ESSAYS ON DEPOSIT INSURANCE, UNDERGROUND BANKING AND BUYING LOCAL PREFERENCE

by

Weining Bao

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This dissertation consists of three self-contained essays that explore three distinct
topics in deposit insurance, underground banking and social preference respectively.
The first essay, “Deposit Insurance, Market Discipline and Consumer Welfare: A
Study of The Recent Financial Crisis”, studies the welfare implication of the recent
change in the deposit insurance coverage in the United States on depositors. I develop
and estimate a structural model which represents the banking industry as a two-sided
market place where banks act as intermediaries between consumers who have funds
and businesses seeking loans. I find that this policy erodes market discipline and
harms consumers. Moreover, market competition magnifies the damage on market
discipline. Counterfactuals indicate that banks reduce their deposit interest rates and
increase their risk caps under the new policy.
The second essay, “Underground Banking in the Emerging Market: Relationship as
Collateral”, which is a joint work with Jian Ni, investigates the mechanism behind
the use of relational contracts in sustaining the borrower-lender relationship in the
underground banking industry. Built on the relational contract framework, we show
that the borrower-lender relationship serves as “collateral” that screens out risky borrowers and countervails the moral hazard over the course of relationship. Contrary to conventional wisdom, the value of “collateral” is determined by future expected transactions. In the optimal contract, as the relationship advances, the loan rate falls and the value of the relationship rises. We find that borrowers have more incentives to repay as the relationship continues.

The third essay, “Buying Local, Consumer Benefit and Social Welfare”, which is coauthored with Baojun Jiang and Jian Ni, examines the welfare implication of the “buying-local” preference in a product market. We consider a market in which consumers are heterogeneous in “buying-local” and product price and quality are endogenous. We derive a non-monotonic relationship between “buying-local” and consumer surplus (social welfare) in a monopoly market. Contrary to conventional wisdom, we find that in a monopoly market, information friction may improve consumer surplus and social welfare. Moreover, we show that a welfare trap emerges when competition is present.

Keywords: Deposit Insurance, Informal Banking, Relational Contract, Buying Local

JEL Classification: D03, D82, D86, G21, G28, G33, L11, L12, L13

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Secondary Advisor: Jian Ni
Acknowledgment

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Chapter 1

Deposit Insurance, Market Discipline and Consumer Welfare: A Study of The Recent Financial Crisis

1.1 Introduction

The recent financial crisis reminds the public of the fragility of the banking system. To strengthen depositors’ confidence in the security of their money at banks and eliminate the chance of panic based bank-runs, many countries generously expanded their deposit insurance schemes after 2008, the year when Washington Mutual Bank
Some high income economies raised insurance coverage. For example, the United States increased the ceiling of insured deposits from $100,000 to $250,000 in 2008 and the European Union doubled its coverage limit by the end of 2009 to 100,000 Euros. Others like Austria and Germany simply provided blanket guarantees on deposits (Feyen & Vittas, 2009). Unlike policymakers who reach a general consensus that deposit insurance is desirable in practice, academics argue that introduction of this safety net weakens market discipline and encourages more opportunistic behaviors in the banking sector, which in turn harms depositors (Demirgüç-Kunt & Kane, 2002).

In this paper, I try to resolve the controversy over the recent extension of deposit insurance coverage in the United States. I explore the market discipline before and after the policy reform and examine the effect of this extra coverage on consumer welfare in deposit markets. I separate the gains and losses to consumers surplus under the new policy and compare them across demographic groups and different market structures.

I develop a structural model of the retail banking industry to study the interactions between depositors and banks. My model of consumer demand for retail bank deposits extends the literature on banking choice by allowing endogenous saving. The amount of deposits that a depositor places at a bank depends on the bank’s deposit interest rate as well as its risk of default and the individual’s outside investment opportunities.

---

1 The failure of Washington Mutual Bank was the largest bank failure in U.S. history in terms of assets.
I model a bank as an expected profit maximizing intermediary that uses deposit interest rate to compete for funds in deposit markets and employs risk cap to shape its loan portfolio in credit markets. I consider the fraction of loans of a bank that are impaired and use the distribution of this fraction to characterize the default risk of a bank’s loan portfolio. Built on these distributions, I bridge the gap between the default risk of a bank’s loan portfolio and its failure rate so the default risk of a bank is endogenously determined by its optimal instruments in both markets.

I utilize a two-year panel data set that contains characteristics of banks in all Metropolitan Statistics Areas (MSA) in the United States before and after the recent financial crisis. It is compiled from the Call Reports and the Summary of Deposits for 2007 and 2009. Information on consumer savings is collected from the Survey of Consumer Finance 2007-2009 two-year panel data set. Since this data set over samples rich people, I use local income distributions generated from the 2006 and 2008 Census to reweigh the corresponding distributions in the survey.

I estimate consumers’ preferences for bank deposit services and risk as well as the distribution of outside investment options. Both consumers’ risk preferences and the distribution of outside investment options are allowed to vary over time because the recent financial crisis crippled financial markets and investors may exhibit more risk aversion in financial decision making if the outcomes of their prior investments were disappointing (Barberis, Huang, & Santos, 2001). Moreover, individuals may be emotionally affected by the air of panic triggered by the stock market crash and
widespread bank failures, causing them to become more reluctant to take risks during the crisis (Guiso, Sapienza, & Zingales, 2013). I separately estimate banks’ loan demands in credit markets for 2007 and 2009 as the economic downturn began in 2008 and dampened the performance of loan markets. I also recover the distribution of the fraction of a loan portfolio that is uncollectible and the probability of failure for each bank before and after the policy change.

Using my model estimates, I explore the welfare implications of the new policy by solving the equilibrium of consumers’ optimal financial decisions, as well as banks’ optimal deposit interest rates and risk caps under both the old deposit insurance regime and the new one. I compare the market discipline and the consumers’ surplus in these two scenarios. I find that banks reduce their deposit interest rates and boost their risk caps after the ceiling of insured deposits increases. I examine this moral hazard under different market structures and the results of my counterfactual experiments indicate that market competition amplifies the negative effect of deposit insurance on market discipline. I separate the consumers’ welfare gains from additional insurance coverage and their welfare losses from weakened market discipline. The results of my policy analysis reveal that the gains are dominated by the losses, and the losses in depositors’ welfare are equivalent to a 0.28% drop in deposit interest rates. Furthermore, market competition accounts for at least 40% of the losses in depositors’ welfare.

This paper lies in the interface of several subjects. First, it is related to the growing
literature on deposit insurance and market discipline. The leading work by Diamond and Dybvig (1983) considers banks as intermediaries and shows that deposit insurance smoothes the coordination frictions among depositors and prevents panic based bank-runs. Matutes and Vives (1996) extends the intermediation theory to imperfect competition by incorporating product differentiation and network effect. They find that at a fairly priced premium, deposit insurance is desirable when banks are local monopolies. Empirical works, however, spend more effort on detecting the downside of deposit insurance. Both Keeley (1990) and Chernykh and Cole (2011) confirm that introduction of deposit insurance weakens market discipline and encourages banks to pursue more risky investments. Apart from the reduced-form analysis, my works contribute to the literature by presenting a structural framework that can quantify the pros and cons of deposit insurance and its influences on depositors’ welfare. Furthermore, I show that deposit insurance favors consumers more if the deposit market is less competitive. This is consistent with the finding in Matutes and Vives (1996).

Second, this paper links to the literature on corporate default. To predict the probability of default, much of the previous literature selects a set of exogenous covariates and then either estimates its correlation with actual defaults (C. Brown & Dinc, 2011), or assumes the value of underlying firm follows some dynamic stochastic process based on those covariates, and computes the likelihood that the firm’s value drops below some threshold at some time in the future (Black & Scholes, 1973; Crobie & Bohn, 2002). Instead of following the conventional wisdom, I derive the bank’s
failure rate from its market conduct. The probability of default is endogenous by nature and depends on the interactions with rival banks. Hence, this paper provides a micro-foundation on bank failure and empirically estimates what a bank’s default rate would be the recent financial crisis.

Third, this paper contributes to the small but growing literature on structural estimation of bank demand. Both Adams, Brevoort, and Kiser (2007) and Dick (2008) adopt the random coefficients discrete choice approach to study the demand in the markets of retail deposits, and they use the number of bank accounts to calculate a bank’s market shares. K. Ho and Ishii (2011) point out that market share computed based on the number of bank accounts could be misleading because banks compete for funds rather than depositors in deposit markets. Ho and Ishii introduce exogenous consumer saving to the banking choice problem and construct market shares based on dollar deposits. I extend their work by endogenizing consumers' saving decisions and nesting the decisions into consumers’ bank choice problem. Deposit interest rates affect not only how likely it is that a consumer will patronize a bank, but also how much she will deposit at that bank. The consumer’s decision to deposit funds does not appear in the prior studies, and it is crucial to my analysis, because people tend to withdraw their money when they believe a financial crisis erodes the investment environment.

Finally, this paper presents an application of indirect inference approach developed by Gourieroux, Monfort, and Renault (1993) and demonstrates the advantage of
that approach when the model complexity prohibits the direct derivation of moment conditions.

The remainder of the paper is organized as follows. Section 1.2 gives an overview of deposit insurance. Section 1.3 describes the data. Section 1.4 and 1.5 present the model and estimation methodology. Results and counterfactual analysis are provided in Section 1.6 and 1.7 respectively. Section 1.8 concludes.

1.2 Deposit Insurance: An Overview

Deposit insurance is a mechanism to insure deposits and protect depositors from losses resulting from a bank failure. The Federal Deposit Insurance Corporation (FDIC), an independent regulatory agency backed by the government of the United States, provides this insurance. As of September 26, 2013, there are 6915 depository institutions insured by the FDIC with the total amount of insured deposits over 10 trillion \(^2\) (Federal Deposit Insurance Corporation, 2013). When an insured bank is unable to pay its debts, the FDIC takes it over and guarantees the payback of deposits up to the insurance limit to all customers of this bank. Due to the financial crisis, the FDIC took over 168 banks between 2007 and 2009. The plague of bankruptcy gradually disappeared after 2011 as the economy started recovering from the crisis. Descending from the state deposit guarantee program in early 20\(^{th}\) century, the FDIC was established, under the Bank Act of 1933, to secure bank customers’ deposits

\(^2\)The data on insured deposit is as of June 30, 2013.
against bank insolvency during the great depression. Starting at $2,500, the deposit insurance limit gradually increased over time and reached $100,000 after the passage of Depository Institutions Deregulation and Monetary Control Act of 1980. To maintain depositors’ confidence in the recent financial crisis, the FDIC temporarily raised the deposit insurance limit to $250,000 in 2008. This increase became permanent on July 21, 2010, when the Dodd-Frank Wall Street Reform and Consumer Protection Act went into effect. The current insurance policy covers all deposit accounts\(^3\) and the coverage limit is $250,000 per depositor for every account ownership category, including individuals, corporations and governments, at each insured bank.

