row is the phase-averaged streamwise velocity; the middle row is the phase-averaged vorticity; and the bottom row is the TKE. The selected phases are highlighted in Fig. 6.22(a).
6.3.2.3 Surface Force Analysis

Previous study \cite{23} has already shown that it is the pressure force fluctuations acting on the lumen wall that creates heart murmurs. More specifically, it is the time-derivative of the wall pressure that serves as the source in the murmur generation. Hence, in this section we focus on the analysis of surface force.

An example of the wall pressure signal is shown in Fig. 6.27(a). The overall
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trend of the signal follows the after load. But increased high frequency fluctuations are observed around the peak systole. The time derivative of this pressure signal is plotted in Fig. 6.27(b). A bandpass filter with passband $[10, 1000] Hz$ is applied to exclude the influence of the fundamental frequency (heart rate) at low frequency range and the high frequency noise. Again, high intensity fluctuations can be seen around peak systole, and this will serve as the source of murmur generation. Moreover, since the low frequency effect is excluded here, the time derivative of pressure is well correlated with the pressure fluctuation in both timing and intensity as made clear by Fig. 6.27(c). Therefore, $p''$ can serve as a surrogate to identify the source location.

The source location of the murmurs is defined as the location on the lumen wall with strong pressure fluctuations. The intensity of fluctuation is quantified by the RMS value calculated by Eq. 6.3. In order to investigate how the source location changes with time, surface distribution of pressure fluctuation intensity is plotted in Fig. 6.28 for all three cases at selected phases. For the most severe stenosis ($AS=47\%$) during phase I, isolated hot spots can already be observed around $0.7D_o$ after the aortic root. As the jet flows downstream, the source locations also move downstream, and strong fluctuations occur between $1.0D_o$ and $2.0D_o$. These source locations maintain in this region during most part of the systole in spite of the constantly changing flow rate, and eventually die down at the beginning of diastole. When compared with Fig. 6.24(c), it is easy to observe that the murmur sources are at locations where there are strong near-wall turbulent fluctuations. An example of
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Figure 6.28: Distribution of intensity of the wall pressure fluctuations for all three cases. The selected phases are highlighted in Fig. 6.22(a), and the same contour level is used to facilitate comparison.
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this connection between near-wall turbulence and wall pressure fluctuation would be phase IV. As shown by the streamwise velocity in Fig. 6.24(a), the diameter of the downstream jet has shrunk considerably at this phase due to the lower inflow velocity and reduced valve opening area. Consequently, the shear layer of the jet shifts away from the lumen wall towards the center of the aorta, leading to relatively weak surface pressure fluctuations at this phase in spite of the strong TKE around the jet. The same is also true for the other two cases. However, the timing of the strong wall pressure fluctuations is different among the three cases. No visible fluctuation is observed in the healthy case until phase IV, while the 31% case is expected to start to create murmurs around peak systole and reaches maximum intensity around phase III. Furthermore, it is noted that the same contour level is used in all three cases, and these plots clearly demonstrate that the 47% stenosis produces the strongest fluctuations while they are barely detectable in the healthy case. In general, the more severe the stenosis is, the earlier the murmurs are generated and the stronger the intensity is expected to be. Interestingly, the source location is less affected by the severity of the stenosis and always locate between 1.0D_o and 2.0D_o during most of the systole for 31% and 47% cases.

Compared with the pulsatile flow study in Section 6.2 where the aortic stenosis is modeled as a smooth, axisymmetric constriction, the inclusion of a valve model produces some similar features. The murmur source generated by incomplete valve opening also moves downstream with time. The source intensity increases with the
increase of severity, and the strongest murmurs are expected to be generated during the deceleration phase of the systole in both cases. Nevertheless, there are also some differences. When the stenosis is fixed and smooth, the wall pressure fluctuations of the mild stenosis (A=50%) are not expected to occur until the flow starts to decelerate. However, the post-valvular flow of the mild stenoses (AS=31% and 47%) created by the valve already shows sign of murmur generation around peak systole. Moreover, an axisymmetric vortical structure is generated immediately downstream of the stenosis in the previous study (see Fig. 6.4). However, the valve generates a rotationally symmetric (every 120°) post-valvular flow, and when coupled with the curved-pipe model in Section 6.2 is expected to generate more complex flow structures.

