Abstract

This dissertation consists of three chapters. In Chapter 1, I study an asset pricing implication of a New Keynesian model. Quantitative New Keynesian models have strong implications for the joint variation of total wealth, nominal bond yields, and real bond yields. I use stock market betas of nominal and real bonds as summaries of this joint variation, and ask whether model-implied betas can be parameterized to be consistent with observed betas. Using UK data, I document that the observed beta of a ten-year nominal bond dropped to zero when the UK government gave operational independence to the Bank of England. The observed beta of a ten-year inflation-indexed bond is close to zero both before and after independence. I show that a broad range of plausible model parameterizations cannot reproduce any of these observed betas.

In Chapter 2, I study the effects of positive trend inflation on the term structure of interest rates. I examine the effects on a number of aspects of the yield curve: the steady state, the mean, the variance, impulse responses to economic shocks, and risk compensation. I find that higher rate of trend inflation leads to more volatile inflation, which in turn increases the volatility of bond prices and the quantity of risk compensation. The quantitative effects of these findings are quite small when we assume the standard log utility of the representative household. However, the quantitative effects become significant when we assume the household with Epstein-Zin preferences. For example, the average slope of the yield curve is 1.02 percent for 2 percent inflation, while it is 1.35 percent for 6 percent trend inflation. Excess returns to 10-year bond are 5.6 percent for 2 percent trend inflation and 6.7 percent for 6 percent trend
Inflation.

In Chapter 3, I study the welfare effects of financial globalization. Debt and foreign direct investment (FDI) flows account for the vast majority of foreign capital going into developing countries. Gourinchas and Jeanne (2006) study the welfare consequences from liberalizing debt flows using the Ramsey growth model and find that the welfare gains is quite limited. This paper studies the welfare gains from liberalizing FDI flows and find much larger gains (four percent increase in permanent consumption in the baseline specifications). The key assumption is that FDI flows bring a superior technology into host countries. Additional gains from liberalizing debt flows besides FDI flows are limited. This paper conducts a number of sensitivity analysis. For example, it examines the consequences of a decrease in the world real interest rate, the phenomena observed in recent decades.

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Chapter 1

Evaluating a New Keynesian Model Using Stock Market Betas of Bond Returns

1.1 Introduction

New Keynesian models have strong implications for the joint variation of total wealth, nominal bond yields, and real bond yields. For example, in a simple New Keynesian model, a positive technology shock lowers nominal and real bond yields, whereas it increases total wealth.\footnote{See, for example, the model presented in Chapter 3 of Galí (2008).} The joint variation of bond yields and total wealth translate into a certain relationship of bond returns and return on total wealth, which can be summarized by stock market betas of nominal and real bonds.\footnote{Here I use the equity return as an approximation of total wealth.}

In this paper, I point out that the bond-equity return relationship in data is still puzzling from the perspective of a New Keynesian model, which can account for the average slope of the nominal yield curve, as well as several other aspects of the term structure. More specifically, using the time-varying stock
market betas of long-term nominal and real UK government bonds as measures of the bond-equity return relationship, I find that the model fails to replicate the levels of both the nominal and real bond betas, even considering a wide range of parameters.\textsuperscript{3} I include real bonds in the analysis because it allows us to break down shocks to nominal bonds into shocks that are specific to nominal bonds (primarily inflation shocks) and other shocks.\textsuperscript{4}

Figure 1.1 shows quarterly estimated stock market betas of 10-year nominal and real UK government bonds. I choose to use UK data because they offer the longest sample period for real bonds among advanced economies. The vertical line in the figure indicates the point at which the UK government announced that it would grant operational independence to the Bank of England (May 6, 1997).

I highlight three features about the bond-equity return relationship that can be seen in Figure 1.1. First, the nominal bond beta falls sharply from about 0.4 to zero around the time of the announcement of operational independence. To emphasize, the positive return relationship between the 10-year nominal bond and the equity disappeared following the announcement. Second, the real bond beta also showed a similar change, although, statistically speaking, the beta was zero during most of the sample period. That is, the long-term real bond return and the equity return scarcely moved together in the sample. Third, both the nominal and real bond betas were equal (and zero) after the announcement, meaning that the two types of government bonds are essentially the same in

\textsuperscript{3}Although this is the same beta that shows up in the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), this paper is not tied to the CAPM.

\textsuperscript{4}Formally the real bond is called indexed-linked gilts in UK. For simplicity, I call it the real bond in this paper.
terms of their relationship to the equity, although the two bond prices are not perfectly correlated (i.e., they do not move in lockstep). This suggests that shocks that are specific to nominal bonds (inflation shocks) still exist but are idiosyncratic and that they do not show up in equity returns after independence.

The figure suggests that changes in UK monetary policy brought about by the independence of the Bank of England may have led to changes in the nominal and real bond betas. Adopting this interpretation, I examine whether a New Keynesian model with Epstein-Zin recursive preferences can explain the features about the bond-equity return relationship mentioned above, with some changes in the monetary policy rule. More specifically, I interpret the announcement about the Bank of England’s independence as an event that changed several parameters associated with monetary policy in the model: the coefficients of inflation-gap and output-gap in the monetary policy rule, as well as the level of trend inflation, which largely determines the long-run inflation rate in the model.

I use a New Keynesian model rather than a consumption-based asset pricing model, so that I can explicitly discuss the link between monetary policy and asset prices. I use a New Keynesian model with recursive preferences, since models with recursive preferences have recently succeeded in explaining well-known facts about the nominal yield curve, such as the average slope of the curve (Piazzesi and Schneider, 2007; Rudebusch and Swanson, 2012; van Binsbergen et al., 2012). These authors examine whether the model can explain univariate features of the term structure, such as the average slope of the yield curve, whereas I examine a bivariate feature (the bond-equity return relationship) of the term structure.
The analysis leads to two main results that shed light on the aspects of the bond-equity return relationship that the model fails to explain, as well as the reasons for the model’s failure. First, model-implied betas of long-term real bonds are close to one for a wide range of monetary policy parameters. The reason is that equity in the model (modeled as a claim to aggregate output) is similar to a real bond because output volatility is low. However, observed betas of inflation-indexed bonds are close to zero.

Second, model-implied betas of long-term nominal bonds are greater than those of real bonds and thus greater than one. The wedge between nominal and real bonds is driven by inflation expectations. In typical New Keynesian frameworks, shocks to output are negatively correlated with shocks to inflation, hence the positive relationship between nominal bond prices and equity prices exceeds that for real bond prices. However, prior to the UK announcement of independence, observed betas of nominal bonds are substantially less than one. Moreover, after the announcement, betas for both nominal and real bonds are indistinguishable from each other (and equal to zero).

In principle, post-announcement betas of nominal and real bonds could match if investors interpreted the announcement as the end of inflation in the UK. Then nominal and real bonds would move in lockstep. However, returns to nominal bonds are more volatile than returns to inflation-indexed bonds both before and after the announcement. Thus in the data, the wedge between nominal and real bonds is unrelated to equity returns. This property is inconsistent with New Keynesian joint dynamics of inflation and output.
Related Literature  This paper relates to three strands of literature. First, the paper builds on the literature on dynamic general equilibrium modeling of the term structure of interest rates. An early discussion in the literature is that of Backus et al. (1989), who show that the standard consumption-based asset pricing model generates a downward sloping yield curve, rather than the upward-sloping curve seen in the data. den Haan (1995) finds that the standard real business cycle model has the same implication.

Among a large number of the studies, this paper is particularly cognate with Piazzesi and Schneider (2007), Rudebusch and Swanson (2012), and van Binsbergen et al. (2012), all of whom use recursive preferences to attempt to resolve the puzzle in slightly different settings. The key idea of these papers is that investors with recursive preferences are afraid of shocks that move inflation and future consumption growth in opposite directions, because then the nominal bond loses its value when investors particularly favor consumption.\textsuperscript{5} Piazzesi and Schneider (2007) empirically find, in the U.S. data, that positive inflation surprises indeed tend to lead to lower future consumption growth, whereas Rudebusch and Swanson (2012) find that technology shocks have the same feature.

Second, this paper relates to the literature on the bond-equity return relationship. Baele et al. (2010) study the sources of the U.S. bond-equity return relationship using a dynamic factor model. Observing that the CAPM beta

\textsuperscript{5}These models still imply a downward-sloping real yield curve. The U.S. data suggest an upward-sloping real yield curve, whereas UK data suggest a downward-sloping real yield curve. Greenwood and Vayanos (2010) argue that the UK real yield curve is downward sloping because of the special demand from pension funds and life insurance companies due to regulations. Kısacıkçı (2013) examines the properties of the real term structure in a variety of New Keynesian models.
of 10-year nominal U.S. Treasury bonds switches signs over time, Campbell et al. (2013a) build a dynamic term structure model that allows the bond-stock covariance to change signs. Campbell et al. (2013b) examine the time-varying U.S. bond-equity return relationship, using a New Keynesian model with habit formation.

Third, this paper relates to the literature on positive trend inflation; i.e., a positive inflation rate in the deterministic steady state, as opposed to zero, as is normally assumed in the literature. Recent work on positive trend inflation shows that it significantly affects the properties of the model: the steady state (Ascari, 2004), the stochastic mean of macro variables in the model (Amano et al., 2007), the determinacy of the model (Ascari and Ropele, 2009; Coibion and Gorodnichenko, 2011), and the optimal monetary policy (Ascari and Ropele, 2007). This paper contributes to the literature on positive trend inflation by examining its effects on asset prices. Using a new Keynesian model with recursive preferences, Goto (2013) examines in detail the relationship between positive trend inflation and asset prices, as in this paper.

1.2 Stock Market Betas of UK Nominal and Real Government Bonds

In this section, I present empirical findings about the stock market betas of UK 10-year nominal and real government bonds (henceforth, the nominal bond beta and the real bond beta), which I later use to evaluate the model presented in Section 1.3.
**Estimation** I assume that both the nominal and real betas are constant within a quarter but can vary over quarters.\(^6\) Under this assumption, we can consistently estimate the betas by running the following regression for each quarter:

\[
R_{i,t} = \alpha_i + \beta_i R_{m,t} + \varepsilon_{i,t}
\]

where \(R_{i,t}\) denotes the daily return on either the 10-year nominal or real bond in excess of the return on the 1-year nominal bond.\(^7\) \(R_{m,t}\) denotes the excess return on the UK aggregate stock market index (the FTSE all-share index). Lewellen and Nagel (2006) take this approach to estimate the time-varying CAPM betas of various portfolios.

The assumption that the beta is constant within a quarter seems reasonable. The bulk of the research finds that expected returns vary over time but do so over several years.\(^8\) Thus, if the beta varies monthly or weekly, expected returns \((\beta_i R_{m,t})\) become much more volatile than previous research suggests.

To address a potential issue of nonsynchronous trading, I also estimate the betas including a lead and a lagged market return in the regression, as in Dimson (1979). This does not affect the results reported in this paper.

**Data** Before turning to the estimation results, I describe the bond and equity data used in the estimation. The data sample is 1986Q1 through 2010Q4.

I use the FTSE all-share index to construct the market return. The FTSE

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\(^6\) I obtain qualitatively the same results, even if I assume the betas are constant within a month. The results are statistically less precise, since there are only 22 trading days per month on average.

\(^7\) In the Appendix, I describe in detail how I construct the nominal return series for the real bond.

\(^8\) See, for example, a review article by Cochrane (2008).
all-share index is a market capitalization-weighted index representing the performance of all eligible companies listed on the London Stock Exchange’s main market. The index covers approximately 98 percent of the UK’s market capitalization. These data come from Bloomberg.


Results I now turn to the estimation results. Figure 1.1 plots the point estimates of the nominal and real bond betas with 95 percent confidence intervals. The two vertical lines indicate two major UK monetary policy changes that took place in the 1990s: the introduction of inflation targeting in October 1992 and the independence of the Bank of England in May 1997.

There are two things to note here. First, the nominal bond beta fell to zero after the Bank of England obtained independence. That is, 10-year nominal bond returns and equity returns were previously positively correlated, and this correlation disappeared after independence. Note that we do not observe a similar break when inflation targeting was introduced.

Second, the real bond beta also shows similar behavior, although the real bond beta was not statistically different from zero throughout most of the sample period. That is, 10-year real bond returns and equity returns were barely correlated throughout the sample period. I attempt to interpret these results through the lens of the model presented in the next section.
1.3 A New Keynesian Model with Recursive Preferences and Trend Inflation

1.3.1 Overview

The model is a simple New Keynesian model with two features: recursive preferences and positive trend inflation (i.e., a positive rate of inflation in the deterministic steady state). As shown by Rudebusch and Swanson (2012) and van Binsbergen et al. (2012), a dynamic general equilibrium model with recursive preferences (Kreps and Porteus, 1978; Epstein and Zin, 1989; Weil, 1989) can account for several moments of the nominal yield curve, such as the average slope of the curve.