1.3 Data

The data used in my analysis is a 2007-and-2009 two-year panel data set compiled from several sources. The data on bank characteristics come from financial statements in Call Reports from the Federal Reserve Board and information on branch deposits is taken from Summary of Deposits from the FDIC. I collect the data on a household’s financial status and demographic information from the 2007-and-2009 Survey of Consumer Finance. The 2006 and 2008 census provide local income distributions for each MSA in the United States based on which I generate local distribution of demographics.

\(^3\)Checking accounts, savings accounts and money market accounts, and certificates of deposit.
1.3.1 Banks: Deposit Markets

A market is defined geographically by a Metropolitan Statistical Area and the product is dollar deposits, including checking, savings and time deposits. Market share is calculated based on the amount of deposits a bank collects in a market. Table 1.1 describes market concentration in deposit markets.\(^4\) It seems that market structure is quite stable in deposit markets and banks bid aggressively for deposits as HHI is low.

\[
\text{Table 1.1: Market Summary Statistics}
\]

<table>
<thead>
<tr>
<th></th>
<th>Year 2007</th>
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<th>Year 2009</th>
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<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank Number</td>
<td>5.785</td>
<td>6</td>
<td>1.491</td>
<td>6.033</td>
<td>6</td>
<td>1.483</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C(1)</td>
<td>0.2441</td>
<td>0.2304</td>
<td>0.0865</td>
<td>0.2431</td>
<td>0.2259</td>
<td>0.0876</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HHI</td>
<td>1263</td>
<td>1157</td>
<td>604.7</td>
<td>1273</td>
<td>1174</td>
<td>592.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Number</td>
<td>365</td>
<td></td>
<td></td>
<td>364</td>
<td></td>
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<td></td>
<td></td>
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</tbody>
</table>

Deposit interest rate is computed by dividing a bank’s interest expense on deposits over its total deposits. Since this is the 6-month rate,\(^5\) I compound it to get the annual interest rate. A bank’s geographic focus of business is denoted by the single

\(^4\)C(1) is the one-firm concentration ratio. HHI is calculated by summing the square of market shares and then multiplying it by 10,000.

\(^5\)Because it is calculated based on the financial statements in the second quarter Call Report.
market indicator which is one if it raises more than 85% of its deposits from a market\textsuperscript{6} and zero otherwise. To facilitate my estimation, I drop observations that have zero deposits, zero premises expenses and missing values. I also drop observations whose market share is less than 5%. The remaining data set contains 741 banks and 1,950 bank-market observations in 2007 and 729 banks and 2,027 bank-market observations in 2009. Table 1.2 summarizes bank characteristics in deposit markets. A quick inspection of deposit rates suggests that market discipline is weakened after increasing the deposit insurance coverage because, on average, deposit rates went down by more than a half in 2009.

\begin{table}[h]
\centering
\begin{tabular}{lcccccc}
\hline
 & \textbf{Year 2007} & & \textbf{Year 2009} & \\
 & Mean & Median & Std Error & Mean & Median & Std Error \\
\hline
Deposit Rate & 0.0281 & 0.0278 & 0.0058 & 0.0134 & 0.0130 & 0.0060 \\
Bank Age & 102.2 & 103 & 50.66 & 105.5 & 105 & 51.86 \\
Branch Density & 16.86 & 7 & 35.72 & 17.15 & 7 & 40.20 \\
Single Market & 0.1677 & 0 & 0.3737 & 0.1524 & 0 & 0.3595 \\
Bank Obs & 741 & & & 729 & & \\
Bank-Market Obs & 1950 & & & 2027 & & \\
\hline
\end{tabular}
\caption{Bank Characteristics Summary Statistics I}
\end{table}

\textsuperscript{6}This definition is consistent with that in K. Ho and Ishii (2011) and Cohen and Mazzeo (2007), except for the former’s threshold is 90% and latter’s is 80%.
1.3.2 Banks: Credit Markets

This subsection introduces variables related to a bank’s behaviors in credit markets. Loan return is the ratio of a bank’s loan income to total loans. Similar to deposit rate, it is annualized before model estimation. Loan losses is a bank’s actual charge-offs normalized by total loans. The ratio of allowance for loan and lease losses to total loans, denoted by ALLL* measures expected loan losses percentage. I define a bank’s geographical diversification indicator to be one if it operates in multiple markets and zero otherwise. This index distinguishes inter-state banks from local banks in portfolio risk management. Table 1.3 demonstrates bank characteristics in

<table>
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<tr>
<th></th>
<th>Year 2007</th>
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<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td></td>
</tr>
<tr>
<td>Loan Return</td>
<td>0.0775</td>
<td>0.0766</td>
<td>0.0118</td>
<td>0.0596</td>
<td>0.0589</td>
<td>0.0089</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loan Losses</td>
<td>0.0026</td>
<td>0.0017</td>
<td>0.0062</td>
<td>0.0120</td>
<td>0.0057</td>
<td>0.0172</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALLL*</td>
<td>0.0247</td>
<td>0.0234</td>
<td>0.0104</td>
<td>0.0358</td>
<td>0.0299</td>
<td>0.0203</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(TA)</td>
<td>13.74</td>
<td>13.35</td>
<td>1.580</td>
<td>13.79</td>
<td>13.46</td>
<td>1.582</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC / TA</td>
<td>0.1002</td>
<td>0.0924</td>
<td>0.0305</td>
<td>0.0840</td>
<td>0.0788</td>
<td>0.0286</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Failed Banks</td>
<td>1</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
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Note: log(TA) = log(total asset); EC / TA = \(\frac{\text{equity capital}}{\text{total asset}}\).

credit markets. The negative impact of a financial crisis on a bank’s credit market performance is obvious as the plague of bankruptcy diffuses in the banking sector.
Average loan return fell by nearly 30% from 7.47% in 2007 to 5.40% in 2009 while both expected loan losses and actual bad debts soared up in the same periods.

1.3.3 Depositors

In this subsection, I discuss variables used in a consumer’s optimal saving problem. A depositor’s bank deposit is the sum of her checking, savings and certificates of deposit at banks. Her portfolio size is determined by the amount of liquid assets she owns, including bank deposits, bonds, stocks, mutual funds and investment in brokerage accounts. The income from her asset position is calculated as the total of her interest income, dividend income and capital gains (losses). Risk-free rate is the annualized return on a 3-month treasury bill and market return is the weighted average of bond

Table 1.4: Depositors’ Characteristics Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Year 2007</th>
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<th></th>
<th></th>
<th>Year 2009</th>
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<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td>Mean</td>
<td>Median</td>
<td>Std Error</td>
<td></td>
</tr>
<tr>
<td>Portfolio Size</td>
<td>9322</td>
<td>761</td>
<td>33800</td>
<td>7498</td>
<td>593</td>
<td>30100</td>
<td></td>
</tr>
<tr>
<td>Bank Deposits</td>
<td>753</td>
<td>79</td>
<td>3599</td>
<td>673</td>
<td>85</td>
<td>2432</td>
<td></td>
</tr>
<tr>
<td>Portfolio Income</td>
<td>938</td>
<td>21</td>
<td>4864</td>
<td>442</td>
<td>7</td>
<td>2845</td>
<td></td>
</tr>
<tr>
<td>Risk-free Rate</td>
<td>0.0435</td>
<td></td>
<td></td>
<td>0.0015</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Return</td>
<td>0.0440</td>
<td></td>
<td>0.0886</td>
<td>0.1833</td>
<td>0.2550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>1454</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Note: Portfolio size (income) and bank deposits are measured in thousands dollars.
market return and stock market return where weights are given by the relative market size in both markets. Bond market returns and stock market returns are computed from BofA Merrill Lynch US Corp Master Total Return Index Value and S&P 500 index respectively. Observations with missing values are dropped. Summary statistics are exhibited in table 1.4. The sample distribution of portfolio size, bank deposits and portfolio income are highly skewed, because the Survey of Consumer Finance over-sampled the rich people.

1.4 Model

To explore the effect of the recent change in deposit insurance policy, I develop a model on the behaviors of consumers and banks. Consumers are assumed to maximize their utility in a two-stage process. First, they decide whether to use a bank deposit service and if so, at which bank they will place their money. Then each consumer forms an investment portfolio by allocating funds over bank deposits and other financial instruments to maximize her expected utility. Banks are profit-maximizing intermediaries that transfer funds absorbed in deposit markets into financial instruments in credit markets. I assume each bank operates as a portfolio manager that trades off risk of failure and expected return when constructing a consumer’s loan portfolio. A bank’s portfolio selection is explicitly modeled and is directly linked to its decisions on both sides of the markets.
1.4.1 Consumers

This subsection characterizes a consumer’s demand for depository services and investment decisions in details. Following Dick (2008) and Ho & Ishii (2011), I assume each consumer chooses only one bank at which to place her deposit. The demand side of model is built on random coefficients discrete choice framework developed by Berry, Levinsohn, and Pakes (1995).

At the time $t$, consumer $i$ in market $m$ is endowed with a package of money $w_{imt}$ and an investment outside option $I_{i0mt}$. This outside option summarizes all the other investment opportunities which consumer $i$ can access, such as public stocks and corporate bonds. Investment in these assets risky. Let $\beta_{imt} = \theta_{mt,1} + \theta_{mt,2} \log(w_{imt})$ be the systematic risk of $I_{i0mt}$. Assuming $I_{i0mt}$ is well diversified, applying the capital asset pricing model, the expected return of $I_{i0mt}$ is

$$E[r_{i0mt}] = r_{ft} + \beta_{imt}(E[r_{mt}] - r_{ft}),$$

where $r_{ft}$ and $r_{mt}$ represent risk-free rate and market return on date $t$ respectively. The variance of $r_{i0mt}$ is $\beta_{imt}^2 \text{Var}[r_{mt}]$. Consumer $i$ is risk adverse and her risk coefficient is $a_{imt}$, drawn from the exponential distribution with mean $\theta_{at}$. Denote bank $j$’s observed characteristics and demand shock by $X_{jmt}$ and $\xi_{jmt}$ correspondingly and write $\epsilon_{ijmt}$ as consumer $i$ and bank $j$ idiosyncratic preference shock. Bank $j$’s observable characteristics $X_{jmt}$ include indicators denoting whether she raises most of her deposits from this market, branch density and bank age. The first characteristic tells
bank $j$’s geographic focus and provides a measure of its effort in delivering deposit services and the other characteristics describe its business capacity and operation experience, which are proxies for customer service quality.