Our previous study (Chapter 4) already shows that, in the steady flow case, the fluctuation of the wall shear stress (WSS) is much weaker than the wall pressure fluctuation. It would be useful to reexamine if this is still true for the current case. Fig. 6.29 plots the WSS fluctuation intensity at the same phase as Fig. 6.28. The timing and location of the appearance of strong WSS fluctuations are quite similar to the wall pressure fluctuations. However, their intensity is negligible compared to the pressure fluctuations, especially for the stenosed cases. Therefore, the pressure fluctuations are still the dominant factor in generating the murmurs.
Figure 6.29: Distribution of intensity of the wall pressure fluctuations for all three cases. The selected phases are highlighted in Fig. 6.22(a), and the same contour level is used to facilitate comparison.
6.3.3 Conclusion

In this study, a simple RDOF valve model is introduced to study the fluid-structure interaction in the aortic stenosis. A canonical straight aorta model is employed here, and the valve is modeled after a commercial bioprosthetic valve. Instead of solving a PDE for the valve motion, a set of ODEs is used to move the valve with a prescribed mode shape. By comparing with results from another valve model with more complicated constitutive relations, this simple model is proved to be able to capture the first-order valve motion (open/close) quite accurately.

This valve model is then used to study the aortic stenosis, which is caused by the incomplete opening of the aortic valve. A healthy case, along with two stenosed cases with \( AS = 31\% \) and \( 47\% \) are studied. The healthy case serves as the baseline, and it shows extremely weak murmur generation. The incomplete opening is induced by increasing the stiffness coefficient of the valve model, and only rotationally symmetric stenosis is studied here. The results show that the near wall turbulent fluctuations are responsible for the strong wall pressure fluctuations, which serve as the source of murmur generation in the stenosed cases. The intensity of the sources increases as the severity increases. In both stenosed cases, the strongest wall pressure fluctuations occur in the deceleration phase of the systole. Moreover, the source locations are observed to locate between \( 1.0D_o \) and \( 2.0D_o \) after the aortic root for the two severities studied here.
Chapter 7

Thesis Summary

Cardiac auscultation is a century-old technique that has great potential due to its inexpensive and non-invasive nature. Coupled with recent development of ultra-sensitive, low-power, low-cost compact acoustic sensors, advanced signal analysis algorithms and powerful portable computers, automated cardiac auscultation could have a profound impact on the early detection as well as monitoring of heart conditions.

A lack of clear understanding between cause (abnormal flow) and effect (murmur) as well as the limitations of the electronic stethoscope based auscultation, have severely limited the success of past attempts at automated cardiac auscultation. In an effort to tackle this problem, we use a novel multiphysics computational approach to investigate the causal mechanism(s) behind heart murmurs from first principles.

To ensure efficient solution of the cardiovascular flows, a previously developed ghost-cell method based sharp-interface immersed boundary flow solver is upgraded in two ways. First, a biconjugate gradient stabilized method based pressure Poisson solver is used to replace the previous multigrid solver. This enables explicit coupling of the ghost cell formula, and reduces error accumulation around the immersed
boundary. A strong scaling study shows that the upgraded flow solver achieves eight time speed-up. Second, the majority of the grid points in the cardiovascular flow problems lie in the solid region, which are in principal wasted. By employing a graph-partitioning based parallel framework, these wasted grid points can be effectively ignored during the solution process, and the computational cost, including the wall time and allocated memory, is significantly reduced.

An important aspect of our study is to understand the role of shear and compression waves in heart murmurs. To achieve this, a classic vector decomposition is applied to the abnormal sound generated by a two-dimensional canonical arterial stenosis. Close inspection of the results reveals that the shear wave and the compression wave are almost out of phase in the low frequency range, resulting in a weaker total signal. In addition, the intensity of compression wave on the skin surface can accurately predict the source location of the murmurs, while shear wave can lead to false predictions. Thus, it is critical to include both bulk modulus and shear modulus in the material model in order to gain correct understanding of the murmur propagation.

The majority of the previous experimental and numerical investigations of heart murmurs have only studied either the generation (hemodynamics) or the propagation (acoustics) of the murmurs. In the current research, we use a one-way coupled hemodynamic-acoustic approach to investigate the generation and propagation of murmurs associated with the aortic stenosis. A steady flow assumption is adopted
CHAPTER 7. SUMMARY

in our first model. Direct numerical simulation is used to explore the hemodynamics of the jets formed at the stenosis, and temporal history of the wall pressure are recorded. Subsequently, the recorded pressure is used as the source in the acoustic study, and structural wave propagation in the tissue is resolved by a high-order, linear viscoelastic wave solver in order to explore the propagation of the murmurs through a tissue-like material. The hemodynamic study finds that the source locations is not co-located with the site of disease, i.e. the stenosis, but locates in the ascending aorta. Moreover, this location changes only slightly with stenosis severity. A characteristic frequency related to the periodic vortex shedding is observed in the spectra of wall pressure signal in the form of break frequency. However, the acoustic study shows that the propagation process alters the source spectrum significantly, and this characteristic frequency is not of particular interest in the murmur signal detected on the skin surface. We also test two source localization method using only the murmurs signal. Both distribution of murmur intensity on the skin surface and free-space Green’s function can provide accurate prediction of source location in the current study.