I include positive trend inflation without indexation into the model, because one of my interpretations of the Bank of England’s independence is as an event that lowered the level of trend inflation. Trend inflation affects the properties of asset prices in the model. For example, the inflation risk premium is slightly larger when trend inflation is higher, because the uncertainty (proxied by the volatility) of inflation rises with the level of trend inflation. This is broadly consistent with the empirical findings about the relationship between inflation uncertainty and term premia reported by Wright (2011). Goto (2013) examines the relationship between the level of trend inflation and asset prices. In the remainder of this section, I describe the model in detail.

1.3.2 Households

The representative household maximizes lifetime utility:

$$U_t = \left( u_t^{1-1/EIS} + \beta (CE_{t+1})^{1-1/EIS} \right)^{1-1/EIS}$$
where $u_t \equiv C_t^\alpha (1 - N_t)^{1-\alpha}$ denotes the current utility flow that consists of consumption, $C_t$, and leisure, $1 - N_t$. $\text{CE}_{t+1}$ denotes the certainty equivalent of expected future lifetime utility:

$$\text{CE}_{t+1} \equiv (E_tU_{t+1}^{1-\gamma})^{(1-\gamma)^{-1}}$$

where $\gamma$ denotes the coefficient of relative risk aversion.$^9$ $\text{EIS}$ denotes the elasticity of intertemporal substitution. The lifetime utility consists of the current utility flow and the certainty equivalent of the expected future lifetime utility, aggregated by the CES function. If $\gamma = 1/\text{EIS}$, the preferences revert to the standard constant relative risk aversion (CRRA) preferences.

### 1.3.3 Final Good Firms

Competitive firms combine intermediate goods to produce the final good, $Y_t$, using the following technology:

$$Y_t = \left( \int_0^1 Y_{i,t}\frac{\varepsilon+1}{\varepsilon} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1, \tag{1.1}$$

where $Y_{i,t}, i \in [0,1]$ denotes the intermediate good $i$ and $\varepsilon$ denotes the elasticity of substitution across intermediate goods. Profit maximization leads to the following demand for intermediate goods:

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t \tag{1.2}$$

$^9$The specification here follows van Binsbergen et al. (2012) and Gourio (2012). As shown by Swanson (2012), $\gamma$ is not in general equal to the coefficient of relative risk aversion, when the household can vary its labor supply in response to shocks. In the current specification, however, $\gamma$ happens to be equal to the coefficient of relative risk aversion. See Example 2 in Section 3.3 of Swanson (2013).
where \( P_{i,t} \) denotes the price of the intermediate good \( i \). Substitute (2.4) into (2.3) to express the aggregate price in terms of the prices of intermediate goods:

\[
P_t \equiv \left( \int_0^1 P_{i,t}^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}.
\]

### 1.3.4 Intermediate Goods Firms

There is a continuum of monopolistically competitive intermediate goods firms indexed by \( i \in [0, 1] \). Firm \( i \) produces the good using the linear technology

\[
Y_{i,t} = A_t N_{i,t},
\]

where \( A_t \) denotes the level of aggregate technology, which evolves according to the following law of motion:

\[
\log A_t = \rho_a \log A_{t-1} + \sigma_a \varepsilon_{a,t}
\]

where \( \sigma_a \varepsilon_{a,t} \) is an independently and identically distributed technology shock with a mean of zero and a standard deviation of \( \sigma_a \). Firm \( i \) supplies its product to meet the demand according to (2.4), and sets the price as in Calvo (1983):

\[
P_{i,t} = \begin{cases} 
P_{i,t-1} & \text{with probability } \theta \\
\tilde{P}_t & \text{with probability } 1 - \theta,
\end{cases}
\]

where \( \tilde{P}_t \) denotes the reset price under which the firm maximizes the discounted expected future profits:

\[
E_t \sum_{j=0}^{\infty} \theta^j M_{i,t+j} \left( \tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right).
\]

Here, \( \theta^j \) is the probability that the reset price \( \tilde{P}_t \) remains effective in time \( t + j \), \( s_t \) the real marginal cost, and \( M_{i,t+j} \) the product of the nominal stochastic
discount factors:

\[ M_{t,t+j}^s \equiv M_{t,t+1}^s \times \cdots \times M_{t,j+1,t+1}^s \]

\[ M_{t,t+1}^s \equiv \beta \left( \frac{u_{t+1}}{u_t} \right)^{1-\gamma} \frac{C_t}{C_{t+1}} \left( \frac{V_{t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{1-\frac{1}{\delta}} \frac{P_t}{P_{t+1}} \]

\[ \equiv M_{t,t+1} \frac{P_t}{P_{t+1}}. \]

1.3.5 Monetary Policy and Long-Run Inflation

Monetary policy rule  The monetary authority sets the short-term nominal interest rate, \( R_{t+1} \), according to the Taylor (1993) rule:

\[ \frac{R_{t+1}}{R^*} = \left( \frac{R_t}{R^*} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\Pi} \left( \frac{Y_t}{Y^*} \right)^{\phi_y} \left( 1 - \rho_r \right) \exp(\sigma m^t), \]

where \( R^* \) is the steady-state level of the short-term interest rate, \( Y^* \) is the steady-state level of output, and \( \Pi^* \) is the steady-state level of gross inflation, which can be larger than 1.

Trend Inflation without Indexation  In introducing positive trend inflation, I assume no indexation. That is, firms that do not re-optimize their prices continue to use their existing prices, rather than increasing them by the rate of inflation. The assumption of no indexation is consistent with microeconomic evidence on price adjustments documented by Bils and Klenow (2004) and Nakamura and Steinsson (2008). Cogley and Sbordone (2008) also find empirical evidence against indexation; they find that indexation is unnecessary to explain the inflation dynamics using a New Keynesian model once the model allows variations in trend inflation.
1.3.6 Equilibrium

The economy’s resource constraint is $Y_t = C_t$. An equilibrium is a collection of stochastic processes,

$$(C_t, N_t, W_t, Y_t, P_t, R_{t+1}, P_i, Y_i, N_i, \Pi_t)_{t=0}^\infty,$$

such that household and firm problems are solved, markets clear, the policy rate is set according to the monetary policy rule, and the resource constraint is satisfied.

1.3.7 Asset Prices

In this subsection I describe the asset price implication of the model. All asset prices satisfy the same asset pricing equation, which is derived from the first-order condition of the household problem. Suppose that the payoff of an asset is $x$. Then the price of the asset satisfies the following asset pricing equation:

$$p_t = E_t(M_{t,t+1}x_{t+1}),$$

where $M_{t,t+1}$ is the stochastic discount factor defined above. In the following I describe how I compute the price of equity and long-term nominal and real bonds, which I use in turn to compute the stock market betas of the nominal and real bonds.

**Equity** The equity is modeled as a claim to aggregate output. The ex-dividend price of the equity is given by

$$W_t = \sum_{j=1}^\infty E_t(M_{t,t+j}C_{t+j})$$

$$= E_t(M_{t,t+1}(C_{t+1} + W_{t+1})). \tag{1.3}$$
The equity return is given by

\[ R_{w,t+1}^{w} \equiv \frac{C_{t+1} + W_{t+1}}{W_{t}}. \]

**Bond**  The government bond is modeled as a default-free zero-coupon bond. I consider both nominal and real bonds. The price of an \( n \)-period zero-coupon nominal bond satisfies

\[ Q_{n,t}^{s} = E_{t}\left( M_{t,t+1}^{s} Q_{n-1,t+1}^{s} \right), \]

where \( M_{t,t+1}^{s} \) denotes the nominal stochastic discount factor. The holding period return of the bond between time \( t \) and \( t + 1 \) is given by

\[ R_{n,t+1}^{n,s} = \frac{Q_{n-1,t+1}^{s}}{Q_{n,t}^{s}}. \]

The price of an \( n \)-period zero-coupon real bond satisfies

\[ Q_{t}^{(n)} = E_{t}\left( M_{t+1} Q_{t+1}^{(n-1)} \right), \]

and the corresponding holding period return is

\[ R_{t,t+1}^{n} = \frac{Q_{n-1,t+1}}{Q_{n,t}}. \]

**Beta**  The stock market beta of an \( n \)-period nominal bond is given by

\[ R_{i,t+1}^{s} - R_{i,t+1}^{f} = \alpha + \beta (R_{w,t+1}^{w} - R_{i,t+1}^{1}) + \varepsilon_{t+1}, \]

and the beta of a \( n \)-period real bond is given by:

\[ R_{i,t+1}^{1} = \alpha + \beta (R_{w,t+1}^{w} - R_{i,t+1}^{1}) + \varepsilon_{t+1}. \]
1.3.8 Interpreting the Bank of England’s Independence as a Parameter Change

In this section, I discuss my interpretation of the consequences of the Bank of England’s independence in May 1997. I interpret the policy change as an event that alters one or more of the following parameters in the monetary policy rule: the inflation gap coefficient, the output gap coefficient, and the level of trend inflation. In the following, I first give the relevant historical background and then explain why I interpret the policy change in this manner.

On May 6, 1997, the UK government announced that it would grant operational independence to the Bank of England. Since then, the newly created Monetary Policy Committee (MPC) has been responsible for independently setting the policy rate.

The purpose of granting independence was to increase the credibility of the Bank of England’s inflation targeting policy, which was adopted in October 1992 (McCafferty, 2013) but had not been regarded as fully credible because the Chancellor of the Exchequer was responsible for interest rate decisions, suggesting that short-term political pressures could influence decisions (Martijn and Samiei, 1999; Gürkaynak et al., 2010). Indeed, Lane and Samiei (1997) examine minutes of monetary policy meetings between the Chancellor of the Exchequer and the Governor of the Bank of England and document multiple occasions when the Chancellor either cut or kept the policy rate constant, contrary to the Governor’s suggestion to raise it.

The announcement was made four days after the Labour Party’s election win, and it was a surprise to most, if not all, people. Even a news report a few
hours before the announcement failed to mention anything about independence but instead mooted a likely increase in the policy rate by 25 basis points, which turned out to be true (Loftus, 1997). As discussions regarding the Bank of England’s independence were held before the announcement (Roll Committee, 1993), its content might have been expected. However, as Spiegel (1998) argues, the timing of the announcement was certainly unexpected.

Financial markets reacted strongly to the announcement. The 10-year break-even inflation rate dropped 28 basis points on the date. The reaction was huge, compared with the average daily change of the break-even inflation during this period: 0.4 basis points. Gürkaynak et al. (2010) and Wright (2011) document similar large declines in long-term nominal forward rates. Wright (2011) also documents little change in the real forward curve. These market reactions suggest that the announcement lowered inflation expectations and the term premium.

Given the surprising nature of the announcement, I interpret the policy change as sudden changes in some parameter(s) in the model. The object of and the actual reactions for the policy change suggest some structural changes in the monetary policy rule. I thus consider changes in one or more of the following parameters in the monetary policy rule: the inflation gap coefficient, the output gap coefficient, and the level of trend inflation. As shown in Figures 1.6, 1.7, and 1.8, these parameter changes are consistent with the immediate changes in the break-even inflation documented in the literature.

\[10\text{The average is from January through May 1997.}\]
1.4 Parameterization and the Solution Method

In this section I discuss parameterization and the solution method.

1.4.1 Parameterization

Table 1.1 lists the baseline parameter values. These values are in line with those used in the literature. The model is a quarterly model. I set $\beta = 0.995$ so that the steady-state annualized real interest rate is two percent. I set $\gamma = 75$. As shown by Swanson (2012), this means that the coefficient of relative risk aversion is $\alpha \times \gamma = 0.5 \times 75 = 37.5$. The Calvo parameter is $\theta = 0.75$ so that the average duration of a price is four quarters, which is broadly consistent with the empirical evidence reported by Nakamura and Steinsson (2008). The annual rate of trend inflation is two percent. The standard deviation of technology shocks is 0.005, while that of monetary policy shocks is 0.003.

1.4.2 Solution Method

I solve the model using a second-order perturbation method around its deterministic steady state, as in Schmitt-Grohé and Uribe (2004). A second-order approximation is necessary for the model to generate a non-zero risk premium.\(^{11}\) An advantage of using perturbation methods is their efficiency. Though the model has four state variables, it takes under 10 minutes to solve the model using a standard laptop.