The utility received by consumer $i$ when depositing at bank $j$ is

$$u_{ijmt} = \max_{0 \leq s_{ijmt} \leq w_{imt}} v_{ijmt}(s_{ijmt}) + X_{jmt} \theta_{imt}^D + \xi_{jmt} + \epsilon_{ijmt}$$

(1.1)

where $v_{ijmt}(s_{ijmt})$ measures consumer $i$’s utility from her pecuniary return when saving $s_{ijmt}$ at bank $j$ and $\theta_{imt}^D$ is a vector representing her preference towards deposit services. Suppose her risk preference can be characterized by mean-variance utility function, then

$$v_{ijmt}(s_{ijmt}) = w_{imt}\left[E\left[\frac{s_{ijmt}}{w_{imt}} R_{ijmt} + (1 - \frac{s_{ijmt}}{w_{imt}}) r_{i0mt}\right]\right]$$

return to consumer $i$’s

(1.2)

asset position

$$-\frac{1}{2} \text{Var}\left[\frac{s_{ijmt}}{w_{imt}} R_{ijmt} + (1 - \frac{s_{ijmt}}{w_{imt}}) r_{i0mt}\right]$$

where $R_{ijmt}$ is the net return of saving at bank $j$. A deposit exceeding deposit insurance coverage is subject to bank default risk. To keep the model tractable, I assume consumers $i$ can recover her deposits up to the amount that is protected by deposit insurance when her bank becomes insolvent. Let $p_{jmt}$ be bank $j$’s failure rate, the derivation of which is provided in next subsection. Then $R_{ijmt}$ can be written as
\[ R_{ijmt} = \begin{cases} r_{jmt}, & \text{if } s_{ijmt} \leq \frac{l_t}{1+r_{jmt}} \\ \frac{l_t}{s_{ijmt}} - 1, & \text{with probability } p_{jmt}, \text{if } s_{ijmt} > \frac{l_t}{1+r_{jmt}} \\ r_{jmt}, & \text{with probability } 1 - p_{jmt}, \text{if } s_{ijmt} > \frac{l_t}{1+r_{jmt}} \end{cases} \]

where \( r_{jmt} \) is bank \( j \)'s deposit interest rate and \( l_t \) is deposit insurance coverage. If consumer \( i \) does not deposit at all, her utility is

\[ u_{i0mt} = v_{i0mt} + \epsilon_{i0mt} \quad (1.3) \]

where \( v_{i0mt} = w_{imt}(E[r_{i0mt}] - \frac{a_{imt}}{2} Var[r_{i0mt}]) \).

Suppose the random coefficients can be decomposed as

\[ \theta_{int}^D = \theta^D + \nu_{int}^D, \quad \nu_{int}^D \sim N(0, \Sigma) \]

where \( \Sigma \) is a diagonal matrix and individual preference shock \( \nu_{int}^D \) is assumed to be independent across time and markets. Let \( s_{ijmt}^* \) maximize (1.2). Following Berry, Levinsohn, and Pakes (1995), the probability that consumer \( i \) chooses bank \( j \) is

\[ \frac{\exp(\delta_{jmt} + \mu_{ijmt})}{\sum_j \exp(\delta_{jmt} + \mu_{ijmt})} \]

where \( \delta_{jmt} = X_{jmt} \theta^D + \xi_{jmt} \) is the part of utility that does not vary with consumer characteristics and \( \mu_{ijmt} = v_{ijmt}(s_{ijmt}^*) + X_{jmt} \nu_{int}^D \) is the interaction term.

Market share is defined by the amount of dollar deposits. Therefore, bank \( j \)'s market share is
\[ m_{s_{jmt}} = \frac{\int \int s^*_{ijmt} \frac{\exp(\delta_{jmt} + \mu_{ijmt})}{\sum_j \exp(\delta_{jmt} + \mu_{ijmt})} dF(a_{imt})dF(w_{imt})}{\sum_j \int \frac{\exp(\delta_{jmt} + \mu_{ijmt})}{\sum_j \exp(\delta_{jmt} + \mu_{ijmt})} dF(a_{imt})dF(w_{imt})} \] (1.4)

### 1.4.2 Banks

This subsection describes the bank’s profit maximization problem. Each bank solves its portfolio optimization problem by simultaneously choosing relevant instruments in deposit markets and credit markets. I first construct the bank’s loan portfolio from its deposit interest rate and risk cap and then derive the bank’s default risk from the loan loss distribution of its loan portfolio. Finally, I present the bank’s expected profit while taking into account the limited liability in the situation of bankruptcy.

This paper focuses on the demand side of retail banking industry, so the bank-side model only incorporates the core aspects of the bank’s decision and abstracts away from the details of individual transactions.

**Loan Portfolio**

At the time \( t \), bank \( j \) is endowed with a set of loan demands. Each loan demand has the same size and is characterized by a pair \((K, \rho)\), where \( K \) is the expected return of loan and \( \rho \) is the default risk. To keep the problem tractable, I assume banks will not be able to collect loan repayments if borrowers default. The return of loan \((K, \rho)\) is therefore \( \frac{1+K}{1-\rho} \) with probability \( 1 - \rho \) and \(-1\) with probability \( \rho \).

Suppose bank \( j \)’s loan demand endowments are uniformly distributed over the triangle bounded by horizontal axis, vertical line \( \rho = 1 \) and line \( K = \text{slope}_{jt} \cdot \rho \). The slope
\( \text{slope}_{jt} \) measures bank \( j \)'s loan profitability at each default risk level in the sense of stochastic dominance. Fixing \( \rho \), bank \( j \) is more likely to make high return deals when \( \text{slope}_{jt} \) is large. I assume \( \text{slope}_{jt} \) can be parameterized by

\[
\text{slope}_{jt} = \exp(X^{L}_{jt} \phi + \omega_{jt})
\]

where \( X^{L}_{jt} \) is a vector of bank \( j \)'s observable characteristics that affect its loan profitability, including the log of bank size and the equity capital to total asset ratio. The log of bank size proxies bank \( j \)'s screening methodology and the ratio tells its business strategy, all of which influence the performance of its loan portfolio. The error term \( \omega_{jt} \) captures the effect of bank \( j \)'s unobserved characteristics, such as special business campaigns, on its credit market profitability. I assume the following independence condition.

\[
E[\omega_{jt}|X^{L}_{jt}] = 0
\]

The probability density function of bank \( j \)'s loan endowment is

\[
f(K, \rho) = \frac{2}{\exp(X^{L}_{jt} \phi + \omega_{jt})}, \quad 0 \leq K \leq \rho \exp(X^{L}_{jt} \phi + \omega_{jt}), 0 \leq \rho \leq 1
\]
Figure 1.1: Bank $j$’s loan portfolio

Triangle 0A1 in figure 1.1 illustrates the domain of $f$. When constructing a loan portfolio, bank $j$ trades off the probability of default and expected return. To control the overall riskiness of its asset position, it may decline loan applications that appear insecure. For estimation tractability, I assume that bank $j$ only accepts profitable loan applications whose default risk is less than $\rho_{jt}$. Cost of capital is the sum of deposit interest rate $r_{jt}$ and operation cost $\eta_{jt}$, so bank $j$ rejects funding applications below line $K = r_{jt} + \eta_{jt}$. By picking her loan risk cap $\rho_{jt}$, bank $j$ never considers loan demands to the right of line $\rho = \rho_{jt}$. Hence, bank $j$’s deposit interest rate $r_{jt}$ and risk cap $\rho_{jt}$ shape its loan portfolio, represented by triangle $BCD$ in figure 1.1.

The probability density function of loan endowments in portfolio $BCD$ is
\[ g(K, \rho | r_{jt}, \rho_{jt}) = \frac{2 \exp(X_{jt}^{L} \phi + \omega_{jt})}{(\rho_{jt} \exp(X_{jt}^{L} \phi + \omega_{jt}) - (r_{jt} + \eta_{jt}))^{2}} \]

\[ r_{jt} + \eta_{jt} \leq K \leq \rho \exp(X_{jt}^{L} \phi + \omega_{jt}), \quad \frac{r_{jt} + \eta_{jt}}{\exp(X_{jt}^{L} \phi + \omega_{jt})} \leq \rho \leq \rho_{jt} \]

(1.8)

**Default Risk**

The derivation of bank \( j \)'s failure rate from its loan portfolio follows two steps. First, I consider the fraction of bad loans for loans in bank \( j \)'s loan portfolio and construct the distribution of this fraction for bank \( j \)'s loan portfolio. Then I define bank \( j \)'s failure rate as the probability that the proportion of bad debts in bank \( j \)'s loan portfolio exceeds some threshold. The threshold is allowed to vary across banks and time, because banks are heterogeneous in their buffers against loan defaults.

For bank \( j \)'s loan portfolio characterized by \((r_{jt}, \rho_{jt})\), consider the loans whose default risk is \( \rho \). Let \( \tau(\rho) \) be the fraction of these loans that are uncollectible. Then for each \( \rho \), \( \tau(\rho) \) is a random variable distributed over the unit interval and \( E[\tau(\rho)|\rho] = \rho \). On the other hand, \( \tau(\rho) \) describes how bad loans are distributed in bank \( j \)'s loan portfolio. I assume \( \tau(\rho) = H^{-1}(z|\rho, \theta_{\tau}) \), where \( H(\cdot|\rho, \theta_{\tau}) \) is the cumulative distribution function of a beta random variable with parameters \( \theta_{\tau}\rho \) and \( \theta_{\tau}(1 - \rho) \), and \( z \) is the standard uniform random variable. The fraction of bad loans of bank \( j \)'s loan portfolio is therefore
\[ \bar{\tau}_{jt}(r_{jt}, \rho_{jt}) = \int \frac{H^{-1}(z|\rho, \theta)}{dG_{\rho}(\rho|r_{jt}, \rho_{jt})} \]  

fraction of bad loans for loans whose default risk is \( \rho \)

where \( G_{\rho}(\rho|r_{jt}, \rho_{jt}) \) is the marginal distribution of \( \rho \) given bank \( j \)'s loan portfolio \( BCD \). It is clear that \( \bar{\tau}_{jt}(r_{jt}, \rho_{jt}) \) is a random variable distributed over the unit interval too. Let \( \bar{H}(\bar{\tau}_{jt}(r_{jt}, \rho_{jt})|r_{jt}, \rho_{jt}) \) be its cumulative distribution function. Notice that \( H(\cdot|\rho, \theta) \) is monotonic increasing in \( z \), for every \( \bar{\tau}_{jt} \in [0, 1] \), there is a unique \( z^{*} = z^{*}(\bar{\tau}_{jt}) \in [0, 1] \) solving the equation above. Hence, the cumulative distribution function of \( \bar{\tau}_{jt} \) is

\[ \bar{H}(a|r_{jt}, \rho_{jt}) = \Pr(\bar{\tau}_{jt} \leq a) = \Pr(z^{*}(\bar{\tau}_{jt}) \leq z^{*}(a)) \]

\[ = \Pr(z \leq z^{*}(a)) = z^{*}(a), \quad \forall a \in [0, 1] \]  