Last but not least, the effects of the pulsatility and the valve on the hemodynamics are investigated individually. Compared with the steady flow case, the pulsatile inflow and aortic valve lead to a traveling source location, and the sources are expected to be strongest during the deceleration phase of the systole.
Appendix A

Profile for the Stenosis

The stenosis has the following prescribed shape:

\[
r(z) = \frac{D}{2} \left\{ 1 - \frac{D - D_j \exp(-a(z - z_0)) \sin \left[ \frac{\pi(z - z_0)}{L_s} \right]} {\exp(-aL_p) \sin \left( \frac{\pi L_p}{L_s} \right)} \right\},
\]

(A.1)

where \(D_j\) is the minimum diameter of the constriction (jet diameter), \(z_0\) marks the starting location of the stenosis in \(z\) direction, \(L_s\) is the length of the stenosis, and \(L_p\) measures the distance between the starting point of the constriction and the location with the minimum diameter. Here, \(L_s\) and \(L_p\) are set to 1\(D\) and 0.8\(D\), respectively.
Appendix B

Grid Convergence Study

B.1 Steady Flow

The flow in this study is resolved using the DNS. It is therefore important to establish that the mesh employed is able to capture all the relevant scales. The grid convergence study is conducted on the 75% stenosis case, since it is expected to generate the jet with the highest intensity. Three different meshes, coarse (96 × 288 × 278), baseline (128 × 384 × 370) and fine (128 × 640 × 576), are used to solve the same problem. The ratios of average computational cell volume between different meshes are $V_{coarse}/V_{baseline} \approx 2.37$ and $V_{baseline}/V_{fine} \approx 2.59$. Data over 2.5 flow-through-times are collected and analyzed.

Fig. B.1 shows the comparison of the mean streamwise velocity and the Root-Mean-Square (RMS) of the streamwise velocity obtained from the aforementioned three meshes. The velocities are plotted along the pipe diameter at $\theta = 45^\circ$, $90^\circ$, $135^\circ$, and $180^\circ$ on the frontal plane. One can see that the velocity profiles for the three meshes are quite close to each other at these selected locations. In Fig. B.2 we show
APPENDIX B. GRID CONVERGENCE STUDY

Figure B.1: (a) Non-dimensionalized mean streamwise velocity and (b) non-dimensionalized RMS of streamwise velocity plotted at $\theta = 45^\circ$, $90^\circ$, $135^\circ$, and $180^\circ$ on the frontal plane. In the first plot, unit velocity corresponds to $0.25D$, and in the second plot, unit velocity corresponds to $D$.

Figure B.2: Non-dimensionalized TKE averaged over the cross-sectional area and plotted against the angle, $\theta$. 
APPENDIX B. GRID CONVERGENCE STUDY

Figure B.3: (a) Non-dimensionalized RMS of pressure and (b) non-dimensionalized RMS of wall shear stress plotted along the anterior surface.

the distribution of the cross-sectional averaged turbulent kinetic energy (TKE) along the aortic arch. It is noted that the results from the baseline and the fine meshes agree reasonably well, while the result from the coarse mesh deviates from the others significantly. Apart from the flow properties inside the modeled aorta, we are also highly interested in the surface force distribution. Thus, the wall pressure fluctuation and the wall shear stress (WSS) fluctuation along the monitor points on the anterior surface from different meshes are plotted in Fig. B.3. We note that the results of the baseline and fine meshes match well. Table B.1 lists the percentage of difference of the same variables in Figs. B.2 and B.3 with regard to the results from the fine mesh, and the definition is

\[
E(f) = \frac{\sqrt{\frac{1}{\pi} \int_0^\pi (f - f_{\text{fine}})^2 d\theta}}{f_{\text{fine,max}}},
\]

where \( f \) represents the value from coarse mesh or baseline mesh. The results from

\[
\text{B.1}
\]
Table B.1: Percentage of error with regard to the fine mesh.