Since perturbation methods are local approximations, there is a possibility

\(^{11}\)With a third- and higher-order approximation, the model generates a risk premium that depends on the state variables.
that accuracy may be sacrificed in exchange for computational efficiency. Caldana et al. (2012), however, find that the accuracy of perturbation methods is comparable with global methods, such as Chebyshev polynomials in the case of a real business cycle model with recursive preferences, whose structure is similar to the model presented in this paper. Nonetheless, I should point out that their results may not necessarily apply to the model in this paper, since the accuracy of local methods can, in principle, differ for each application.

In some simulations that I conduct below, I simulate a time-series path of the approximate solution. In that case, I simulate a path by pruning out the extraneous high-order terms, as in Kim et al. (2008).

1.5 Model Evaluation

In this section, I evaluate via simulation the new Keynesian model presented above. The criteria for evaluation is whether the model can account for the level of nominal and real bond betas, both before and after the Bank of England’s independence, as seen in Figure 1.1. In doing so, I simulate the model and compute the two betas for different values of the coefficients of inflation gap and output gap in the monetary policy rule, as well as the level of trend inflation. (As argued in Section 1.3.8, I interpret the Bank of England’s independence as the event that has changed one or more of these three parameters.) I present simulation results by plotting the model-implied betas against each parameter, as in Figure 1.4.
1.5.1 Bond and Equity Pricing Implications of the Model

Before turning to the model-implied betas, I briefly discuss other bond and equity pricing implications of the model. Table 1.2 lists several basic model-implied bond and equity pricing moments along with the corresponding empirical moments for UK data. The empirical moments are based on UK data from 1986 to 2010. The table shows that the model generates reasonably realistic bond pricing moments: the slope of the nominal yield curve, the mean excess returns to 10-year nominal bonds, and the standard deviation of excess returns to 10-year nominal bonds. The model, on the other hand, cannot match equity pricing moments. Although the mean UK equity premium is low during this period, the long-run equity premium is much larger. According to Dimson et al. (2008), the average UK equity premium for 1900 to 2005 is 5.29%, which is much larger than the model-implied 0.17%. Also, the model predicts a much smaller standard deviation for the equity premium.

1.5.2 Monetary Policy Rule and the Betas

Here I examine the role of the monetary policy rule on the nominal and real bond betas. Specifically, I examine how the level of the nominal and real betas varies for different values of the inflation gap coefficient, $\phi_\pi$, and the output gap coefficient, $\phi_y$, in the monetary policy rule. The analysis uncovers how the central bank’s reaction to the inflation gap and the output gap is related to the nominal and real bond betas.

**Inflation Gap Coefficient** Figure 1.3 shows how the level of the nominal and real bond betas changes as the inflation gap coefficient increases from 1.2
to 2.4. Changes in the inflation gap coefficient alter the dynamics of macro aggregates. Most importantly, the volatility of inflation decreases, whereas the volatility of output/consumption slightly increases.

There are three things to note in Figure 1.3. First, the level of the real bond beta is close to 1 for the entire range of the inflation gap coefficient considered here. This results from the fact that aggregate consumption does not vary so much across different states of the model economy, and thus the behavior of the equity price resembles the behavior of the real bond price. Although the volatility of consumption is slightly higher for larger values of the inflation gap coefficient, its quantitative effects are negligible, as can been in the figure.

Second, the level of the nominal bond beta is larger than 1, regardless of the value of the inflation gap coefficient. This is because a technology shock is the main shock in the model economy. When a positive technology shock hits the economy, inflation falls, because firms face lower marginal costs. Since a technology shock is persistent, a positive technology shock also lowers future inflation expectations. Lower (unexpected) future inflation in turn raises the nominal bond price. Thus, the nominal bond return changes more than the equity return, with the result that the nominal bond beta is larger than 1.

Third, the model predicts that the nominal bond beta becomes smaller, as the monetary authority reacts more aggressively to the short-run deviation of inflation from the target (i.e., as the inflation gap coefficient increases). A higher inflation gap coefficient reduces the volatility of inflation. In other words, inflation changes to a lesser extent in response to shocks as the monetary authority becomes more aggressive toward inflation development. Thus, the nominal bond price also varies less in response to shocks, leading to the lower level of
the nominal bond beta.

**Discrepancies between the model and the data** There are two discrepancies between the model and the data. The first discrepancy is the level of the nominal and real betas after the Bank of England gained its operational independence. While the betas are zero in the data, the model predicts much higher values for the betas. The zero betas in the data indicate that shocks that affect equity returns do not affect bond returns. In the model, however, the same set of shocks (technology shocks and monetary policy shocks) affect bond and equity returns, and so some co-movement between bond and equity returns, i.e., non-zero betas, is inevitable.

The second discrepancy relates to the ratio between the nominal and real bond betas. The model predicts that the level of the nominal bond beta is larger than the level of the real bond beta. In the data, however, there is no statistically significant difference between the levels of the two betas. In other words, the data suggest that the nominal bond and the real bond are essentially the same for their return association with the equity, suggesting that shocks to inflation affect nominal bond returns but not equity returns. The two economic shocks in the model, however, affect both consumption and inflation at the same time. Shocks to inflation, therefore, cannot affect only the nominal bond, and thus a difference always exists between the levels of the nominal and real beta bonds. In contrast to the nominal bond beta, the real bond beta varies little for different values of the output gap coefficient, again resulting from the fact that aggregate consumption does not vary much across different states of the economy even for a low value of the output gap coefficient.
Output gap coefficient The relationship between the output gap coefficient and the nominal and real betas can be understood exactly the same way as above, as long as it is recognized that there is a trade-off between inflation stabilization and output stabilization. As the monetary authority becomes more responsive to the output gap (i.e., as the output gap coefficient increases), the volatility of output decreases and the volatility of inflation increases. This leads to a larger response of inflation and thus a larger response of the nominal bond price to a technology shock, leading to a higher level of the nominal bond beta.

1.5.3 Trend Inflation and the Betas

Here I examine the role of trend inflation on the nominal and real bond betas. Specifically, I examine how the model-implied betas vary when trend inflation changes from zero to six percent.

Figure 1.5 displays the effects of trend inflation on the nominal and real bond betas. There are several things to note here. First, the real bond beta is about one for the range of trend inflation considered here, and the logic is the same as explained in the previous subsection. The only difference here is that the real bond beta decreases slightly when the level of trend inflation exceeds five percent. This occurs because the volatility of output increases non-linearly as trend inflation rises (Amano et al., 2007). As a result, the cash flow of the equity becomes more volatile, and the equity looks less similar to the real bond, which leads to a lower level of the real bond beta. However, its quantitative effects are not large as can be seen in the figure.

Second, the level of the nominal beta is higher for a higher level of trend
inflation. As just explained, higher volatility of output due to high trend inflation leads to a lower level of the real bond beta. This also applies to the nominal bond beta. However, higher trend inflation increases the volatility of inflation at a greater rate. The volatility of inflation is larger for higher trend inflation, because the dispersion of prices set by intermediate goods firms increases with the level of trend inflation (without indexation). As a result, the nominal bond price responds more to shocks under high trend inflation than under low trend inflation, leading to a higher level of the beta. As an illustration, Figure 1.11 displays dynamic responses of the 10-year nominal yield to a positive one-standard-deviation technology shock for different levels of trend inflation.

**Discrepancies between the model and the data** As in the case of the coefficients in the monetary policy rule, the model is unable to explain the dynamics of the nominal and real bond betas in Figure 1.1. As the analysis in this section shows, the model cannot explain the data with changes in trend inflation, and the same set of discrepancies remains. The first discrepancy is the level of the nominal bond beta before the Bank of England’s independence. The second discrepancy is the fact that both the nominal and real betas are zero after independence. The third discrepancy is that the ratio of the nominal and the real bond betas is 1 after independence. A detailed explanation of this is given in Section 1.5.2.
1.6 Sensitivity Analysis

The results reported above hold for a wide set of the parameter values. In this section, I report the sensitivity of the nominal and real bond betas to several parameters in the model.

1.6.1 Equity as a Claim to Levered Output

The equity can also be modeled as a claim to a levered output, as in Abel (1999). The ex-dividend price of the equity in this case is given by

\[ W_t = \sum_{j=1}^{\infty} E_t \left( M_{t,t+j} C_{t+j}^\lambda \right) \]

\[ = E_t \left( M_{t,t+1} (C_{t+1}^\lambda + W_{t+1}) \right) , \]

where \( \lambda \) is the degree to which an equity is levered with respect to wealth. The equity return is given by

\[ R_{t,t+1}^w \equiv \frac{C_{t+1}^\lambda + W_{t+1}}{W_t} . \]

A higher value of \( \lambda \) leads to a higher equity premium. For example, with \( \lambda = 3 \), the model-implied average excess equity return is 6.36% with the standard deviation of 4.04%. As shown in Figure 1.12, with the leverage parameter of eight, both the nominal and real betas are close to 0, as we see in the UK data since 1997. However, the typical leverage parameter used in the literature is between two and three, and the leverage parameter of eight implies too volatile dividend process.
1.6.2 Other Parameters

CRRA parameter \( \gamma \) for the beta Figure 1.9 shows how the nominal and real bond betas are related to the CRRA parameter \( \gamma \). The figure shows that high CRRA parameters do not change the levels of either the nominal or real bond betas. Since the CRRA parameter affects the price of the risk, the quantity of risk (betas) stays the same.

Elasticity of Intertemporal Substitution Figure 1.10 shows how the nominal and real bond betas are related to the elasticity of intertemporal substitution parameter. The figure shows that higher elasticity of substitution lowers the level of the nominal bond beta but barely affects the level of the real bond beta. The nominal beta is lower for higher EIS, because the volatility of inflation also becomes higher. The real bond beta, on the other hand, is not sensitive to the value of the EIS.

1.7 Conclusion

In this paper, I evaluate a New Keynesian model using the time-varying stock market betas of nominal and real government bonds. Using UK data, I document a sharp change in the beta of the 10-year nominal bond when the UK government gave operational independence to the Bank of England. Interpreting the Bank of England’s independence as an event that changed the parameters in the monetary policy rule, I examine whether a New Keynesian model with recursive preferences can account for the level of nominal and real bond betas both before and after independence. I find that the model generates much larger
betas than we find in the data. Also, the model has difficulty in generating the zero betas that we see in the data.
### Table 1.1: Baseline parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Subjective time discount factor</td>
<td>0.995</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>CRRA parameter</td>
<td>75</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Intertemporal elasticity of substitution</td>
<td>0.3</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Elasticity of substitutions among intermediate goods</td>
<td>10</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Calvo parameter</td>
<td>0.75</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Monetary policy inertia</td>
<td>0.7</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Coefficient of inflation gap</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Coefficient of output gap</td>
<td>0.1</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>AR(1) coefficient of a technology shock</td>
<td>0.95</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Standard error of a technology shock</td>
<td>0.005</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Standard error of a monetary policy shock</td>
<td>0.003</td>
</tr>
</tbody>
</table>

### Table 1.2: Empirical and model-implied asset pricing moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>UK data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean slope of the nominal curve</td>
<td>0.37</td>
<td>0.17</td>
</tr>
<tr>
<td>Mean excess return to 10-year nominal bonds</td>
<td>3.68</td>
<td>0.83</td>
</tr>
<tr>
<td>Std Dev of the excess return</td>
<td>10.05</td>
<td>10.90</td>
</tr>
<tr>
<td>Mean equity premium</td>
<td>0.61</td>
<td>0.17</td>
</tr>
<tr>
<td>Std Dev of equity premium</td>
<td>17.91</td>
<td>3.54</td>
</tr>
</tbody>
</table>

Note: The table lists the mean and the standard deviation of some bond and equity pricing moments from the data and the model. Empirical moments are based on UK data from 1986 to 2010. The unit is annualized return. Section 1.3 describes. Table 1.1 lists the parameter values.
Figure 1.1: Quarterly estimated stock market betas of UK government bonds

Note: The figure plots quarterly estimated stock market betas for 10-year nominal and real UK government bonds. The sample is 1986Q1 through 2010Q4. The first line indicates the timing of the introduction of inflation targeting (October 1992), and the second line indicates the timing of the independence of the Bank of England (May 1997).
Figure 1.2: UK 10-year break-even inflation rate

Note: The figure plots the UK 10-year break-even inflation rate: the difference between the yield of 10-year nominal gilt and the yield of 10-year inflation-indexed gilt.
Figure 1.3: Inflation gap coefficient and stock market betas of government bonds