(1.9)

A bank fails if a substantial amount of loans become uncollectible. Suppose such threshold can be parameterized by \( \Phi(X_{jt}^{F}|\theta_{F}) \), where \( \Phi(\cdot) \) is the cumulative distribution function of a standard normal random variable and \( X_{jt}^{F} \) is a vector of bank \( j \)'s characteristics including log of bank size and a time indicator. Bank \( j \) becomes insolvent if \( \bar{\tau}_{jt}(r_{jt}, \rho_{jt}) \geq \Phi(X_{jt}^{F}|\theta_{F}) \). Hence, bank \( j \)'s failure rate is

\[ p_{jt} = 1 - \bar{H}(\Phi(X_{jt}^{F}|\theta_{F})|r_{jt}, \rho_{jt}) \]  

(1.10)
Profit Maximization

To complete bank $j$’s profit maximization problem, I have to bridge the gap between its bad debt percentage and the realized return of its loan portfolio. Recall that the proportion of loans with default risk $\rho$ that are uncollectible is $H^{-1}(z^*(\tau_{jt}(r_{jt}, \rho_{jt}))|\rho, \theta)$, when fraction $\tau_{jt}(r_{jt}, \rho_{jt})$ of bank $j$’s outstanding loans will not be repaid. Then bank $j$’s realized return is

$$K(\bar{\tau}_{jt}(r_{jt}, \rho_{jt})|r_{jt}, \rho_{jt}) = \int \int [(1 - H^{-1}(z^*(\bar{\tau}_{jt}(r_{jt}, \rho_{jt}))|\rho, \theta)(\frac{1+K}{1-\rho} - 1)]$$

$$ \sum_{m \in j(m)} \int_0^{x_{jt}^F} (K(\bar{\tau}_{jt}(r_{jt}, \rho_{jt})|r_{jt}, \rho_{jt}) - (r_{jt} + \eta_{jt}))D_{jmt} d\bar{H}(\tau_{jt}(r_{jt}, \rho_{jt})|r_{jt}, \rho_{jt})$$

I assume that bank $j$ receives zero payoff when it becomes insolvent. Bank $j$’s portfolio optimization problem is therefore to choose its deposit interest rate $r_{jt}$ and loan risk cap $\rho_{jt}$ to maximize its expected profit.

$$\max_{\{r_{jt}, \rho_{jt}\}} \sum_{m \in j(m)} \int_0^{x_{jt}^F} (K(\bar{\tau}_{jt}(r_{jt}, \rho_{jt})|r_{jt}, \rho_{jt}) - (r_{jt} + \eta_{jt}))D_{jmt} d\bar{H}(\tau_{jt}(r_{jt}, \rho_{jt})|r_{jt}, \rho_{jt})$$

where $j(m)$ is the set of markets in which bank $j$ operates and $D_{jmt}$ denotes the amount of deposits it raised in market $m$. 
1.5 Estimation

The model is estimated by generalized method of simulated moments. Due to data limitations and the consistency of estimation methodology, I adopt the indirect inference approach (Gourieroux, Monfort & Renault, 1993) to estimate model primitives when moment conditions cannot be directly derived.

1.5.1 Deposit Market

This subsection provides the estimates of demand for bank depository services \( \{\theta^D, \Sigma\} \), the systematic risk of individual investment outside option \( \{\theta_{mt,1}, \theta_{mt,2}\} \) and a consumer’s risk preference \( \theta_{at} \). Two sets of moments are used. One comes from moment conditions in Berry, Levinsohn, and Pakes (1995) and the other one is based on a depositor’s optimal saving decision.

Using the fact that a bank’s unobserved demand shock \( \xi_{jmt} \) has zero mean conditional on instruments, the first set of moment conditions is given by

\[
E[\xi_{jmt}|Z_{jmt}] = 0 \tag{1.13}
\]

I use instruments for an indicator on whether bank \( j \) collects most of its deposits in market \( m \), because by definition it is correlated with unobserved bank quality \( \xi_{jmt} \) for all banks operating in multiple markets. Two types of instruments are employed. As argued in Berry, Levinsohn, and Pakes (1995), in oligopoly competition, a competitor’s characteristics correlate with a bank’s market share but not with its unobserved...
demand shock. Type one instruments are the average of competitors’ main office indicators and the mean of competitors’ age, branch density and employees per branch. Cost-shifters reveal another way to find demand-side instruments. The second type of instruments include wage and expense on premises and fixed assets. The former is calculated by dividing a bank’s labor expense over the number of equivalent full-time employees and the latter is normalized by a bank’s assets.

The second set of moments is built on the following identity.

\[
\hat{\text{inc}}_{i\text{mt}} \quad = \quad \frac{s^*_{ij\text{mt}}}{w_{i\text{mt}}} \hat{R}_{ij\text{mt}} \quad + \quad \frac{(1 - s^*_{ij\text{mt}})\hat{r}_{i0\text{mt}}}{w_{i\text{mt}}}
\]

realized return of realized return from realized return from
consumer i’s investment bank deposit investing on outside option

(1.14)

where \(\hat{\text{inc}}_{i\text{mt}}\) is consumer i’s portfolio income, including income from interest-earning assets, stock dividends and capital gains. I do not observe consumer i’s actual savings return from bank j \(\hat{R}_{ij\text{mt}}\), because neither her bank affiliation nor her deposit interest income is available in the data set. On the other hand, the data set supplies bank deposit interest rates and the list of failed banks. By aggregating over all banks, I am able to get around detecting deposit returns received by each consumer in the data because matching every depositor to her bank is no longer a step that must be walked through during estimation. Taking the expected value of both sides of this identity conditional on \(w_{i\text{mt}}\) only gives
\[
E\left[\frac{\hat{\text{inc}}_{ijmt}}{w_{imt}}|w_{imt}\right] = E\left[\frac{s^*_{ijmt}}{w_{imt}}\hat{R}_{ijmt} + (1 - \frac{s^*_{ijmt}}{w_{imt}})\hat{r}_{i0mt}|w_{imt}\right] \tag{1.15}
\]

The expectation on the right hand side is computed by integrating over all bank \(j\)s, consumer \(i\)’s risk preference \(a_{imt}\) and actual return of outside option \(\hat{r}_{i0mt}\). Since \(\hat{r}_{i0mt}\) is a realization of \(r_{i0mt}\) and \(E[r_{i0mt}|w_{imt}] = r_{ft} + (\theta_{mt,1} + \theta_{mt,2} \log(w_{imt})) (E[r_{m,t}] - r_{ft})\), it is clear that

\[
E\left[\frac{s^*_{ijmt}}{w_{imt}}\hat{R}_{ijmt} + (1 - \frac{s^*_{ijmt}}{w_{imt}})E[r_{i0mt}|w_{imt}]|w_{imt}\right]
\]

Let \(\lambda(w_{imt};j,m)\) be the choice probability implied by my demand model that consumer \(i\) with portfolio size \(w_{imt}\) selects bank \(j\) in market \(m\). Then the average ex-post investment return on the right hand side of the equation above can be written as

\[
E_m[\sum_{j \in m(j) \cup \{0\}} \lambda(w_{imt};j,m) E_{a_{imt}}\left[\frac{s^*_{ijmt}}{w_{imt}}\hat{R}_{ijmt} + (1 - \frac{s^*_{ijmt}}{w_{imt}})E[r_{i0mt}|w_{imt}]|w_{imt}\right]|w_{imt}]
\]

mean realized portfolio return for consumers depositing at bank \(j\) in market \(m\)

where \(m(j)\) denotes the set of banks being active in market \(m\). By (1.15), I have

\[
E\left[\frac{\hat{\text{inc}}_{imt}}{w_{imt}}|w_{imt}\right] = E_m[\sum_{j \in m(j) \cup \{0\}} \lambda(w_{imt};j,m) E_{a_{imt}}\left[\frac{s^*_{ijmt}}{w_{imt}}\hat{R}_{ijmt} + (1 - \frac{s^*_{ijmt}}{w_{imt}})E[r_{i0mt}|w_{imt}]|w_{imt},j,m\right]]
\]

If \(\theta_{mt,1}, \theta_{mt,2}\) and \(\theta_{at}\) are known, the right hand side of (1.16) becomes a non-linear function of \(w_{imt}\) alone. However, its complexity prevents me from constructing mo-
ments on the model primitives directly so I the adopt indirect inference approach to derive moment conditions. The left hand side of (1.16) is the conditional mean investment return exhibited in the data and the right hand side is the corresponding return implied by the model. Under the true parameters, projecting both sides of (1.16) onto the space spanned by 1, log$(w_{imt})$ and log$^2(w_{imt})$ should leave the same projections. Let $\hat{\pi}_0$, $\hat{\pi}_1$ and $\hat{\pi}_2$ be the coefficients from regressing $E[\hat{inc}_{imt}/w_{imt}]$ on 1, log$(w_{imt})$ and log$^2(w_{imt})$. Similarly, denote the coefficients from regressing $E_m[\sum_{j \in m(\cup\{0\})} \lambda(w_{imt},j,m)E_{a_{imt}}[\hat{s}_{ijmt}/w_{imt}\hat{R}_{ijmt} + (1 - \hat{s}_{ijmt}/w_{imt})E[r_{i0mt}|w_{imt}]]w_{imt},j,m]$ on 1, log$(w_{imt})$ and log$^2(w_{imt})$ by $\tilde{\pi}_0$, $\tilde{\pi}_1$ and $\tilde{\pi}_2$. Then following moments help me estimate $\theta_{mt,1}$, $\theta_{mt,2}$ and $\theta_{at}$.

$$E[\hat{\pi}_k - \tilde{\pi}_k] = 0, \quad k = 0, 1, 2$$

(1.17)

1.5.2 Loan Market

In this subsection, I recover bank-side parameters $\{\phi, \theta_r, \theta_F\}$ from three sets of moments. To identify $\phi$, I need $slope_{jt}$ for each bank. According to Generally Accepted Accounting Principles, allowance for loan and lease losses (ALLL) is an estimate of a bank’s expected credit losses. Hence,
bank $j$’s ALLL = bank $j$’s total loans \cdot \ E[\rho|\tau_{jt}, \rho_{jt}]

credit risk of bank $j$’s loan portfolio

To solve for $slope_{jt}$, I have to provide another equation on $\rho_{jt}$ and $slope_{jt}$, because both of them are unknowns in the equation above. Notice that the data set contains bank $j$’s realized loan return and the percentage of bad debts, using (1.11) I have

bank $j$’s realized loan return = $K(\overline{\tau}_{jt}|\tau_{jt}, \rho_{jt})$

where $\overline{\tau}_{jt}$ is bank $j$’s actual fraction of uncollectible loans. Once $slope_{jt}$ is known, by (1.5) and (1.6), $\phi$ is identified by the following moment.

$$E[\log(slope_{jt}) - X_{jt}^L\phi|X_{jt}^L] = 0 \quad (1.18)$$

The remaining parameters are estimated via method of moments. I use the variance of bank’s bad debt percentage $Var[\overline{\tau}]$ to figure out $\theta_{\tau}$. Integrating this variance over all banks produces the model implied variance of a bank’s bad debt percentage $Var[\overline{\tau}] = E_j[Var[\overline{\tau}_{jt}|r_{jt}, \rho_{jt}]]$. Write the corresponding empirical variance from data as $\widehat{Var}[\overline{\tau}]$. The following moment condition holds under the true value of $\theta_{\tau}$.

$$E[Var[\overline{\tau}] - \widehat{Var}[\overline{\tau}]] = 0 \quad (1.19)$$

It is left to determine $\theta^F$. Recall bank $j$’s failure rate is $p_{jt} = 1 - H(\Phi(X_{jt}^F\theta^F)|r_{jt}, \rho_{jt})$. 