<table>
<thead>
<tr>
<th></th>
<th>E(TKE)</th>
<th>E($P_{rms}$)</th>
<th>E($WSS_{rms}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>15.85%</td>
<td>9.79%</td>
<td>11.94%</td>
</tr>
<tr>
<td>Baseline</td>
<td>3.17%</td>
<td>2.54%</td>
<td>4.19%</td>
</tr>
</tbody>
</table>

Figure B.4: Instantaneous velocity profile along the diameter of $\theta = 40^{\circ}$ at $t = 0.15T$.

Since the same inlet velocity profile is employed in all three cases, the case with 75% stenosis is expected to have the strongest post-stenotic jet and is most susceptible to the baseline case are reasonably close to those of the fine mesh, while the results from the coarse mesh deviate noticeably. Based on this grid convergence study, we employ the baseline mesh ($128 \times 384 \times 370$) for all the simulations.

**B.2 Pulsatile Flow**

Since the same inlet velocity profile is employed in all three cases, the case with 75% stenosis is expected to have the strongest post-stenotic jet and is most susceptible...
APPENDIX B. GRID CONVERGENCE STUDY

Figure B.5: Time history of the total kinetic energy.

to the resolution issues. Therefore, the grid convergence study is conducted on the 75% stenosis with three grids with different resolutions: coarse (96 × 288 × 278), baseline (128 × 384 × 369) and fine (128 × 640 × 576). The results over one cardiac cycle are collected and compared. Fig. B.4 shows the instantaneous velocity profile (t = 0.15T) along the diameter of the cross section at $\theta = 40^\circ$. As can be seen here, while the results from the coarse mesh deviates from the others significantly, the velocity profiles from the baseline and the fine meshes are sufficiently close. Furthermore, the time variation of the total kinetic energy is plotted in Fig. B.5. The results from the baseline and the fine meshes overlap with each other through the whole cycle, with around 0.5% difference near the peak. Again, there is noticeable difference between the plot from the coarse mesh and the other two. Based on this study, the baseline mesh (128 × 384 × 369) is used to study all three cases, i.e. $AS = 50\%, 62.5\%$ and
APPENDIX B. GRID CONVERGENCE STUDY

Figure B.6: Instantaneous velocity magnitude profile along the diameter of cross section $s = 0D_o$ at $t = 0.08T$.

B.3 Reduced Degree-of-Freedom Valve Model

Given the flow is driven by the same velocity profile in all three cases, the case that generates the strongest post-stenotic jet is expected to be most affected by any grid resolution issues. Hence, a grid convergence study is carried out on the $AS=47\%$ stenosis. Three uniform grids with different resolutions: coarse ($120 \times 128 \times 200$), baseline ($154 \times 160 \times 256$) and fine ($190 \times 200 \times 320$) are employed, and the ratios of the computational cell volume are $V_{\text{coarse}}/V_{\text{baseline}} \approx 2.05$ and $V_{\text{baseline}}/V_{\text{fine}} \approx 1.93$. The simulations are carried out for one cardiac cycle and the results are compared here. Fig. B.6 plots the instantaneous velocity magnitude ($t = 0.08T$) across the
diameter at cross section $s = 0D_0$. It clearly shows that the velocity profile from the baseline mesh agrees well with the fine mesh, while the result from coarse mesh deviates significantly from the other two cases, especially near the valve. Moreover, the time history of the total kinetic energy over one cardiac cycle is shown in Fig. B.7. The kinetic energy calculated on the baseline and the fine meshes overlaps well with each other over the whole cycle. However, the results from the coarse mesh is higher than the other two cases. Based on this study, the baseline mesh ($154 \times 160 \times 256$) is used to study all three cases.
Appendix C

Green’s Function for the Elastic Wave Equation

The governing equation for the elastic wave generation by a point source at the origin can be expressed as follows:

\[ \rho_s \frac{\partial^2 v_i}{\partial t^2} - \frac{\partial}{\partial x_j} \left[ \lambda \frac{\partial v_k}{\partial x_k} \delta_{ij} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \right] = s_i(t) \delta(\vec{x}), \]  

(C.1)

where \( v_i \) is the displacement vector, \( s_i(t) \) is the source vector, and \( \delta(\vec{x}) \) is the Dirac delta function used to localize the source. If the wave is propagating freely in the space, its behavior can be described in the frequency domain with the help of the free-space Green’s function \( G_{ij}(\vec{x}, \omega) \)

\[ \langle v_i \rangle(\vec{x}, \omega) = G_{ij}(\vec{x}, \omega) \langle s_i \rangle(\omega), \]  