Note: The figure plots the simulated stock market betas of the 10-year nominal and real government bonds for the inflation gap coefficient in the monetary policy rule from 1.2 to 2.4. The betas are simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I simulate a time series path of the approximated solution by pruning out the extraneous high-order terms as in Kim et al. (2008). Third, I estimate the betas by running regressions of either returns to nominal or real government bonds on the equity return. I repeat this exercise for different inflation gap coefficients of the monetary policy rule.
Figure 1.4: Output gap coefficient and stock market betas of government bonds

Note: The figure plots the simulated stock market betas of the 10-year nominal and real government bonds for the output gap coefficient in the monetary policy rule from 1.2 to 2.4. The betas are simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I simulate a time series path of the approximated solution by pruning out the extraneous high-order terms as in Kim et al. (2008). Third, I estimate the betas by running regressions of either returns to nominal or real government bonds on the equity return. I repeat this exercise for different inflation gap coefficients of the monetary policy rule.
Figure 1.5: Trend inflation and stock market betas of government bonds

Note: The figure plots the simulated stock market betas of the 10-year nominal and real government bonds for annual rate of trend inflation from 0 to 6 percent. The betas are simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I simulate a time series path of the approximated solution by pruning out the extraneous high-order terms as in Kim et al. (2008). Third, I estimate the betas by running regressions of either returns to nominal or real government bonds on the equity return. I repeat this exercise for different annual rate of trend inflation.
Figure 1.6: Trend inflation and the 10-year break-even inflation

Note: The figure plots the simulated 10-year break-even inflation for the annual rate of trend inflation from 0 to 6 percent. The break-even inflation is simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I compute the model-implied average 10-year nominal and real yield. The break-even inflation is the difference between the nominal and the real yield.
Figure 1.7: Inflation gap coefficient and the 10-year break-even inflation

Note: The figure plots the simulated 10-year break-even inflation for the inflation gap coefficient in Taylor rule from 1.2 to 2.4. The break-even inflation is simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I compute the model-implied average 10-year nominal and real yield. The break-even inflation is the difference between the nominal and the real yield.
Figure 1.8: Output gap coefficient and the 10-year break-even inflation

Note: The figure plots the simulated 10-year break-even inflation for the output gap coefficient in Taylor rule from 0.1 to 0.5. The break-even inflation is simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I compute the model-implied average 10-year nominal and real yield. The break-even inflation is the difference between the nominal and the real yield.
The figure plots the simulated stock market betas of the 10-year nominal and real government bonds for the CRRA parameter from 5 to 125. The betas are simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I simulate a time series path of the approximated solution by pruning out the extraneous high-order terms as in Kim et al. (2008). Third, I estimate the betas by running regressions of either returns to nominal or real government bonds on the equity return. I repeat this exercise for different CRRA parameters.
Figure 1.10: EIS and stock market betas of government bonds

Note: The figure plots the simulated stock market betas of the 10-year nominal and real government bonds for the elasticity of intertemporal substitution parameter from 0.3 to 1.5. The betas are simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I simulate a time series path of the approximated solution by pruning out the extraneous high-order terms as in Kim et al. (2008). Third, I estimate the betas by running regressions of either returns to nominal or real government bonds on the equity return. I repeat this exercise for different CRRA parameters.
Figure 1.11: Dynamic response of 10-year nominal yield to technology shock

Note: The figure plots the dynamic response of 10-year nominal yield to positive one standard deviation technology shock for different rates of trend inflation. The vertical axis is changes in the nominal yield in the annualized percentage point.
The figure plots the simulated stock market betas of the 10-year nominal and real government bonds for the leverage parameter $\lambda$ from 1 to 8 as well as the corresponding equity premium. The betas and the equity premium are simulated as follows. First, the model (see Section 3) is solved by a second-order perturbation method. Second, I simulate a time series path of the approximated solution by pruning out the extraneous high-order terms as in Kim et al. (2008). Third, I estimate the betas by running regressions of either returns to nominal or real government bonds on the equity return. I compute the average equity return in excess of one-period risk-free rate. I repeat this exercise for different leverage parameters.
Chapter 2

Trend Inflation and the Term Structure of Interest Rates

2.1 Introduction

This paper studies the effects of trend inflation on the term structure of interest rates. I examine the effects on a number of aspects of the yield curve: the steady state, the mean, the variance, impulse responses to economic shocks, and risk compensation. I consider three different levels of trend inflation 2, 4, and 6 percent.

I find that higher rate of trend inflation leads to more volatile inflation, which in turn increases the volatility of bond prices. The volatility of bond prices increase regardless of the maturity of the bond. In addition, impulse response of bond yields are larger for higher trend inflation. However, I also find that the quantitative effects of these findings are quite small when we assume the standard log utility of the representative household.

The quantitative effects of positive trend inflation become significant when we assume the household with Epstein-Zin preferences. For example, the average slope of the yield curve is 1.02 percent for 2 percent inflation, while it
is 1.35 percent for 6 percent trend inflation. Excess returns to 10-year bond are 5.6 percent for 2 percent trend inflation and 6.7 percent for 6 percent trend inflation.

The main conclusion of the paper is that the model implies that higher trend inflation leads to more volatile inflation and bond yields. Higher trend inflation leads to more risk premium of long term bonds. Many central banks in the world have been struggling with the zero lower bound problem after the financial crisis of 2007-2008.¹ In response to this, some economics, notably Blanchard et al. (2010) and Ball (2013), argue that central banks should target higher rate of inflation. More risk premium in the long-term bond means more room to be hit by the zero lower bound problem. However, my research shows that the benefits come at the costs of more volatile inflation as well as bond yields.

Related research Previous studies find that trend inflation alters the dynamics of macro aggregates in the model and thus the properties of the model.² Ascari (2004) studies the effects on the deterministic steady state as well as the impulse responses of macroeconomic variables to economic shocks. Amano et al. (2007) study the implications on the means and variances of macroeconomic variables. Ascari and Ropele (2007) study the implications on the optimal monetary policy. Nakata (2013) studies the welfare costs of time-varying trend inflation. Kurozumi (2014) examines the implications on expectational stability of rational expectations equilibrium.

This paper proceeds as follows. Section 2 presents the model. Section 3

¹An exception is the Bank of Japan which started facing the zero lower bound problem in 1990s.
²See Ascari and Sbordone (2013) for a review of the literature on trend inflation.
discusses the solution method and the parameterization. Section 4 presents the results. Section 5 concludes.

2.2 The Model

The model is an otherwise simple New Keynesian model except that the steady state level of inflation (henceforth, trend inflation) can be different from zero.\footnote{See, for example, Section 2 of Christiano et al. (2010) for the simple New Keynesian model without non-zero trend inflation.}

The model consists of a representative household, goods-producing firms, and a monetary authority. Bond prices satisfy the standard asset pricing equation.

2.2.1 Households

A representative household chooses consumption, labor supply, and bond holdings to maximize the lifetime utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right),
\]

subject to the budget constraint for each period

\[
P_tC_t + B_{t+1} \leq W_t N_t + R_{t-1,t} B_t + T_t.
\]

Here \(C_t\) is consumption, \(N_t\) is labor supply, \(P_t\) is the final goods’ price (the aggregate price level), \(B_{t+1}\) is the dollar value of one-period nominal bonds held by the household at the end of time \(t\), \(W_t\) is the nominal wage, \(R_{t-1,t}\) is the gross nominal interest rate, and \(T_t\) is nominal profits from intermediate goods firms. Besides the budget constraint, the household’s optimal decision satisfies the solvency condition: \(\lim_{T \to \infty} E_t B_T \geq 0\) for all \(t\).
The first-order conditions give us the optimal consumption-labor decision rule

\[ C_t N_t^p = \frac{W_t}{P_t}, \quad (2.1) \]

and the asset pricing equation

\[ 1 = E_t (M_{t,t+1} R_{t,t+1}), \quad (2.2) \]

where \( M_{t,t+1} \) denotes the nominal stochastic discount factor (SDF)

\[ M_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}}. \]

### 2.2.2 Final Goods Firm

Competitive firms combine intermediate goods to produce the final good, \( Y_t \), using the CES (constant elasticity of substitution) technology

\[ Y_t = \left( \int_0^1 Y_{i,t} \frac{\epsilon+1}{\epsilon} \, di \right)^{\frac{1}{\epsilon}}, \quad \epsilon > 1, \quad (2.3) \]

where \( Y_{i,t}, i \in [0, 1] \) denotes the intermediate good \( i \), and \( \epsilon \) denotes the elasticity of substitution across intermediate goods. Profit maximization leads to the following demand function for intermediate goods

\[ Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t, \quad (2.4) \]

where \( P_{i,t} \) denotes the price of the intermediate goods \( i \). Substitute (2.4) into (2.3) to express the aggregate price in terms of the prices of intermediate goods:

\[ P_t = \left( \int_0^1 P_{i,t}^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}. \quad (2.5) \]
2.2.3 Intermediate Goods Firms

There is a continuum of monopolistically competitive intermediate goods firms indexed by \(i \in [0, 1]\). Firm \(i\) produces the good using the following linear technology

\[
Y_{i,t} = A_t N_{i,t},
\]

where \(A_t\) denotes the level of aggregate technology which evolves according to the following stationary process:

\[
\log A_t = \rho_a \log A_{t-1} + \sigma_a \varepsilon_{a,t},
\]

where \(\varepsilon_{a,t}\) is an iid technology shock with mean zero and standard deviation \(\sigma_a\). Firm \(i\) supplies its product to meet the demand according to (2.4) and sets the price as in Calvo (1983):

\[
P_{i,t} = \begin{cases} 
P_{i,t-1} & \text{with probability } \theta \\
\tilde{P}_t & \text{with probability } 1 - \theta,
\end{cases}
\]

where \(\tilde{P}_t\) denotes the reset price under which the firm maximizes the discounted expected future profits:

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j M_{t,t+j} \left( \tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right).
\]

\(\theta^j\) is the probability that the reset price \(\tilde{P}_t\) remains effective in time \(t + j\), \(s_t\) is the real marginal cost,\(^4\) and \(M_{t,t+j}\) is the product of the nominal SDFs:

\[
M_{t,t+j} \equiv M_{t,t+1} \times \cdots \times M_{t+j-1,t+j}
\equiv \beta^j \frac{C_t}{C_{t+j}} \frac{P_t}{P_{t+j}}.
\]

\(^4\)Since Firm \(i\)'s real total cost is given by \((W_t Y_{i,t})/(P_t A_i)\), the real marginal cost is given by \(s_t \equiv W_t/(P_t A_t)\), which is common across intermediate firms.
Profit maximization leads to the following expression of the relative reset price, 
\[ \tilde{p}_t \equiv \tilde{P}_t / P_t: \]
\[ \tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta \theta)^j X_{t,j} \frac{\varepsilon}{\varepsilon - 1} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \theta)^j X_{1,t,j}^{1-\varepsilon}} \equiv \frac{K_t}{F_t}. \]  
(2.6)

Both the numerator \( K_t \) and the denominator \( F_t \) have a recursive structure:
\[ K_t = \frac{C_t N_t^\varepsilon}{A_t} + \beta \theta E_t \Pi_{t+1}^{\varepsilon} K_{t+1} \]  
(2.7)
\[ F_t = 1 + \beta \theta (E_t \Pi_{t+1}^{\varepsilon})^{1-\varepsilon} F_{t+1}. \]  
(2.8)

Here \( \Pi_{t+1} \) is the gross rate of inflation between dates \( t \) and \( t + 1 \).

Since the fraction \( 1 - \theta \) of intermediate firms reset their prices, the price index (2.5) can be rewritten as follows
\[ P_t = \left( (1 - \theta) \tilde{P}_t^{1-\varepsilon} + \theta P_t^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \]

Dividing both sides by the price level \( P_t \) and then solving for the relative reset price \( \tilde{p}_t \) yields
\[ \tilde{p}_t = \left( \frac{1 - \theta \Pi_{t+1}^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}}. \]  
(2.9)

Yun (1996) shows the following relationship between the aggregate output and the aggregate inputs:
\[ Y_t = \left( \frac{P_t^*}{P_t} \right)^{\varepsilon} A_t N_t, \]  
(2.10)
where \( P_t^* \) is the alternative price index defined by
\[ P_t^* = \left( \int_0^1 P_{t,-\varepsilon} d\tilde{t} \right)^{-\frac{1}{\varepsilon}}, \]
which can be rewritten as
\[ p_t^* \left( (1 - \theta) \frac{\tilde{P}_t^{1-\varepsilon}}{1 - \theta} + \theta \frac{\Pi_t^{\varepsilon}}{P_t^{1-\varepsilon}} \right) = 1. \]  
(2.11)
2.2.4 The Monetary Authority

The monetary authority sets the one-period nominal interest rate, \( R_{t,t+1} \), according to the Taylor rule

\[
\frac{R_{t,t+1}}{R^*} = \left( \frac{R_{t-1,t}}{R^*} \right)^{\phi_r} \left( \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_y} \left( \frac{Y_t}{Y^*} \right)^{\phi_y} \right)^{1-\phi_r} \times \exp(\sigma_m \varepsilon_{m,t}) \tag{2.12}
\]

where \( \sigma_m \varepsilon_{m,t} \) is an iid monetary policy shock with mean zero and standard deviation \( \sigma_m \). \( R^* \) and \( Y^* \) denote the steady-state levels of nominal interest rate and output, when the steady-state level of gross inflation is \( \Pi^* \).