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Define the indicator that bank \( j \) becomes insolvent by \( 1_{jt} \). Then \( 1_{jt} \) is a Bernoulli random variable with parameter \( p_{jt} \). Consider the model implied expected value of interaction between \( X_{jt}^F \) and \( 1_{jt} \), I have

\[
E[X_{jt}^F \cdot 1_{jt}] = X_{jt}^F (1 - H(\Phi(X_{jt}^F \theta^F)|r_{jt}, \rho_{jt}))
\]

Let \( E[X_{jt}^F \cdot 1_{jt}] \) be the relevant empirical expectations computed from data. The following moment provides the estimate of \( \theta^F \).

\[
E[E[X_{jt}^F \cdot 1_{jt}] - E[X_{jt}^F \cdot 1_{jt}]] = 0 \quad (1.20)
\]

### 1.6 Results

Parameter estimates are listed in table 1.5. In deposit markets, both branch density and bank age have positive effects on demand. Branch density measures the scope of a bank’s network in a market. When a bank opens many branches in a market, consumers may find it convenient to use its banking services. Bank age counts for business experience. Depositors prefer banks that have a long history because their accumulated knowledge about customer services helps to facilitate business transactions and communications. Negative coefficients on \( \theta_{2009,1} - \theta_{2007,1} \) and \( \theta_{2009,2} - \theta_{2007,2} \) indicate that the reward to risk taking diminished during the financial crisis. For the median investor, my model predicts the systematic risk of her investment outside option is 1.05 in 2007 and 0.75 two years afterwards. Estimates of risk preferences
Table 1.5: Estimation Results

<table>
<thead>
<tr>
<th>Variables</th>
<th>Banking Choice (Mean)</th>
<th>Banking Choice (S.E.)</th>
<th>Optimal Saving ($\theta_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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<tr>
<td>Banking Choice (Mean)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Branch</td>
<td>3.0015*</td>
<td>0.4228*</td>
<td>0.9954*</td>
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<td>Age</td>
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<td>(0.1075)</td>
<td>(0.0841)</td>
</tr>
<tr>
<td>Single</td>
<td>−12.7655*</td>
<td>1.0638*</td>
<td>0.0035*</td>
</tr>
<tr>
<td></td>
<td>(2.1437)</td>
<td>(0.3591)</td>
<td>(0.0011)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>−0.0020*</td>
</tr>
</tbody>
</table>

Note: * means significant at 1% level.
Variables

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<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std. Error</th>
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</thead>
<tbody>
<tr>
<td>$\theta_{2009,1} - \theta_{2007,1}$</td>
<td>$0.0215^*$</td>
<td>$(0.0006)$</td>
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<tr>
<td>$\theta_{2009,2} - \theta_{2007,2}$</td>
<td>$4.4718^*$</td>
<td>$(0.0067)$</td>
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<tr>
<td>$\theta_{2007,a}$</td>
<td>$5.0662^*$</td>
<td>$(1.2162)$</td>
</tr>
<tr>
<td>$\theta_{2009,a}$</td>
<td>$1.3071$</td>
<td>$(1.3071)$</td>
</tr>
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</table>

Loan Portfolio ($\phi$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2009</td>
<td>$-0.8472^*$</td>
</tr>
<tr>
<td>log(TA)</td>
<td>$-0.1048^*$</td>
</tr>
<tr>
<td>EC / TA</td>
<td>$-1.9589^*$</td>
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</table>

Bad Loans ($\theta_r$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_r$</td>
<td>$8.3241^*$</td>
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</table>

Bank Failure ($\theta^F$)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2009</td>
<td>$-0.3564$</td>
</tr>
</tbody>
</table>

Note: * means significant at 1% level.
shows that people become more cautious when engaging risky investment. The model-implied required return for an investor with the mean risk aversion to participate in the market portfolio is 1.77% before the crisis and 16.47% thereafter. Both of them are less than the corresponding expected market return so a person with the mean risk aversion will be happy to invest in the market portfolio.

In loan markets, the negative and significant effect of year 2009 on $slopes_{jt}$ confirms the recent financial crisis is a disaster to loan business. The inverse relationship between bank size and the chances of getting high return loans comes from the fact that large banks usually have many hierarchies in their organizations so they mainly rely on hard information to process loan applications. On the other hand, profitable loans may not always have enough hard information to signal their quality. As a result, firms with promising projects may seek funding from local small banks when their business potential cannot be reflected in financial documents. The capital asset to total asset ratio reflects a bank’s attitude toward risk taking. A high capital asset to total asset ratio means a bank operates conservatively and may not be enthusiastic about seeking high return loans. This has a negative impact on the $slopes_{jt}$. Using
the estimate of $\theta_r$, I know the actual loan loss percentage of a moderately risky bank\(^7\) in 2007 follows a distribution with mean 0.0241 and standard deviation 0.0483. The corresponding mean and standard deviation in 2009 are 0.0364 and 0.0551. Hence; both the magnitude and the volatility of bad loans fraction rises during financial crisis. Intuitively, a bank is like a soldier moving in a minefield and financial crisis increases the density of mines. Without any protective equipment, the soldier will easily get himself killed. To successfully get through the bad days, banks use reserves and equity to construct the firewall that can sustain them under the assaults of charge-offs. The estimate of $\theta^F$ shows that large banks tend to be more resistant to loan defaults and the effect is significant.

Using the estimated parameters, my model predicts the average loan return is 0.0782 in 2007 and 0.0608 in 2009. The actual loan returns from the data have mean 0.0775 in 2007 and 0.0596 in 2009.\(^8\) Moreover, my model predicts that an average bank's failure rate is 0.0015 in 2007 and 0.0339 two years later. The actual probability of a bank failure computed from the data is 0.0013 in 2007 and 0.0329 in 2009.\(^9\) It is clear that my model performs fairly well in capturing the key aspects of the retail banking industry.

\(^7\)Let the bank's expected loan loss be the mean of ALLL$^*$.

\(^8\)Table 1.3.

\(^9\)Computed using the actual number of failed banks in both years.
1.7 Counterfactual Analysis

In this section, I perform policy simulation to estimate the effect of the recent change in deposit insurance coverage on depositors’ welfare. Using my model estimates, I decompose the policy impact into several components and analyze them separately. Comparison of each piece with the status quo ante shows the benefits and losses on consumers’ surplus and explains the mechanism of deposit insurance in deposit markets. Given the rich variations on market heterogeneities, the policy fitness may differ across markets. For this reason, I also investigate the welfare effect under different market structures.

1.7.1 Policy Effect

Policy experiments are carried out in the market Charlotte-Gastonia-Rock Hill, NC-SC\textsuperscript{10}, 2009. There are two active banks in it, denoted as bank A and bank B. Using the estimated consumer preferences and market conditions, I solve for market outcome and compute consumers’ welfare for each of the following three scenarios.\textsuperscript{11}

S\textsubscript{1}. Deposit insurance coverage is $100,000.

S\textsubscript{2}. Deposit insurance coverage is $250,000 and banks’ deposit interest rates and risk caps are the same as those in S\textsubscript{1}.

S\textsubscript{3}. Deposit insurance coverage is $250,000 and banks re-optimize.

\textsuperscript{10}Charlotte-Gastonia-Rock Hill, NC-SC is the name of MSA.

\textsuperscript{11}I take 1000 draws when calculating consumers welfare.
The difference between consumer surplus in $S1$ and $S2$ is the direct effect which summarizes the benefits of extended coverage. The gap between consumer welfare in $S2$ and $S3$ is the indirect effect that measures the welfare losses from the induced downgrade of market discipline. The sum of these two effects yields the impact of the new policy on depositors’ welfare. To obtain a dollar-valued measure of welfare change, I create the equivalent deposit interest rate subsidy (tax) with which depositors’ welfare under the original coverage is the same as that under the new coverage. All effects are calculated at the individual level. Results are presented in table 1.6.

Under the old deposit insurance coverage ($S1$), bank A’s equilibrium deposit interest rate and risk cap is $(0.0151, 0.0425)$ while bank B’s is $(0.0078, 0.0339)$. After the new coverage comes into effect ($S3$), bank A reduces its deposit interest rate to 0.0068 and inflates its risk cap to 0.1184. Bank B responds similarly by lowering its deposit interest rate to 0.0071 and boosting its risk cap to 0.0964. It is clear that raising the deposit insurance coverage deteriorates market discipline which makes a bank a less appealing place to put money.

A quick inspection on column 1 in table 1.6 tells us that the recent policy change hurts depositors since the benefits from extra coverage fails to cover the losses from the deterioration of market discipline. On average, the losses in depositors’ welfare is equivalent to a 0.28% drop in deposit interest rates. Since all depositors suffer from the moral hazard of deposit insurance, whether an individual is able to gain from this policy change depends on the extent to which she is able to enjoy the surplus
from extended coverage. I repeat the policy experiment for depositors whose portfolio size are $50,000, $200,000, $350,000 and $500,000 and list the simulation results in column 2 to column 5 correspondingly. It is clear that welfare effect is heterogeneous across depositors and unfortunately, all depositors in this market lose in this policy reform. Column 2 provides an example that people do not profit from the extra coverage because their portfolio sizes are so small that the original insurance limit is enough to protect their funds at banks. These people are purely victims of the new policy. Indeed, for investors with a portfolio size of $50,000, the coverage change costs them 0.77% of their bank deposits. The last three columns display the cases in which the gain from additional coverage cannot compensate the loss from reduced market discipline, so depositors are worse-off. In general, people who have large portfolios are less likely to be against this policy because their funds at banks become safer.