(C.2)
APPENDIX C. GREEN’S FUNCTION

where \( \langle \cdot \rangle \) represents the Fourier transform of the function, and \( \omega \) is the angular frequency. The Green’s function in the three-dimensional space corresponding to Eq. \( \text{C.1} \) has the following expression

\[
G_{ij}(\vec{x}, \omega) = \frac{ik_p}{12\pi(\chi + 2\mu)} \left[ h^1_0(k_pr)\delta_{ij} + \left( \delta_{ij} - \frac{3x_ix_j}{r^2} \right) h^1_2(k_pr) \right] - \frac{ik_s}{12\pi\mu} \left[ -2h^1_0(k_sr)\delta_{ij} + \left( \delta_{ij} - \frac{3x_ix_j}{r^2} \right) h^1_2(k_sr) \right],
\]

where \( h^1_n \) is the Hankel function of the first kind, and \( k_p, k_s, r \) are defined as

\[
k_p = \omega/c_p, \quad k_s = \omega/c_s, \quad r = |\vec{x}|.
\]

\[\text{(C.3)}\]

\( k_p \) and \( k_s \) can be interpreted as the compression and shear wave number, respectively. It is easily observed that the first term of the Green’s function describes the propagation of the generated compression wave, while the second term is solely responsible for the propagation of the shear wave. In the above equation, the source is located at the origin, while in this study, the source is the pressure force distributed on the lumen wall of the aorta.

The acceleration can be expressed as \( \langle a_i \rangle(\vec{x}, \omega) = (i\omega)^2 G_{ij}(\vec{x}, \omega) \langle s_j \rangle(\omega) \). Thus, for a general two-dimensional surface \( A \) in a three-dimensional space, the acceleration at any location can be written as

\[
\langle a_p \rangle(\vec{x}, \omega) = \int_A (i\omega)^2 G_{pq}(\vec{x} - \vec{X}, \omega) \langle P \rangle(\vec{X}, \omega) M_q(\vec{X})dA,
\]

\[\text{(C.5)}\]
APPENDIX C. GREEN’S FUNCTION

where \( p, q \) are indices ranging from 1 to 3, \( \vec{x} \) is a random point in the space and \( \vec{X} \) is a point on the lumen surface \( A \). \( P(\vec{X}) \) is the hydrodynamic pressure force at \( \vec{X} \) and \( \vec{M}(\vec{X}) \) is the unit normal vector at the same location. The effect of viscosity can be included by replacing the second Lame constant \( \mu \) with \( \mu_{\text{effective}} = \mu - i\omega\eta \).
Appendix D

Flow Rate Profile and After-Load Model

A cardiac cycle can be divided into systole and diastole. The blood flow is pumped out of left ventricle through aorta during the systole. The aortic valve is closed during the diastole, and there is no blood flow through aorta during this time period. A blood flow rate that mimics the physiological trend is modeled as follows:

\[
Q = \begin{cases} 
SV \frac{\pi (T_s - t)}{T_s} \sin \left( \frac{\pi t}{T_s} \right) & t \leq T_s; \\
0 & T_s < t \leq T.
\end{cases} \tag{D.1}
\]

Here, \(SV\) is the stroke volume; \(T_s\) is the length of the systole; \(T\) is the duration of the cardiac cycle.

The current modeled aorta is coupled with a lumped element model (LEM) to provide the after load. The model adopted here is proposed by Watanabe et al. [119]
APPENDIX D. FLOW RATE & AFTER LOAD

Table D.1: Parameters used in Eqs. D.1 and D.2

<table>
<thead>
<tr>
<th>Section</th>
<th>SV (ml)</th>
<th>$T_0$ (s)</th>
<th>$T$ (s)</th>
<th>$R_1$ (Pa $\cdot$ s/ml)</th>
<th>$R_2$ (Pa $\cdot$ s/ml)</th>
<th>$C$ (ml/Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2</td>
<td>40.5</td>
<td>0.33</td>
<td>0.77</td>
<td>18.0</td>
<td>220.0</td>
<td>0.02</td>
</tr>
<tr>
<td>6.3</td>
<td>70.0</td>
<td>0.33</td>
<td>0.9375</td>
<td>8.0</td>
<td>160.0</td>
<td>0.02</td>
</tr>
</tbody>
</table>

and the after load is calculated based on the following equations

\[
P_1 = R_1 Q, \\
\frac{dP_2}{dt} = \frac{1}{C} \left( Q - \frac{P_2}{R_2} \right), \quad (D.2)
\]

\[
P_o = P_1 + P_2.
\]

Here, $P_o$ is the pressure at the outlet (after load); $Q$ is the flow rate provided in Eq. D.1 and $R_1$, $R_2$ and $C$ are parameters listed in Table D.1.
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