2.2.5 Equilibrium

The economy’s resource constraint is

\[ Y_t = C_t. \tag{2.13} \]

A symmetric equilibrium is a sequence of \((C_t, N_t, W_t, Y_t, P_t, R_{t,t+1}, P_{t,t}, Y_{i,t}, N_{i,t}, \Pi_t)_{t=0}^{\infty}\) such that household and firm problems are solved, all markets clear, the policy rate is set according to the monetary policy rule, and the resource constraint is satisfied. The equilibrium conditions are (2.1), (2.2), (2.6), (2.7), (2.8), (2.9), (2.11), (2.10), (2.12), and (2.13).

2.2.6 Bond prices, yields, and returns

Bond prices satisfy the asset pricing equation (2.2) which is reproduced here in a slightly different form:

\[ P_t = E_t \left( M_{t,t+1} x_{t+1} \right), \]

where \( P_t \) is the price of the asset, \( M_{t,t+1} \) is the SDF, and \( x_{t+1} \) is the payoff of the asset. The price of a default-free \( n \)-period zero-coupon bond that pays a
dollar at maturity is then given by

\[ P_t^{(n)} = E_t \left( M_{t,t+1} P_{t+1}^{(n-1)} \right). \]

The corresponding continuously compounded yield to maturity is given by

\[ y_t^{(n)} = -\frac{1}{n} \log P_t^{(n)}. \]

Note that \( y_t^{(1)} = -\log R_{t,t+1}. \)

The risk-neutral bond price is defined as the price discounted by the current and expected future short rates as in previous studies in the literature: \(^6\)

\[ \hat{P}_t^{(n)} \equiv \exp \left( -y_t^{(1)} \right) E_t \hat{P}_{t+1}^{(n-1)} \]

\[ = P_t^{(1)} E_t \hat{P}_{t+1}^{(n-1)}. \]

The corresponding continuously compounded risk-neutral yield is given by

\[ \hat{y}_t^{(n)} \equiv -\frac{1}{n} \log \hat{P}_t^{(n)}. \]

In studying how the level of trend inflation affects the term structure, I examine the implications on three measures of risk compensation embedded in bond prices: the slope of the yield curve and the excess returns on long-term bonds. The slope of the yield curve is given by

\[ y_t^{(40)} - y_t^{(1)}. \]

The excess return to holding the long-term bond for one period is given by

\[ ER_t^{(n)} \equiv \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} - y_t^{(1)}. \]

In reporting yields and returns, I multiply by 400 to make it in units of annualized percentage points.

\(^6\)For example, see Rudebusch and Swanson (2012).
2.3 Solution method and parametrization

I solve the model using a second-order perturbation method around its deterministic steady state as in Schmitt-Grohé and Uribe (2004).\textsuperscript{7} A second-order approximation is necessary for the model to generate a non-zero risk premium under the current setting. Although perturbation methods are local approximations, Caldara et al. (2012) find that the accuracy of perturbation methods is comparable with several global methods such as Chebyshev polynomials in the case of a real business cycle model with recursive preferences, whose structure is quite similar to the present model.

Table 2.2 lists the baseline parameter values which are in line with those used in the literature. The model is a quarterly model.

2.4 Results

2.4.1 Results for household with log utility

This subsection summarizes the results for household with log utility. The main conclusion of this part is that while trend inflation affects the dynamics of macro variables as well as bond prices, its quantitative effects are fairly small. Table 2.1 lists the steady state values, means, and standard deviations of consumption, inflation, policy rate, 5-year rate, and 10-year for three different levels of trend inflation: 2, 4, and 6 percent per year. There are four observations from the table worth emphasizing. First, the steady state value of consumption goes down as trend inflation increases. This is because the price dispersion,

\textsuperscript{7}See Table 2.1 for the detail about the deterministic steady state.
the source of inefficiency in the model, is higher for higher level of trend inflation (Ascari, 2004). Second, the volatility of inflation is larger for higher trend inflation. As the level of trend inflation increases, the dispersion of prices increase, leading to more volatile inflation. Third, the model-implied yield curve is hump-shaped. The average 5-year slope is 0.01 percent for 2 percent trend inflation and 0.02 percent for 6 percent trend inflation. The average 10-year slope is $-0.021$ percent for 2 percent trend inflation and $-0.025$ percent for 6 percent trend inflation. Fourth, while the level of trend inflation does affect both macroeconomic variables and bond prices, its quantitative effects are fairly small for the model presented above.

Figure 3.5 delivers the same message from a different angle. The figure shows the impulse responses to positive technology shock for consumption, inflation, policy rate, 5-year rate, and 10-year rate. For all the variables, the figure shows that impulse responses are larger for higher trend inflation. For example, an increase in consumption is larger for the case of 6% trend inflation than 4% trend inflation. However, the quantitative differences in impulse responses are fairly small.

### 2.4.2 Results for household with Epstein-Zin preferences

In this section, I study how the effects of trend inflation changes when the representative household has Epstein-Zin preferences (Epstein and Zin, 1989). I focus on the effects on asset prices, as Epstein-Zin preferences does not alter the first-order dynamics of macroeconomic variables in the model (Rudebusch and Swanson, 2012).

Instead of the log utility, the representative household now has the following
form of lifetime utility

\[ U_t = \left( u_t^{1-1/EIS} + \beta (CE_{t+1})^{1-1/EIS}\right)^{(1-1/EIS)^{-1}} \]

where \( u_t \equiv C_t^\alpha (1 - N_t)^{1-\alpha} \) denotes the current utility flow that consists of consumption \( C_t \) and leisure \( 1 - N_t \). \( CE_{t+1} \) denotes the certainty equivalent of expected future lifetime utility:

\[ CE_{t+1} \equiv (E_t U_{t+1}^{1-\gamma})(1-\gamma)^{-1} \]

where \( \gamma \) denotes the coefficient of relative risk aversion.\(^8\) \( EIS \) denotes the elasticity of intertemporal substitution. The lifetime utility consists of the current utility flow and the certainty equivalent of the expected future lifetime utility, aggregated by the CES function. If \( \gamma = 1/EIS \), the preferences reduces to the standard Constant Relative Risk Aversion (CRRA) preferences. The nominal SDF is now give by

\[ M_{t,t+1} \equiv \beta \left( \frac{u_{t+1}}{u_t} \right)^{1-\gamma} \frac{C_t}{C_{t+1}} \left( \frac{V_{t,t+1}^{1-\gamma}}{E_t V_{t+1}^{1-\gamma}} \right)^{1-\delta} \frac{P_t}{P_{t+1}}. \]

Table 2.4 summarizes the results and lists the means and standard deviations of policy rate, 5-year rate, and 10-year rate for 2 percent, 4 percent, and 6 percent trend inflation. There are \( x \) observations from the table worth emphasizing. First, all rates become more volatile for higher rate of trend inflation. For example, the standard deviation of 10-year rate is 1.5 percent for 2 percent trend inflation and 2.2 percent for 6 percent trend inflation. Second, the average slope of the yield curve is higher for higher level of trend inflation. The

\(^8\)The specification here follows van Binsbergen et al. (2012) and Gourio (2012). As shown by Swanson (2012), \( \gamma \) is not in general equal to the coefficient of relative risk aversion, when the household can vary its labor supply in response to shocks. In the current specification, however, \( \gamma \) happens to be equal to the coefficient of relative risk aversion. See Example 2 in Section 3.3 of Swanson (2013).
average slope is 1.02 percent for 2 percent trend inflation and 1.35 percent for 6 percent trend inflation. Third, the average excess returns of 10-year bond are also higher for higher rate of trend inflation. The average excess returns are 5.6 percent for 2 percent trend inflation, while the excess returns are 6.7 percent for 6 percent trend inflation (see Figure 2.2).

2.5 Conclusion

This paper studies the effects of trend inflation on the term structure of interest rates. I find that trend inflation increases the volatility of inflation as well as that of bond yields. Higher trend inflation also increases the risk premium. However, the quantitative effects are not significant unless we assume the household with Epstein-Zin preferences.
Variable Description Value
---
$C$ Consumption $p^* N$ 
$N$ Labor supply $\left(\frac{1-\theta(\Pi^*)^{\psi}}{\rho}p^*\right)^{1/\psi}$ 
$R$ Gross nominal interest rate $\beta^{-1}\Pi^*$ 
$p^*$ Measure of price dispersion; see Equation (2.10) $\frac{1-\theta(\Pi^*)^{\psi}}{1-\theta(\Pi^*)^{\psi-1}}$ 
$K$ See Equation (2.6) $\tilde{p} F$ 
$F$ See Equation (2.6) $N^{\varphi-1}/p^*$ 
$W/P$ Real wage $\left(\frac{1-\theta(\Pi^*)^{\psi-1}}{1-\theta}\right)^{1/\varphi}$ 
$\tilde{p}$ Relative reset price

Table 2.1: Deterministic steady state

Parameter Description Value
---
$\beta$ Subjective time discount factor 0.998 
$\varphi$ Frisch elasticity of labor supply 10 
$\varepsilon$ Elasticity of substitutions among intermediate goods 1.5 
$\theta$ Calvo parameter 0.75 
$\rho_r$ Monetary policy inertia 0.8 
$\phi_\pi$ Coefficient of inflation gap 1.8 
$\phi_y$ Coefficient of output gap 0.1 
$\rho_a$ AR(1) coefficient of a technology shock 0.9 
$\sigma_a$ Standard error of a technology shock 0.01 
$\sigma_m$ Standard error of a monetary policy shock 0.005 
$\Pi^*$ Steady state level of trend inflation $1.04^{0.25}$

Table 2.2: Baseline parameter values
<table>
<thead>
<tr>
<th>S.S.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>S.S.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>S.S.</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
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<tr>
<td>2 percent annual inflation</td>
<td>4 percent inflation</td>
<td>6 percent inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.000</td>
<td>0.998</td>
<td>0.032</td>
<td>0.999</td>
<td>0.997</td>
<td>0.033</td>
<td>0.998</td>
<td>0.995</td>
</tr>
<tr>
<td>Inflation</td>
<td>2.000</td>
<td>2.305</td>
<td>2.664</td>
<td>4.000</td>
<td>4.343</td>
<td>2.703</td>
<td>6.000</td>
<td>6.388</td>
</tr>
<tr>
<td>Policy rate</td>
<td>2.820</td>
<td>3.028</td>
<td>1.471</td>
<td>4.836</td>
<td>5.080</td>
<td>1.504</td>
<td>6.852</td>
<td>7.139</td>
</tr>
<tr>
<td>5-year rate</td>
<td>2.820</td>
<td>3.028</td>
<td>1.320</td>
<td>4.836</td>
<td>5.080</td>
<td>1.352</td>
<td>6.852</td>
<td>7.139</td>
</tr>
<tr>
<td>10-year rate</td>
<td>2.820</td>
<td>2.472</td>
<td>0.640</td>
<td>4.836</td>
<td>4.90</td>
<td>0.657</td>
<td>6.852</td>
<td>6.510</td>
</tr>
</tbody>
</table>

Table 2.3: Steady states, means, and variances (Log utility)

Note: The table lists steady states, means, and standard deviations of consumption, inflation, and bond yields for various trend inflation. Inflation and bond yields are expressed annualized percentage points.
Table 2.4: Means, and variances (Epstein-Zin)

Note: The table lists means and standard deviations of various bond yields for three different levels of trend inflation. Bond yields are expressed annualized percentage points.
Figure 2.1: Impulse responses to positive technology shock

Note: The table lists steady states, means, and standard deviations of consumption, inflation, and bond yields for various trend inflation. Inflation and bond yields are expressed annualized percentage points.
Figure 2.2: Average excess returns to 10-year bond (annualized percentage points) in the case of Epstein-Zin preferences.
Chapter 3

FDI and the Welfare Gains from Financial Integration

3.1 Introduction

Foreign direct investment (FDI) flows account for more than 40 percent of foreign capital going into developing countries. Together with debt flows, the two types of financial flows constitute almost all the foreign capital that developing countries accept.\(^1\) The benefits and costs of FDI and debt flows have naturally become an active research topic in international macroeconomics.\(^2\) This paper aims to contribute to this literature by studying the welfare consequences of foreign capital flows, in particular FDI flows.