<table>
<thead>
<tr>
<th></th>
<th>Market</th>
<th>$w = 50k$</th>
<th>$w = 200k$</th>
<th>$w = 350k$</th>
<th>$w = 500k$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direct Effect</strong></td>
<td>1656</td>
<td>0</td>
<td>587</td>
<td>3402</td>
<td>6984</td>
</tr>
<tr>
<td><strong>Indirect Effect</strong></td>
<td>−2920</td>
<td>−869</td>
<td>−2521</td>
<td>−5922</td>
<td>−10144</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>−1264</td>
<td>−869</td>
<td>−1934</td>
<td>−2520</td>
<td>−3160</td>
</tr>
<tr>
<td><strong>Equivalent Rate</strong></td>
<td>−0.28%</td>
<td>−0.77%</td>
<td>−0.65%</td>
<td>−0.35%</td>
<td>−0.23%</td>
</tr>
</tbody>
</table>

Note: $w = \text{portfolio size}$.
1.7.2 The Effect of Market Competition

This subsection explores the relationship between market competition and the policy impact on depositors welfare. I repeat the previous policy experiment by allowing only one bank to be active in the market. Since the market has two banks, I conduct the simulation twice. Table 1.7 compares the simulated welfare effects with that when both banks are present.

The first column denotes the welfare effect when competition is present while the rest denote welfare when market competition disappears. When only bank A is active, the policy reform makes it drop its deposit interest rate by 0.0021 and increase its risk cap by 0.0124. When only bank B serves the market, the new policy regime makes it decrease its deposit interest rate by 0.0014 and raise its risk cap by 0.0212. Comparing these changes with those in my previous experiment where the markets are competitive, it is clear that market competition exaggerates the moral hazard of deposit insurance. Intuitively, the opportunity cost of bankruptcy is a bank’s profit which diminishes as a market becomes more competitive. When competition is absent from the market, the cost of moral hazard is high. In fact, the desire to capture the monopoly profit counterbalances the incentive for boosting risk cap which leads to a greater chance of bankruptcy. When only one bank is active, the policy change costs depositors no more than 0.17% of their bank deposits. However, depositors lose 0.28% of their money at banks under the updated policy if both banks open their businesses. Hence, at least 40% of the loss in consumer welfare is attributed to market
competition.

<table>
<thead>
<tr>
<th></th>
<th>Market</th>
<th>Bank A only</th>
<th>Bank B only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Effect</td>
<td>1656</td>
<td>153</td>
<td>297</td>
</tr>
<tr>
<td>Indirect Effect</td>
<td>−2920</td>
<td>−920</td>
<td>−613</td>
</tr>
<tr>
<td>Total</td>
<td>−1264</td>
<td>−767</td>
<td>−316</td>
</tr>
<tr>
<td>Equivalent Rate</td>
<td>−0.28%</td>
<td>−0.17%</td>
<td>−0.07%</td>
</tr>
</tbody>
</table>

1.8 Conclusion

This paper investigates the impact of the recent change in deposit insurance coverage on depositors welfare and market discipline. It finds that raising the coverage limit to $250,000 weakens market discipline and disadvantages depositors in general. It shows that the new policy harms people with small portfolios more than to those who have large portfolios. Moreover, competition exaggerates the moral hazard of deposit insurance which makes the policy reform even less appealing to consumers.
Chapter 2

Underground Banking in the Emerging Market: Relationship as Collateral

2.1 Introduction

Underground banking, operating parallel to the formal banking system, describes the informal banking practice in an economy. Unlike regular banks which operate under state charters, the underground banks do not have state charters and conduct businesses through their owner’s personal network with other agents in the economy. The connections with local firms and residents help the underground bank to facilitate the money transfer from the depositors to the investors. In emerging markets like China,
the underground banks, such as private moneylenders or informal banks that are not chartered, are considered to be an illegal practice and therefore they are not protected by law. However, the actual execution of the law banning these underground banks by the government varies over time. Restrictions on the bank lending to small firms and the business inefficiency coming from the attendant bureaucracy of the regular banks make small businesses look for alternative sources of funding outside the formal banking system. This huge demand for credit generates a boom in China’s informal banking sector and accelerates the development of underground banks in China. While the existence of underground banking greatly alleviates the pain to small firms from credit rationing in the formal banking system, not every small enterprise is friendly toward underground banks. Due to the nature of underground banks, lenders seldom ask for collateral when granting loan offers. Since the underground banking is considered to be an illegal business practice, contracts between firms and underground banks are not enforced by law. Business fraud and intentional default, both of which are nightmares for lenders, arise in the context of underground lending. However, the underground banking market in China has not collapsed. Rather; it has been growing over time. In this paper, we use the relational contract to resolve this dilemma and show how the self-enforcing relational contract helps to sustain the lender-borrower relationship in the underground loan market. In fact, the difference in mechanisms that induce repayment is the fundamental difference between the regular banks and the underground banks. The former use third party actions, such as
court actions, while the latter rely on self-enforcing contracts. Regular banks usually require collateral, such as physical assets, when making loan offers. We interpret the ongoing lender-borrower relationship as the collateral collected by an underground bank from its borrower. The value of this collateral is the option value of funding future investments from the underground bank. Contrary to conventional wisdom, the collateral here is intangible and its value is endogenously determined by the expected future transactions between the firm and the underground bank. Nevertheless, this collateral ensures the self-enforcement of the loan contract with the underground bank when protection by the legal system is absent.

We develop a model of non-stationary relational contract that addresses the dynamic interactions between the entrepreneur (he) and the lender (she) in the context of underground banking. We identify the difference between regular banking and underground banking, and incorporate this difference into our framework by allowing the entrepreneur to borrow either from the underground bank or from the regular bank. Along the course of the lender-borrower relationship, we demonstrate how self-enforcing contracts alleviate the problem of hidden information and moral hazard so that transactions between these two parties are sustainable. We interpret the lender-borrower relationship as the intangible collateral kept by the lender and link the value of it to the loan rates of the optimal relational contract. We show how the lender uses this collateral when dealing with the entrepreneur.

We show that the framework of relational contract is not applicable if the entrepreneur
is a one-shot borrower. We find that the sequence of loan rates implied by the optimal relational contract satisfies the fastest price property, and the contract becomes in favor of the entrepreneur as the relationship advances. The lender uses the collateral to discipline the entrepreneur and to screen out the risky one. The value of the collateral grows over time and may eventually surpass the cost of repaying, so the entrepreneur who may default at the early stage of the relationship will choose to repay after the relationship passes some threshold. We show that there is a tension between the effectiveness of the collateral policy in achieving each goal for the lender. When the lender successfully uses the collateral to discipline the entrepreneur, she is not able to screen out the entrepreneur who is risky, and vice versa.

Our work is related to the literature of relational contracts. Baker and Gibbons (2002) study the optimal relational contracts within firms and between firms and how these two types of contracts differ. Levin (2003) studies the self-enforced relational contracts in the context of hidden information, moral hazard and subjective performance measures and shows that the optimal contract can be stationary. Bentley (2003) studies relational contracts in a principal-agent model in which subjective evaluation is allowed. M. Brown, Falk, and Fehr (2004) investigate how the absence of third-party enforcement affects relational contracts and the nature of market interactions. Fuchs (2007) studies how optimal relational contracts resolve the moral hazard problem in the context of the labor market where private monitoring is present. Board (2011) studies how relational contracts can be used to overcome the holdup problem.
Mukherjee and Vasconcelos (2011) study the optimal relational contract in a multi-task environment. Halac (2012) explores how information structure on the value of the relationship affects the optimal relational contract in a principal-agent model. Li and Matouschek (2013) study how relational contracts help to resolve the conflict between the manager and the worker in a company. Yang (2013) studies how the non-stationary relational contracts determine the wage dynamics inside a firm. We contribute to this active area of research by examining the optimal relationship contract in underground banking in the emerging markets. We interpret the value of the lender-borrower relationship as the collateral collected by the lender to ensure repayment. In contrast to the collateral requirement in the static lender-borrower setting, we show that the value of collateral is increasing in the lender’s belief on the probability that the entrepreneur is a good borrower.

Our work contributes to the literature on informal banking. Allen, Qian, and Qian (2005) study how the informal financial system in China supports the fast growth of private firms. Straub (2005) examines the market condition under which the informal banking is more efficient than the formal banking. Anderson, Baland, and Moene (2009) study the enforcement issue in the context of rotating savings and credit associations in the developing economy. Ayyagari, Demirgüç-Kunt, and Maksimovic (2010) study the nature of both the formal banking system and the informal banking system in China and investigates how they affect the financing patterns of firms. (Turvey & Kong, 2010) provide empirical evidence that the majority of farm
households tend to use informal banking to finance their businesses. Ordonez (2013) explores how a combination of regulation and reputation can improve the shadow banking system. Gennaioli, Shleifer, and Vishny (2013) study how the shadow banking system improves welfare under rational expectation and shows that the system may collapse when tail risks are neglected. Madestam (2014) studies a credit market in which both formal banks and informal banks exist and shows that these two types of credits can be complement or substitute each other. Our approach allows us to examine the enforcement mechanism in underground banking in a dynamic context. We find that the lender-borrower relationship is sustainable in the dynamic context even when government regulation does not exist in the market.

Our work also contributes to the literature of emerging markets. Mahmood and Mitchell (2004) and Chang, Chung, and Mahmood (2006) study the relationship between business groups and innovation in the emerging market. Shen and Xiao (2014) study the entry and demand expansion in the fast-food chain industry in China. Bertrand, Mehta, and Mullainathan (2002), Fisman and Wang (2010) and Jia, Shi, and Wang (2013) study the relationship of coinsurance in business groups in emerging markets such as China and India. Our work contributes to this literature by examining the underground banking system in the emerging market. The lack of legal protection and the strict controls of the formal bank’s credit provision generate the special nature of the informal banking system in China. We show how the mechanism of reputation makes this system sustainable.
The remainder of this paper is organized as follows. Section 2 presents the model setup. Section 3 gives the preliminary analysis for our model. Section 4 shows the optimal contract and relates it to the value of the collateral. Section 5 concludes. All missing proofs are in the appendix.