Gourinchas and Jeanne (2006) study the welfare effects of liberalizing debt flows using the Ramsey-Cass-Koopmans growth model and find that the liberalization increases permanent consumption by only one percent.\(^3\) When a country borrows from abroad, it can accumulate capital more rapidly and shorten the time to reach the steady state than otherwise, resulting in higher economic

\(^1\)See Table 1 in Kose et al. (2009).
\(^2\)See, for example, Kose et al. (2009) and papers cited therein.
\(^3\)Ramsey (1928), Cass (1965), Koopmans (1965).
growth and welfare. However, the growth effects are short-lived and small, because the liberalization of debt flows does not change the steady state itself, and the benefits disappear in the long-run.

Given the results about debt flows, this paper studies the welfare gains from liberalizing FDI flows using an extended Ramsey growth model. The key assumption which is grounded in empirical studies is that FDI flows bring more advanced technology to a host country, which enables workers employed by foreign firms earn more than those employed by domestic firms. This paper then studies additional welfare gains from liberalizing debt flows as well as FDI flows to assess the relative importance of FDI flows and to evaluate the overall benefits from financial integration (i.e., liberalizing all kinds of foreign capital flows). This paper also conducts a number of sensitivity analysis given the large differences among developing countries and a few notable changes in the world economic environment. For example, I examine the consequences of a decline in the world real interest rate, the recent phenomena that have occurred in many parts of the world.

There are three main conclusions from the analysis. First, liberalizing FDI flows can increase permanent consumption substantially. Under the baseline parameter values, the liberalization increases permanent consumption more than four percent. On the other hand, liberalizing debt flows under the same specification only increases permanent consumption by half a percent. That is, this paper highlights the relative importance of FDI flows over debt flows. Second, the welfare gains from financial integration (liberalizing all kinds of foreign capital) can be substantial, although further gains from liberalizing debts flows in addition to FDI flows are limited. Under the baseline specifications, 80 percent
of gains come from the liberalization of FDI flows. Third, a decline in the world real interest rate increases the welfare gains from financial integration, which suggests that the recent change in the world economic environment actually have increased the value of financial integration.

This paper is most closely related to Gourinchas and Jeanne (2006) as discussed above. This paper is also closely related to Hoxha et al. (2013). They study welfare gains from financial integration using the otherwise standard neoclassical growth model in which capital varieties exist as in Romer (1990). With a value of the elasticity of substitution among capital that is consistent with empirical literature, they find that financial integration can increase permanent consumption by more than nine percent.

This paper is also related to a large empirical literature on financial globalization and in particular on FDI. Kose et al. (2009) provide a comprehensive review of the literature on financial globalization. Alfaro and Johnson (2012), Javorcik (2012), and Kalemli-Ozcan and Villegas-Sanchez (2012) review the literature on the effects of FDI flows. Morarn (2006) provides numerous case studies of foreign direct investments conducted in developing countries.

This paper proceeds as follows. Section 2 presents the model. Section 3 discusses calibration. Section 4 presents the main results. Section 5 presents the results of various sensitivity analysis. Section 6 concludes.
3.2 The models of financial integration

I use a variant of the small open economy version of the Ramsey-Cass-Koopmans growth model to study the welfare gains from various kinds of financial integration. In the following subsections, I consider four different cases of financial integration (including financial autarky) and present the corresponding four models. First, I present the model of financial autarky. Second, I present the model for the first case of financial integration where the economy accepts only debt flows from abroad. Third, I present the model for the second case of financial integration where the economy accepts only foreign direct investment (FDI) flows. Finally, I present the model for the final case of financial integration where the economy accepts both debt and FDI flows. Besides describing the models, I discuss how I obtain the sequence of optimal consumption to compute the lifetime utility.

3.2.1 Financial autarky

In this subsection, I consider the case of financial autarky (i.e., the closed economy) which serves as the benchmark for assessing the welfare gains from financial integration. The economy is populated by a representative household. Time is discrete \((t = 0, 1, 2, \ldots)\), and there is no uncertainty. The representative household has CRRA preferences and maximizes the lifetime utility

\[
\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1 - \gamma} \quad (3.1)
\]
subject to the resource constraint

\[ C_t + I_t = Y_t \]

\[ \Rightarrow C_t + K_{t+1} - (1 - \delta)K_t = K_t^\alpha (A_t L_t)^{1-\alpha}, \]

given the initial level of capital \( K_0 > 0. \) Here \( C_t \) is consumption, \( I_t \) is investment, \( L_t \) is labor supply, \( Y_t \) is output, and \( A_t \) is the level of technology which grows at rate \( g \equiv A_{t+1}/A_t, \) i.e., \( A_t = A_0 g^t. \) \( \delta \) is the rate of capital depreciation.

The size of population (labor supply) is fixed at 1, i.e., \( L_t = 1 \) for all periods. The household makes consumption-saving decision such that the following Euler equation holds:

\[ C_t^{-\gamma} = \beta R_{t+1} C_{t+1}^{-\gamma} \]

\[ \Rightarrow C_t^{-\gamma} = \beta \left( \alpha K_{t+1}^{\alpha-1} A_{t+1}^{1-\alpha} + 1 - \delta \right) C_{t+1}^{-\gamma}, \]

where \( R_{t+1} \left( = \alpha K_{t+1}^{\alpha-1} A_{t+1}^{1-\alpha} + 1 - \delta \right) \) is the gross return on investment between time \( t \) and \( t+1. \) Since the aggregate consumption grows at rate \( g \) in the steady state, the Euler equation implies that the gross interest rate in the steady state is given by

\[ R^* = \beta^{-1} g^\gamma. \]  

(3.2)

Following Gourinchas and Jeanne (2006), I call this interest rate the natural gross rate of interest.

Let \( c_t \equiv C_t/A_t L_t = C_t/A_t \) denote the consumption per efficiency unit of labor and \( k_t \equiv K_t/A_t \) denote the capital per efficiency unit of labor. The Euler
equation can then be written as

\[
\frac{c_{t+1}}{c_t} = \beta^{\frac{1}{\gamma}} \left( \alpha k_{t+1}^{\alpha-1} + 1 - \delta \right)^{\frac{1}{\gamma}} / g.
\]

Since consumption per efficiency unit of labor \(c_t\) remains constant in the steady state, we can derive the steady state level of capital per efficiency unit of labor from the Euler equation:

\[
k^* = \left( \frac{\alpha}{\beta^{-1}g^\gamma - 1 + \delta} \right)^{1/\alpha} = \left( \frac{\alpha}{R^* - 1 + \delta} \right)^{1/\alpha}.
\] (3.3)

Using the resource constraint, we can express the steady state level of consumption per efficiency unit of labor in terms of capital per efficiency unit of labor:

\[
c^* = (k^*)^\alpha + (1 - \delta - g)k^*.
\]

Given \(k_0 < k^*\), the economy converges to \((c^*, k^*)\) over time by accumulating capital. To compute the actual optimal consumption path, I rely on a numerical method which I describe in the appendix.

### 3.2.2 Financial integration: open to debt flows

In this subsection, I consider a case of financial integration under which the economy accepts only debt flows from abroad. Gourinchas and Jeanne (2006) examine this form of financial integration. The economy can borrow at the world interest rate, and the rest of the world is composed of developed countries that have already achieved their steady states, meaning that the world interest rate is equal to the natural gross rate of interest \(R^*\) defined in Equation (3.2).

In this economy, the representative household chooses the sequence of consumption, capital stock, and net foreign asset \(\{C_t, K_{t+1}, B_{t+1}\}\) to maximize the
lifetime utility (3.1) subject to the resource constraint

\[ B_{t+1} + C_t + I_t = Y_t + R^* B_t \]

\[ \Rightarrow B_{t+1} + C_t + K_{t+1} - (1 - \delta)K_t = K_t^{\alpha} A_t^{1-\alpha} + R^* B_t, \]

given the initial level of capital \( K_0 > 0 \) and net foreign asset \( B_0 = 0 \).

In this form of financial integration, the level of capital per efficiency unit of labor jumps to its steady state level in the initial period, because the representative household has an incentive to borrow money from abroad and accumulate capital until the condition \( R^* = \alpha K_{t+1}^{\alpha-1} A_{t+1}^{1-\alpha} + 1 - \delta \) holds so that the Euler equation holds. Thus, the benefit of this form of financial integration is the acceleration of the time to arrive at the steady state, which involves temporary acceleration of economic growth and hence lead to higher lifetime utility.

Since the capital stock immediately reaches to the steady state level and consumption grows at rate \( g \) for all periods, we can obtain a closed form expression for the initial level of aggregate consumption:

\[ C_0 = (R^* - g)K_0 + (1 - \alpha)(k^*)^\alpha A_0. \] (3.4)

The appendix shows the derivation.

### 3.2.3 Financial integration: open to FDI flows

In this subsection, I consider a case of financial integration under which the economy accepts only FDI flows. A key assumption is that FDI flows into a host country with superior technology developed in advanced countries. To

\(^5\)If the country borrows on net, \( B_t \) is negative.

\(^6\)We also assume the transversality condition: \( \lim_{T \to \infty} R^{*\to T} B_{T+1} = 0 \).
represent this point in the model, I distinguish between domestic and foreign capital which operate with the corresponding technology. The total capital in the economy is thus the sum of domestic capital and foreign capital:

\[ K_t \equiv K^h_t + K^f_t. \]

Foreigners own foreign capital and receive the rental price of capital. Both domestic and foreign capital depreciates at the same rate \( \delta \):

\[ K^h_{t+1} = (1 - \delta)K^h_t + I^h_t \]
\[ K^f_{t+1} = (1 - \delta)K^f_t + I^f_t. \]

\( I^f_t \) is interpreted as foreign direct investments. The total labor in the economy is also divided into foreign compatible labor and domestic labor:

\[ L_t \equiv L^h_t + L^f_t. \]

The foreign compatible labor is more skilled than domestic labor and can work for both foreign and domestic companies. An example of foreign compatible labor is workers in South Africa who work at assembly plants of automobile makes such as Volkswagen or BMW (Morarn, 2006). The foreign compatible labor is scarce and accounts for a fraction \( \theta \) of the total labor:

\[ L^f_t \equiv \theta L_t, \quad 0 < \theta < 1. \]

The aggregate output is the sum of the output from domestic and foreign sectors:

\[ Y_t \equiv Y^h_t + Y^f_t = (K^h_t)^\alpha \left( A^h_t L^h_t \right)^{1-\alpha} + \left( K^f_t \right)^\alpha \left( A^f_t L^f_t \right)^{1-\alpha}. \]

Both the domestic technology and the foreign technology grow at the same rate \( g \) with the initial condition \( A^h_0 > A^f_0 > 0 \). That is, the level is the only
difference between the two kinds of technology. If I instead assumed that the foreign technology grows at a faster rate than the domestic technology, the ratio between the two technologies, $A^f_t/A^h_t$, would go to infinity over time, which is counterfactual.

When a country accepts FDI flows, the foreign compatible workers work for the foreign-owned company, because they can get higher wages from the foreign company than from the domestic company. When a country accepts FDI flows, the foreign compatible workers work for the foreign-owned company, because they can get higher wages from the foreign company than from the domestic company. Their wage is

$$w^f_t = \frac{\partial Y^f_t}{\partial L^f_t} = (1 - \alpha) \left( \frac{K^f_t}{A^f_t \theta} \right)^\alpha A^f_t.$$  

The representative household chooses the sequence of consumption and capital stock $\{C_t, K_{t+1}\}$ to maximize the lifetime utility (3.1) subject to the resource constraint

$$C_t + I^h_t = Y^h_t + w^f_t \theta$$

$$\Rightarrow C_t + K^h_{t+1} - (1 - \delta) K^h_t = (K^h_t)^\alpha (A^h_t(1 - \theta))^{1-\alpha}$$

$$+ (1 - \alpha) \left( K^f_t \right)^\alpha \left( A^f_t \theta \right)^{-\alpha} A^f_t \theta,$$

given $K_0 > 0$.

---

7This is true for the baseline parameter values. The appendix shows the condition that this statement is true.

8A Handbook chapter by Harrison and Rodriguez-Clare (2010) lists a number of papers which find wage gaps between domestic and foreign companies that may reflect higher productivity level of foreign companies. According to Harrison and Rodriguez-Clare (2010), the unconditional wage gap is typically large. For example, the gap is 40% for the case of Hungary and 50% for Brazil. The wage gap after controlling for worker and firm characteristics are much smaller. Harrison and Rodriguez-Clare (2010) report, however, that most studies find at least some wage premium even after controlling for worker and firm characteristics. See papers cited therein for detail.