2.2 Model Setup

We consider the lender-borrower interaction in the underground banking context. There are two risk neutral agents, an entrepreneur and a lender, both of which live forever and share the same discount rate $\delta \in (0, 1)^1$. Time is discrete, indexed by $t = 1, 2, \ldots$. At the beginning of time $t$, the entrepreneur is endowed with some illiquid asset $A_t > 0$ and a risky project which needs a unit of funds to finance. The illiquid asset cannot be used to fund the project unless the entrepreneur liquidizes it in the asset market. If the investment is successful, the project yields the entrepreneur a net return of $R > 0$. Otherwise, the entrepreneur is left with nothing. The probability of a successful investment depends on the entrepreneur’s business talent. The entrepreneur can either be a high ability investor (type $H$) or a low ability investor (type $L$). We assume the entrepreneur’s type is the entrepreneur’s private information. Let $p_i > 0$ be the probability of success when the project is carried out by a type $i$ entrepreneur. A high ability entrepreneur is more likely to succeed than a low ability one. To facilitate our analysis, we normalize $p_H$ to unity, so $0 < p_L < p_H = 1$. 

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1 Throughout the paper, we use “entrepreneur” and “borrower” interchangeably.
Let $\beta \in (0, 1)$ denote the probability that the entrepreneur is type $H$. We denote the realized outcome of the project by $y_t(i), i \in \{H, L\}$. Then

$$y_t(i) = \begin{cases} 1 + R & \text{with probability } p_i \\ 0 & \text{with probability } 1 - p_i \end{cases}$$

There are three ways for the entrepreneur to fund his project. First, he can liquidate his assets in the market and use the money exchanged to finance his project. We assume the entrepreneur incurs a liquidation cost $K > 0$ when selling his assets in the market. In the emerging economy, the capital market may not be well developed, so finding a buyer who is willing to pay for assets is not easy. Even in a well developed capital market, liquidation can be very costly, and sometimes may require the seller to offer a steep discount to complete the deal. Because the bargaining power goes to the buyer especially when the seller needs cash immediately. Thus, here we assume $K > R$, so financing by selling the illiquid asset is unprofitable for the entrepreneur.

Alternatively, the entrepreneur can borrow from a regular bank. In emerging markets, the loan application process at a regular bank can be very complicated and inefficient. The entrepreneur has to spend lots of efforts to get the loan application approved and then wait for weeks before funds are available for the project. We denote the cost incurred during this loan application by $C > 0$. Let $\hat{r}_t$ be the loan interest rate offered by the regular bank. We assume $\hat{r}_t = \hat{r} > 0, \forall t$, so the loan rate at the regular bank is constant over time. Finally, the entrepreneur can borrow from an underground

\[\text{In emerging markets in which the interest rates are regulated, such as China, it is possible that}\]
bank. The loan application process at an underground bank is much more simpler. If the lender knows the borrower, sometimes a phone call is sufficient to complete the deal. Usually the owner of an underground bank is in charge of its loan businesses, so she has incentives to make the loan application process simple and efficient. Hence, we assume the entrepreneur does not incur any transaction cost when borrowing from the underground bank.

The lender can access one unit of funds at the unit cost at the beginning of time $t$. If the relationship continues, she grants the loan offer to the entrepreneur at the rate $r_t \leq R$. Then the entrepreneur makes his offer acceptance decision $a_t \in \{0, 1\}$. If the entrepreneur accepts the offer ($a_t = 1$), the project is funded. Otherwise ($a_t = 0$), the relationship ends and both of them take their outside options forever.

Let $\pi > 0$ denote the value of the lender’s outside option. The outside option for the entrepreneur is either to borrow from a regular bank or to give up the investment opportunity.

Once the investment is made, the project outcome realizes and is observed by the entrepreneur only. If the entrepreneur borrows from a regular bank to fund his project, he has to repay the principal and the interest regardless of the project outcome. Because the loan contracts with regular banks are protected by law, the regular bank can seek third party actions, such as court actions, to ensure the loan repayment. We assume this process is completed by the end of time $t$. If the project fails, the loan rate does not change over time if the performance of the overall economy is stable.
entrepreneur has to sell his illiquid assets to repay the debt. On the other hand, since the transactions with the underground bank are not protected by law, the entrepreneur can default and run away if he borrows from the underground bank. When he chooses to repay, he prepares the money from two sources, the project return and the liquidation of the asset. It is suboptimal for the entrepreneur to repay by borrowing from the regular bank, because doing so incurs additional costs for him and will not defer the actual payment\(^3\). We assume \( A_t = A > 1 + R + K, \forall t \), so it is always feasible for the entrepreneur to liquidate the asset to repay the loan. Let \( d_t \in \{0, 1\} \) denote the entrepreneur’s repay decision.

---

\(^3\)The entrepreneur has to pay the transaction cost \( C \) for borrowing from a regular bank. The amount of money he owes increases because the interest rate asked by the regular bank is positive. Notice that the payment to the regular bank is completed by the end of time \( t \), the entrepreneur now has to pay more to clear his debt.
After learning the entrepreneur’s repay decision, the lender decides whether to continue the ongoing lender-borrower relationship $b_t \in \{0, 1\}$. If the lender terminates the relationship ($b_t = 0$), starting from next period, both of them take their outside options. Figure 2.1 shows the time line of the stage game.

2.3 Preliminary Analysis

Before digging into the dynamic interactions between the lender and the entrepreneur, we first consider the situation where the underlying lender-borrower relationship is a one-shot deal. Since the loan repayment is secured by law, a regular bank is willing to lend money to the entrepreneur. On the other hand, however, the entrepreneur is never able to borrow from the underground bank. Unlike a regular bank, the underground bank stands outside the umbrella of the legal enforcement mechanism, and has to bear the loss if the entrepreneur defaults. Of course, the underground bank can refuse to lend to the entrepreneur in the future if he defaults. However, this will not make the current period loan contract self-enforcing. If we consider the lender-borrower relationship as the collateral that the lender uses to discipline the entrepreneur, we will know that it never works after realizing the value of the collateral is zero. In fact, the value of this collateral is the value of the ongoing lender-borrower relationship, which is determined by the expected future transactions. Since the entrepreneur never borrows again, he does not value the future at all. Hence, the value of the collateral vanishes. The following lemma summarizes this result.
Lemma 2.1. If the entrepreneur is a one-shot borrower, then the value of the collateral (lender-borrower relationship) is zero and the loan contract with the underground bank is not self-enforcing. Hence, the entrepreneur never repays and the lender never lends.

The difference in the mechanisms that induce loan repayment is the fundamental difference between a regular bank and an underground bank. The former uses third party actions while the latter relies on self-enforcing contracts to ensure the loan repayment. When the entrepreneur is a repeat borrower, he values the option of borrowing from the underground bank in the future, so the collateral is valuable to him. The theory of collateral tells that the loan contract is self-enforcing if the value of the collateral is sufficiently large. Thus, it is possible to find a self-enforcing contract for an underground bank when the entrepreneur is a repeat borrower. For the rest of this paper, we explore the interactions between an entrepreneur and an underground lender in a dynamic context.

The solution concept is the Perfect Public Equilibrium. In each period, the lender’s loan offer $r_t$, her decision on the relationship continuation $b_t$, the entrepreneur’s decision on the loan offer acceptance $a_t$ and his repay decision $d_t$ are publicly observed. Write $h_t = \{r_t, b_t; a_t, d_t\}$, then the time $t$ history $H_t = \bigcup_{\tau=1}^{t-1} h_t$. The lender’s strategy $\sigma_L = \{(r_t, b_t)\}_{t=1}^{\infty}$, where $r_t : H_{t-1} \to R$ and $b_t : H_{t-1} \cup \{r_t, a_t, d_t\} \to \{0, 1\}$, specifies the loan rate and the lender’s decision on the relationship continuation in each period as a function of the public history. The entrepreneur’s strategy $\sigma_E = \{(a_t, d_t)\}_{t=1}^{\infty}$,
where \( a_t : H_{t-1} \cup \{ r_t \} \rightarrow \{ 0, 1 \} \) and \( d_t : H_{t-1} \cup \{ r_t, a_t, y_t \} \rightarrow \{ 0, 1 \} \), describes his choice on whether to take the loan offer and whether to repay as a function of the public history. A relational contract is defined to be a strategy profile \((\sigma_L, \sigma_E)\) demonstrating a complete plan for a relationship.

Similar to many other dynamic games, multiple equilibria may exist in our model. Clearly, no offer, no borrowing and no repay in each period is an equilibrium. Given the entrepreneur never repays, the lender does not want to make any loan offer. Given the lender never lends, the entrepreneur has no incentive to accept the offer and repay.

To make our analysis interesting, we focus on the equilibrium contract satisfying a set of properties.

**Property 2.1.** *If the entrepreneur’s ability is high (type H) and both agents never deviate in the past, then \( y_t(H) = R \) implies \( d_t = 1 \) for every \( t \).*

Property 2.1 implies the type \( H \) entrepreneur always repays if the investment is successful. Unlike a regular bank, transactions in the underground banking system are not protected by law. If a default occurs, an underground lender has to bear the loss because she cannot get the money back by filing a lawsuit. Hence, a contract that induces the entrepreneur to repay is important for an underground bank lender.

Moreover, the role of the underground banking system in the emerging market is to finance small businesses which are profitable but cannot get funds from the formal banking system. This property is consistent with the position of the underground banking system in the emerging market.
Property 2.2. For the type i entrepreneur, if \( d_t = 1 \) for some \( t \) and both agents never deviate in the past, then \( y_t'(i) = y_t(i) \) implies \( d_{t'} = 1 \), for every \( t' > t \).

Property 2.2 says if the entrepreneur chooses to repay at time \( t \), then he will also repay in the future when facing the same situation. In other words, the equilibrium behavior of the entrepreneur over the course of the relationship is stable. It is uncommon that the lender designs the loan rate dynamics in a way that is conditional on the project outcome, that is, allowing the entrepreneur to make the payment today but refuse to repay tomorrow. If the lender can use the collateral to discipline the entrepreneur now, then she should be able to use it under the same situation in the future.

Property 2.3. The relationship contract \((\sigma_L, \sigma_E)\) is a grim trigger strategy profile.

Once a deviation occurs, \( d_t = 0 \) and \( b_t = 0 \) for all possible \( t \).

We focus on the grim trigger strategies for both agents because they are tractable and produce the most severe punishment. If the entrepreneur defaults on a loan, the lender never lends him in the future. This is in line with the mechanism of collateral that the borrower loses the collateral if he fails to pay back the loan. Funds at the underground bank usually come from the deposits absorbed from local residents. Similar to a regular bank, an underground bank loses money if the lender cannot receive the loan payment on time. Hence, the lender has incentives to punish the entrepreneur if he fails to return the money. The same story applies to the entrepreneur. The lender wants to use the future interest rate discount to encourage today’s loan repayment. However, she may break the promise when the future actually comes. A threat of
no repayment from the entrepreneur can eliminate the lender’s incentive to renege.