9Note that while the foreign wage is higher than the domestic wage, it is still a competitive wage in the sense that foreign firms pay the marginal product of foreign-compatible labor. As long as foreign firms can freely enter the market, this assumption seems plausible.
Foreign capital flows into the country, until it is paid its marginal product. Thus, foreign capital stock per efficiency unit of labor jumps to the steady state level as soon as the country start accepting FDI flows:

$$\frac{K^f_t}{A^f_t} = \left(\frac{\alpha}{R^* - 1 + \delta}\right)^{1/(1-\alpha)} , \quad t = 0, 1, 2, \ldots$$

The wage for foreign compatible workers thus becomes

$$w^f_t = (1 - \alpha) \left(\frac{\alpha}{R^* - 1 + \delta}\right)^{\alpha/(1-\alpha)} A^f_t$$

$$= (1 - \alpha)(k^*)^\alpha A^f_t.$$

The domestic firm continues to rent the capital inside the country, until the domestic capital is paid its marginal product:

$$\frac{K^h_t}{A^h_t(1 - \theta)} = \left(\frac{\alpha}{R^* - 1 + \delta}\right)^{1/(1-\alpha)}.$$

To obtain the optimal consumption path, I rely on a numerical method. I describe the method in detail in the appendix.

### 3.2.4 Financial integration: open to debt and FDI flows

In this subsection, I consider a case of financial integration under which the economy accepts both debt and FDI flows. In this case, the representative household chooses the sequence of consumption, capital stock, and net foreign
asset \{C_t, K_{t+1}, B_{t+1}\} to maximize the lifetime utility (3.1) subject to the following resource constraint

\[
B_{t+1} + C_t + I_t^h = Y_t^h + w_t^f \theta + R^* B_t
\]

\[
\Rightarrow B_{t+1} + C_t + K_{t+1}^h - (1 - \delta)K_t^h = (K_t^h)^\alpha (A_t^h(1 - \theta))^{1-\alpha}
\]

\[
+ (1 - \alpha) \left( K_t^f \right)^\alpha \left( A_t^f \theta \right)^{-\alpha} A_t^f \theta + R^* B_t,
\]
given \(K_0 > 0\) and \(B_0 = 0\). For the reasons explained in the previous subsections, both domestic and foreign capital per efficiency unit of labor jump to their steady state levels after the country starts accepting foreign capital flows:

\[
\frac{K_t^h}{A_t^h(1 - \theta)} = \left( \frac{\alpha}{R^* - 1 + \delta} \right)^{1/(1-\alpha)} = \frac{K_t^f}{A_t^f \theta},
\]

for \(t = 0, 1, 2, \ldots\) We can show that the initial level of consumption is given by

\[
C_0 = (R^* - g)K_0 + (1 - \alpha)(k^*)^\alpha \left( \theta A_0^f + (1 - \theta)A_0^h \right),
\]

(3.5)

where \(k^*\) is the steady state level of capital per efficiency unit of labor. See the appendix for the derivation. Since consumption grows at rate \(g\), it is easy to obtain the whole sequence of optimal consumption.

### 3.3 Calibration

#### 3.3.1 Parameter values

Table 3.1 lists the baseline parameter values. The values chosen for the discount factor, the coefficient of relative risk aversion, the capital share, the depreciation rate, and the technological growth rate are close to the values often used in the literature.
The discount factor $\beta = 0.96$ and the coefficient of relative risk aversion $\gamma = 2$ together imply that the natural gross rate of interest $R^* = 1.0543$. The world real interest has been recently falling, and I will explore its implication on the welfare gains from financial integration below.

The fraction of foreign-compatible labor, $\theta = 0.2$, as well as the ratio of foreign and domestic technology, $A_f^t/A_h^t = 1.2$, are somewhat difficult parameters to calibrate, since developing countries are substantially different from each other in terms of the share of educated workers (i.e., foreign-compatible labor) as well as the level of technology.\(^{10}\) For example, the share of foreign-compatible labor probably depends on the fraction of educated workers among total workers in the country. The average level of education, however, greatly varies across

\[^{10}\text{The baseline values of these two parameters are determined as follows. In the steady state, the ratio of the domestic wage and the foreign wage is equal to the ratio of the level of domestic technology and the level of foreign technology:}\]

\[
\frac{w_f^t}{w_h^t} = \frac{A_f^t}{A_h^t}.
\] (3.6)

According to a Handbook chapter by Harrison and Rodriguez-Clare (2010), the foreign wage premium estimated by many empirical studies, though the range is wide, is between 5 percent and 50 percent. Suppose it is a mid point of the estimates and 20 percent, i.e., $A_f^t/A_h^t = 1.2$. In the steady state, foreign capital stock per efficiency unit of labor is equal to domestic capital stock per efficiency unit of labor:

\[
\frac{K_f^t}{A_f^t \theta} = \frac{K_h^t}{A_h^t(1 - \theta)},
\] (3.7)

which leads to

\[
\frac{1 - \theta}{\theta} = \frac{K_h^t A_f^t}{K_f^t A_h^t}.
\] (3.8)

The ratio of foreign capital and the domestic capital in developing countries is about 4 according to Kim et al. (2008), leading to

\[
\frac{1 - \theta}{\theta} = 4 \cdot 1.2.
\] (3.9)

Solving this for $\theta$, we get $\theta = 1/5.8 \approx 0.2$. 68
countries (Barro and Lee, 2013). Given the large differences across developing countries, I will discuss in detail how the main results change for other reasonable range of values for these parameters. I will also conduct similar sensitivity analysis for some other parameters.

3.3.2 Measuring the welfare gains from financial integration

As in Gourinchas and Jeanne (2006), I measure the welfare gains from financial integration by the percentage increase in consumption at all periods that equalizes the welfare under autarky and the welfare under financial integration (the Hicksian equivalent variation):

\[
h = \left( \frac{U^i}{U^a} \right)^{1/(1-\gamma)} - 1,
\]

where \( U^i \) and \( U^a \) denote the welfare under financial integration and under autarky.\(^{11}\) In the next section, I discuss how the welfare gains depend on the types of financial integration.

3.4 Main results

In this section, I present the main results. After providing the overview of the results, I explain the sources of welfare gains in detail.

---

\(^{11}\) To derive the expression for the Hicksian equivalent variation, solve the following equation for \( h \):

\[
\sum_{t=0}^{\infty} \beta^t(C^a_t(1+h))^{1-\gamma}/(1-\gamma) = \sum_{t=0}^{\infty} \beta^t(C^i_t)^{1-\gamma}/(1-\gamma)
\]

where \( C^a_t \) and \( C^i_t \) denote the consumption under autarky and under financial integration. In the case of log utility, the Hicksian equivalent variation is given by

\[
h = \exp((1 - \beta)(U^i - U^a)) - 1.
\]
3.4.1 Overview of the main results

In Figure 3.1, I plot the welfare gains from three types of financial integration against the initial level of capital based on the parameter values listed in Table 3.1. The figure confirms the finding by Gourinchas and Jeanne (2006); the welfare gains from liberalizing debt flows are quite limited. Even when the initial level of capital is about 30 percent below the steady state level, i.e., $K_0 = 3$, the welfare gain is only about 0.5 percentage increase in aggregate consumption. The figure, on the other hand, shows that the welfare gains from liberalizing FDI flows can be substantial. With the same level of initial capital, the welfare gain from liberalizing FDI flows is four percent which is eight fold of the welfare gain from liberalizing debt flows. The figure also shows that further gain from liberalizing debt flows in addition to FDI flows is not large. For a wide range of initial level of capital, the additional gain is less than one percent of aggregate consumption.

3.4.2 Detailed explanation of the results

In this subsection, I provide a detailed explanation as to why FDI flows bring large welfare gains as described in the previous subsection. To understand the mechanism, first note that FDI flows bring two benefits. First, FDI flows speeds up the accumulation of total capital in the economy, because foreign-owned capital reaches to the steady state level immediately after the country accepts FDI flows. Second, FDI flows come with advanced technology which enables foreign-compatible workers to earn more, contributing to higher consumption and thus welfare.
Of these two channels, the effects of the latter is significantly more important. To make this point, I plot the sequence of optimal consumption for three kinds of financial integration in Fig 3.2: financial autarky, when the economy accepts only debt flows, and when the economy accepts only FDI flows. Contrary to the baseline parameters, I assume *no* difference in the levels of domestic and foreign technology to show the (un)importance of the first channel. Since there is no difference between domestic and foreign technology, the welfare gain from allowing FDI flows comes entirely from the first channel. In this case, the welfare gains from allowing FDI flows is just 0.91 percent, whereas the welfare gain under the baseline parameters is four percent. That is, 80 percent of welfare gains come from the fact that foreign technology is more superior to domestic technology, and only 20 percent of welfare gains come from the fact that foreign capital reaches instantly to the steady state level. Therefore, the figure shows that the effects of allowing FDI flows are higher, if the flows come with more advanced technology. It also shows that the capital accumulation effects are temporary.

To see the importance of superior technology that comes with foreign capital, in Figure 3.4, I plot the sequence of optimal consumption for $A_0^f = 1.1$ and $A_0^f = 1.2$. The figure shows that FDI with better technology increases aggregate consumption permanently. This permanent effect is the source of the large welfare gain from liberalizing FDI flows. Since the wage for foreign-compatible workers is linear in the level of foreign technology, when the level of foreign workers is linear in the level of foreign technology, when the level of foreign

\footnote{All the other parameters are the same as the baseline values. The initial level of capital is set to $K_0^h = 2.5$.}
technology is higher than the level of domestic technology by \( x \) percent, aggregate consumption increases approximately by \( x\theta \) percent. For example, when \( A_0^f = 1.2 \) and \( \theta = 0.2 \), aggregate consumption increases by about four percent as the baseline result shows.

### 3.5 Sensitivity analysis

In this section I conduct several sensitivity analysis to present how the main results vary with the parameter values.

#### 3.5.1 A decline in the world real interest rate

In this subsection, I examine how a decline in the world real interest rate affects the welfare gains from financial integration. Recent worldwide decline in real interest rates has created numerous discussions among policymakers about its causes and implications (Furceri and Pascatori, 2014). The analysis in this subsection provides additional implication of this actively discussed phenomenon.

The model’s world real interest rate is given by \( R^* = \beta^{-1} g^\gamma \), and thus a decline in the interest rate can happen if the technological growth rate \( g \) declines.\(^{13}\)

Lower technological growth rate decreases the need to smooth consumption over time and so lowers the interest rate. In the following I show how this parameter change affects the welfare gains from financial integration.

---

\(^{13}\)Of course, the model’s world real interest rate also declines if the other two parameters change. First case is an increase in the time discount factor \( \beta \). If households become more patient (higher \( \beta \)), they have more incentive to consume less today and save for the future, leading to a lower interest rate. Second case is a decrease in the coefficient of relative risk aversion \( \gamma \). Since households have CRRA preferences, lower risk aversion means that households are more willing to substitute consumption over time, which lowers the interest rate. Here I focus on the change in the technological growth rate, since there is little evidence of long-term changes in the time discount factor and/or the coefficient of relative risk aversion.
The welfare gains from financial integration increase when the technological growth rate $g$ declines. To see this, first note that the parameter change increases the steady state level of capital per efficiency unit of labor (3.3), meaning that a decrease in the world real interest rate widens the distance between the initial level of capital and the steady state level. Thus, shortening the time to reach the steady state by accepting debt flows and/or FDI flows becomes more valuable when the world real interest rate is lower, i.e., the welfare gains from financial integration are larger.

To show the effects of a decline in the world real interest rate more quantitatively, I plot the welfare gains from financial integration for two different technological growth rates in Figure 3.3. The solid line shows the welfare gains for each type of financial integration when $g$ is 1.01, while the dashed line shows the welfare gains when $g$ is 1.02. The figure shows that one percentage point decrease in the technological growth rate or equivalently about one percentage decrease in the world real interest rate increases the welfare gains from financial integration for about one to one and half percent.

3.5.2 Different values of the share of foreign-compatible workers $\theta$

In this subsection I study how the share of foreign compatible workers is related to the welfare gains from financial integration. In Figure 3.5, I plot the sequence of optimal consumption and welfare gains from various financial integration for four different values for the share of compatible labor: 0.1 to 0.4. The figure shows that higher share of foreign compatible workers increases the level of consumption and hence welfare when the economy accepts FDI flows.
3.5.3 Different values of the initial level of foreign technology $A_0^f$

In this subsection I study how the initial level of foreign technology is related to the welfare gains from financial integration. In Figure 3.6, I plot the sequence of optimal consumption and the welfare gains from various kinds of financial integration for three different initial levels of foreign technology. The figure shows that FDI flows bring large welfare gains for countries with lower level of technology. When the level of foreign technology is 30 percent higher than the level of domestic technology, welfare gains are more than twice as large as when the level of foreign technology is 10 percent higher than the level of domestic technology.

3.6 Conclusion

This paper studies the welfare consequences of liberalizing FDI flows and find that the gains can be substantial. In the baseline specifications, the liberalization increases permanent consumption by four percent, which is a sharp contrast with debt flows whose liberalization increases permanent consumption only by half a percent for the same specifications. The results highlight that the overall welfare gains from financial integration (liberalizing all kinds of foreign capital flows) can be substantial and the relative importance of FDI flows over debt flows. This paper conducts a number of sensitivity analysis. In particular, it examines the implications of a decline in the world real interest rate and a decline in the labor share of income, the two recent phenomena that have occurred in many parts of the world. I find that both changes in the economic environment
increases the welfare gains from financial integration.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
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</tr>
<tr>
<td>$g$</td>
<td>Technological growth rate</td>
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</tr>
<tr>
<td>$\theta$</td>
<td>Fraction of foreign compatible labor</td>
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</tr>
<tr>
<td>$A^b_0$</td>
<td>Initial level of domestic technology</td>
<td>1</td>
</tr>
<tr>
<td>$A^f_0$</td>
<td>Initial level of foreign technology</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Table 3.1: Baseline parameter values
Figure 3.1: Welfare gains from various forms of financial integration

Note: The figure plots the welfare gains from three forms of financial integration: open to debt flows (solid line), open to FDI flows (dashed line), and open to debt and FDI flows (dotted line). The horizontal axis is the initial level of capital, and the vertical axis is the welfare gains (the unit is percentage point). I measure the welfare gains by the percentage increase in consumption at all periods that equalizes the welfare under financial autarky and the welfare under financial integration (the Hicksian equivalent variation).
Figure 3.2: Welfare gains from various forms of financial integration

Note: The figure plots the welfare gains from three forms of financial integration open to debt flows (solid line), open to FDI flows (dashed line), and open to debt and FDI flows (dotted line). The horizontal axis is the initial level of capital, and the vertical axis is the welfare gains (the unit is percentage point). I measure the welfare gains by the percentage increase in consumption at all periods that equalizes the welfare under financial autarky and the welfare under financial integration (the Hicksian equivalent variation).
Figure 3.3: Welfare gains from various forms of financial integration

Note: The figure plots the welfare gains from three forms of financial integration: open to debt flows (solid line), open to FDI flows (dashed line), and open to debt and FDI flows (dotted line). The horizontal axis is the initial level of capital, and the vertical axis is the welfare gains (the unit is percentage point). I measure the welfare gains by the percentage increase in consumption at all periods that equalizes the welfare under financial autarky and the welfare under financial integration (the Hicksian equivalent variation).
Figure 3.4: Sequence of optimal consumption when $A^f_0 = 1.1$ and $A^f_0 = 1.2$

Note: The figure plots the sequence of optimal consumption when the initial level of foreign technology is $A^f_0 = 1.1$ (blue) and 1.2 (red). The initial level of capital is set to $K_0 = 2.5$. The other parameter values are listed in Table 3.1.
Figure 3.5: Sequence of optimal consumption and welfare gains from financial integration for various \( \theta \)

Note: The figure plots the sequence of optimal consumption and welfare gains from financial integration when the share of foreign compatible labor varies from 0.1 to 0.4. The initial level of capital is set to \( K_0 = 2.5 \). The other parameter values are listed in Table 3.1.
Figure 3.6: Sequence of optimal consumption and welfare gains from financial integration for various $A_f^t$.

Note: The figure plots the sequence of optimal consumption and welfare gains from financial integration when the initial level of technology varies from 1.1 to 1.3. The initial level of capital is set to $K_0 = 2.5$. The other parameter values are listed in Table 3.1.
Appendix A

Appendix for Chapter 1

Calculating nominal returns on index-linked gilts  In this appendix, I explain how one computes nominal return on index-linked gilts. Since the yields of index-linked gilts are quoted in terms of the price level on the issuance date, we need to take into account of accrued inflation between the gilt’s first issue date and the date that the investor sells it in order to calculate the nominal return on the gilt. The adopted price index is called the Retail Price Index (RPI). The reference RPI is available from UK Debt Management Office’s website.¹

In UK, all index-linked gilts issued before 2005 have and an eight-month indexation lag, and all indexed-gilts issued after 2005 have a three-month indexation lag. Given the three-month indexation lag was introduced recently, I use an eight-month indexation lag for all the analysis.

I illustrate the calculation of nominal returns by a specific example. Suppose that 10-year zero coupon index-linked gilts are issued on June 1, 2000. Suppose also that an investor buys the bond and sells it on July 1, 2000. Then, the

nominal holding period return is calculated as follows:

\[
\frac{P' - P}{P} \times \frac{\text{RPI for November 1999}}{\text{RPI for October 1999}}
\]

where \( P' \) denotes the price of 9 and 11/12 year index-linked gilt on July 1, 2000 and \( P \) denotes the price of 10 year index-linked gilt on June 1, 2000. RPI denotes the Retail Price Index.

For a day other than the 1st of the month, the reference RPI is calculated by linearly interpolating the two adjacent reference RPIs. For example, the reference RPI for August 20, 2005 is calculated as follows:

\[
\text{Ref RPI}_{\text{August 20, 2005}} = \text{Ref RPI}_{\text{August 1, 2005}} + \frac{19}{31} (\text{Ref RPI}_{\text{September 1, 2005}} - \text{Ref RPI}_{\text{August 1, 2005}}).
\]
Appendix B

Appendix for Chapter 2
Appendix C
Appendix for Chapter 3

C.1 The numerical method

In this appendix, I describe the numerical method to compute the optimal consumption path for the models described in Subsections 2.1 and 2.3.

C.1.1 Financial autarky

Here I describe the method to compute the optimal consumption path for the case of financial autarky. The object of interest is the policy function (optimal consumption function) which maps the state variable (capital per efficiency unit of labor) into the control variable (consumption per efficiency unit of labor): $c_t = c(k_t)$. To find the policy function, I first define an equally spaced grid of the state variable of $n$ points, $k_n$, which includes the steady state value of the state variable $k^*$.\textsuperscript{1} The corresponding policy function thus becomes a vector of dimension $n$. I then make an initial guess of the policy function, $c_1$. Then, for each possible value of the next period’s state variable, $k_{t+1} \in k_n$, I compute this period’s control variable using the Euler equation and this period’s state

\textsuperscript{1}I set $n = 500$ in the actual computation.
variable using the resource constraint:

\[
c_t = \beta^{-1/\gamma} \left( \alpha k_{t+1}^{\alpha-1} + 1 - \delta \right)^{-1/\gamma} c_1(k_{t+1})g
\]

\[
k_t \text{ s.t. } k_t^\alpha - c_t - k_{t+1}g + (1 - \delta)k_t = 0,
\]

which gives us another \( n \) combinations of \((k, c)\). I then form the next guess of the policy function, \( c_2 \), by interpolating the initial guess and this newly obtained combinations. I iterate this procedure, until \( c_j \) and \( c_{j+1} \) become sufficiently close to each other.

**C.1.2 Open to FDI flows**

The method to compute the optimal consumption path for the economy which accepts only FDI flows is essentially the same as the case of financial autarky. The only difference is that the relevant Euler equation and the resource constraint are slightly different:

\[
c_t = \beta^{-1/\gamma} \left( \alpha k_{t+1}^{\alpha-1}(1 - \theta)^{1-\alpha} + 1 - \delta \right)^{-1/\gamma} c_1(k_{t+1})g
\]

\[
k_t \text{ s.t. } k_t^\alpha(1 - \theta)^{1-\alpha} + e - c_t - k_{t+1}g + (1 - \delta)k_t = 0,
\]

where \( e \) is the home-productivity adjusted labor income for foreign compatible worker: \( e \equiv w^f_t \theta/A_t^{h,2} \). The other parts of the method are the same.

\(^2\)See Subsection 2.3 for the detail about the model.
C.2 Mathematical Appendix

C.2.1 Derivation of (3.4)

Iterating forward the household’s resource constraint, we obtain the intertemporal budget constraint
\[
\sum_{t=0}^{\infty} R^{s-t} C_t = R^{s} K_0 + \sum_{t=0}^{\infty} R^{s-t} w_t. \tag{C.1}
\]

The left-hand side is the present value of lifetime consumption, whereas the right-hand side is the present value of the lifetime resources. Since the labor market is competitive, the wage, \( w_t \), is equal to the marginal product of labor:

\[(1 - \alpha)A_0g^t k_t^\alpha. \]

Since the capital per efficiency unit of labor reaches the steady state level from period 0 because of financial integration, the wage in time \( t \) is given by

\[w_t = (1 - \alpha)(k^*)^\alpha A_0 g^t.\]

Thus, the intertemporal budget constraint (C.1) can be written as

\[
\sum_{t=0}^{\infty} R^{s-t} C_t = R^{s} K_0 + (1 - \alpha)(k^*)^\alpha A_0 \sum_{t=0}^{\infty} R^{s-t} g^t.
\]

Since consumption grows at rate \( g \), the equation can be further simplified as follows:

\[
\frac{1}{1 - R^{s-1} g} C_0 = R^{s} K_0 + \frac{(1 - \alpha)(k^*)^\alpha A_0}{1 - R^{s-1} g}.
\]

Multiplying both sides of the equation by \( 1 - R^{s-1} g \) yields Equation (3.4).

C.2.2 Derivation of (3.5)

I obtain the sequence of optimal consumption when the economy accepts both debt and FDI flows by combining the results from when the economy accepts
either debt or FDI flows (but not both). Since both domestic and foreign capital per efficiency unit of labor jump to its steady state level in time 0 and aggregate consumption grows at rate $g$ in all periods, I only need to know the initial level of consumption to obtain the whole sequence of optimal consumption. To obtain the initial level of consumption, note that the representative household has three sources of revenue: interests earned from the initial level of capital $R^*K_0$, labor income from domestic companies $(1 - \theta)w^h_t$, and labor income from foreign companies $\theta w^f_t$. The present value of these resources must be equal to the present value of consumption:

$$\sum_{t=0}^{\infty} R^{t-1}C_t = R^*K_0 + \sum_{t=0}^{\infty} R^{t-1} \left( \theta w^f_t + (1 - \theta)w^h_t \right)$$

$$\Rightarrow \sum_{t=0}^{\infty} R^{t-1}C_0g^t = R^*K_0$$

$$+ \sum_{t=0}^{\infty} R^{t-1} \left( \theta(1 - \alpha)(k^*)^\alpha A^f_0g^t + (1 - \theta)(1 - \alpha)(k^*)^\alpha A^h_0g^t \right)$$

$$\Rightarrow \frac{1}{1 - R^{*-1}g}C_0 = R^*K_0 + \frac{1}{1 - R^{*-1}g} \left( (1 - \alpha)(k^*)^\alpha \left( \theta A^f_0 + (1 - \theta)A^h_0 \right) \right).$$

Multiplying both sides of the equation by $1 - R^{*-1}g$ gives us Equation (3.5).

### C.2.3 The condition under which foreign wage is higher than domestic wage

In this subsection, I provide the condition under which foreign wage is higher than domestic wage when the economy is open to only FDI flows. The foreign wage is given by

$$w^f_t = (1 - \alpha)(k^*)^\alpha A^f_t.$$
The domestic wage is given by

\[ w^h_t = (1 - \alpha) \left( \frac{K^h_t}{A^h_t (1 - \theta)} \right)^\alpha A^h_t. \]

Thus, the foreign wage is higher than the domestic wage, if

\[ \frac{K^h_t}{A^h_t} < (1 - \theta) \left( \frac{A^f_0}{A^h_0} \right)^{1/\alpha} k^*. \]

Since \((1 - \theta)(A^f_0/A^h_0)\) is typically larger than one, the condition says that if the initial level of capital per efficiency unit of labor is sufficiently lower than the steady state level, the foreign wage is higher than the domestic wage.
Bibliography


Curriculum Vitae

Yoichi Goto received the B.A. degree from International Christian University in 2006 and the M.A. degree in Economics from the University of Tokyo in 2008. In 2008 he enrolled in the Economics Ph.D program at Johns Hopkins University.