A detailed discussion of managing conflicts when a deviation occurs is beyond the
scope of this paper, so we will not consider the strategy that alternates between
“cooperation” phases and “punishment” phrases, as in Green and Porter (1984).

2.4 Optimal Contracts

In this section, we identify the optimal relational contracts by following a number
of steps. We first consider the entrepreneur’s equilibrium strategy and then solve
for the lender’s optimal loan offer. We present the value dynamics of the underlying
relationship and link it to the value of the collateral kept by the lender. Moreover, we
show that the theory of collateral in the static setting can be applied to the dynamic
context of underground banking where the lender does not require any collateral (in
the traditional sense) from the entrepreneur.

We assume, \( p_L(1 + R) > 1 + \hat{r} + C + (1 - p_L)K \), the project is always profitable
when financed by the regular bank loan. Then borrowing from a regular bank is
a feasible outside option for both types of entrepreneurs. The lender is unable to
detect the entrepreneur’s type by checking the availability of his outside options. The
only possible way for the lender to identify the entrepreneur’s type is to look at his
decisions on repaying over time.

Notice that the entrepreneur here can always borrow from the regular bank. We
need to restrict the value of the lender’s outside option so that borrowing from the
underground bank is reasonable. The expected cost of borrowing from the regular bank is $1 + \hat{r} + C + (1 - p_i)K$ and the minimum average amount of repayment required by the lender is $1 + \pi$. Under the grim trigger strategy, the entrepreneur can renege at most once, so the discounted expected future project funding cost when the entrepreneur defects is $\delta \frac{1}{1-\delta} (1 + \hat{r} + C + (1 - p_i)K)$. Since the value of the collateral is the expected cost savings in the future project funding, it is equal to the difference between the cost of funding of two banks. The expected cost of future funding at the regular bank is $\delta \frac{1}{1-\delta} (1 + \hat{r} + C + (1 - p_i)K)$ and that at the underground bank is $\sum_{\tau=1}^{\infty} \delta^\tau (1 + r_{t+\tau} + (1 - p_i)K)$. Then the value of the collateral is $\delta \frac{1}{1-\delta} (1 + \hat{r} + C) - \sum_{\tau=1}^{\infty} \delta^\tau (1 + r_{t+\tau})$. We need this value to be greater than the current period minimum possible loan payment $1 + r_t$ to guarantee the existence of the self-enforcing contract. Notice that, $\sum_{\tau=0}^{\infty} \delta^\tau (1 + r_{t+\tau}) \geq \frac{1}{1-\delta} (1 + \pi)$, we assume

\[ (1 + \pi) \leq \delta (1 + \hat{r} + C) \tag{2.1} \]

In emerging markets, due to the lack of business regulation, a regular bank’s loan service is inefficient. When applying for a loan at a regular bank, in addition to the heavy paperwork, an entrepreneur has to do whatever he can to please the loan manager in order to obtain approval of his application. This can be extremely costly for small enterprises, because regular banks usually target the large companies and are reluctant to lend money to small businesses. As a result, small enterprises turn to the underground banking system for their funds. In fact, the high transaction cost
of the regular bank borrowing makes the underground bank borrowing appealing to small companies, and creates the foundation of the lender-borrower relationship in the underground loan market.

When the deal makes money, the lender wants to finance the project whenever possible. According to the three properties mentioned before, the equilibrium strategy for the lender takes the following form. The lender offers the loan in the first period and continues making the offer until the entrepreneur either defaults or rejects the offer. A detailed examination over the entrepreneur’s strategy tells that the borrower will always accept the offer because the associated cost is zero as the repay agreement is not binding. Moreover, the type $H$ entrepreneur’s probability of success is unity. Based on the two properties discussed above, in equilibrium, the type $H$ entrepreneur accepts the offer and repays in the first period, and continues doing so until a deviation occurs. The equilibrium behavior of the type $L$ entrepreneur depends on the evolution of the value of the collateral throughout the relationship, and will be examined later in this section. Indeed, the dynamics of the collateral value tell us how effectively the lender can discipline the entrepreneur over time.

Based on the equilibrium loan repayment patterns, We divide the rest of our analysis into two parts.
2.4.1 The Regular Bank is Moderately Inefficient

In this subsection, we consider that the inefficiency of a regular bank’s loan service is moderate. The value of the collateral, therefore, is not large enough to induce the type $L$ entrepreneur to repay when the project fails throughout the relationship. By property 2.2, we focus on the relational contracts that in equilibrium, the type $L$ entrepreneur does not repay when the project fails until the relationship lasts for $t$ periods. Notice that the cost of repaying for the type $L$ entrepreneur when the project fails at time $t$ is $1 + r_t + K$, and the value of the collateral is $\frac{\delta}{1 - \delta}(1 + \hat{r} + C) - \sum_{\tau=1}^{\infty} \delta^\tau (1 + r_{t+\tau})$, the default occurs if

$$1 + r_t + K > \frac{\delta}{1 - \delta}(1 + \hat{r} + C) - \sum_{\tau=1}^{\infty} \delta^\tau (1 + r_{t+\tau})$$

Using the condition of the lower bound of the loan rates\(^4\), the sufficient condition of default for the type $L$ entrepreneur when the project fails is given by the following inequality.

$$K \geq \frac{\delta}{1 - \delta}(1 + \hat{r} + C) - \frac{1}{1 - \delta}(1 + \pi) \quad (2.2)$$

The left hand side of inequality 2.2 is the cost of liquidation and the right hand side of it is the supremum of the collateral value net of the current loan payment, which measures the benefit of liquidation. When the project is unsuccessful, the only way for the entrepreneur to get the money available for repayment is to sell his illiquid

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\(^4\)The condition is $\sum_{\tau=0}^{\infty} \delta^\tau (1 + r_{t+\tau}) \geq \frac{1}{1 - \delta}(1 + \pi)$ as shown above.
assets. Hence, default occurs if liquidation is too expensive. In this subsection, we study the optimal relational contract under this condition.

At time $t$, the cost of repaying for the type $H$ entrepreneur is the principal and the time $t$ interest rate. Consider the one-shot deviation that the entrepreneur refuses to return the money. The benefit of the deviation is the saving of the time $t$’s repayment $1 + r_t$. The cost of the deviation is losing the collateral, whose value is given by the time $t$ discounted future funding cost saving $\sum_{\tau=1}^{\infty} \delta^\tau (1 + \hat{r} + C - (1 + r_{t+\tau}))$. No one-shot deviation requires the value of the collateral surpasses the benefit of default. Hence, the no renege condition for the type $H$ entrepreneur is

$$\sum_{\tau=0}^{\infty} \delta^\tau (1 + r_{t+\tau}) \leq \frac{\delta}{1 - \delta} (1 + \hat{r} + C) \quad (2.3)$$

Now we look at the no deviation condition for the type $L$ entrepreneur. First, we specify the no renege condition for him when the project is successful. In each period, the lender terminates the relationship with the type $L$ entrepreneur with probability $1 - p_L$. Conditional on today’s repayment, the expected loan payment to the lender in the next period is $p_L(1 + r_{t+1})$ and the probability of resuming the relationship after the next period is $p_L$. Recall that the value of the collateral is the savings of expected future project funding cost, and we now have the time $t$ collateral value, which is

$$\sum_{\tau=1}^{\infty} (p_L\delta)^{\tau-1} \delta (1 + \hat{r} + C + (1 - p_L)K - p_L(1 + r_{t+\tau}))$$

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The single period expected cost savings for the type $L$ entrepreneur $1 + \hat{r} + C + (1 - p_L)K - p_L(1 + r_{t+\tau})$ is higher than that for the type $H$ entrepreneur. Because it is more expensive for the type $L$ entrepreneur to borrow from a regular bank and he can default when borrowing from an underground bank. On the other hand, the type $L$ entrepreneur discounts the expected future cost savings more heavily than the type $H$ entrepreneur does. This comes from the fact that the type $L$ entrepreneur can enjoy the savings only when the project is successful, so he values them less than the type $H$ entrepreneur does, who has a higher probability of project success. The type $L$ entrepreneur does not renege when the project is successful if the value of the collateral dominates the benefit of doing so. Hence, the no renege condition for the type $L$ entrepreneur when the project is successful is

\[ 1 + r_t \leq \sum_{\tau=1}^{\infty} (p_L \delta)^{\tau-1} \delta (1 + \hat{r} + C + (1 - p_L)K - p_L(1 + r_{t+\tau})), \quad \forall t \geq 1 \quad (2.4) \]

In the next step, we compare this no renege condition with the one for the type $H$ entrepreneur.

**Lemma 2.2.** The no deviation condition for the type $H$ entrepreneur implies the no deviation condition for the type $L$ entrepreneur when the project is successful.

Lemma 2.2 says conditional on the project success, the type $L$ entrepreneur is more willing to repay than the type $H$ one. In fact, the value of the collateral is higher for the type $L$ entrepreneur as he gains more from the relationship than the type $H$ entrepreneur does. Consider the off equilibrium strategy for the type $L$ entrepreneur
that he always repays. The value of the collateral depreciates because repaying when
the project fails is suboptimal for him. On the other hand, the value of the collateral
determined by this off equilibrium strategy is exactly the same as that for the type
$H$ entrepreneur. Notice that the cost of repaying is the same for both types of
entrepreneurs when the project is successful, the fact that the type $H$ entrepreneur
repays in the equilibrium implies that the type $L$ entrepreneur is also willing to repay
conditional on the project success.

Following the same argument, the no renege condition for the type $L$ entrepreneur
when the project fails is

$$1 + r_t + K > \sum_{\tau=1}^{\infty} (p_L \delta)^{\tau-1} \delta (1 + \hat{r} + C + (1 - p_L)K - p_L(1 + r_{t+\tau})), \quad \forall t \geq 1$$

Since we focus on the case when the transaction cost $C$ is moderate, so that in
equilibrium, the type $L$ entrepreneur does not repay when the project fails, we assume
$K \geq \frac{\delta}{1-\delta} (1 + \hat{r} + C) - \frac{1}{1-\delta}$. Then the no renege condition for the type $L$ entrepreneur
when the project fails always holds.

Now we look at the lender’s no deviation condition. As mentioned before, the lender
may refuse the offer the promised loan rate discount to the entrepreneur when the
time actually comes. It is important for the lender to keep her promise so that the
relationship is sustainable. Let $\alpha_t$ be the probability of repaying at time $t$ and $\beta_t$ be
the lender’s time $t$ belief of the probability that the entrepreneur is type $H$. Then we
have $\alpha_t = \beta_t + (1 - \beta_t)p_L$, $\beta_{t+1} = \frac{\beta_t}{\alpha_t}$, and $\beta_1 = \beta$. It follows that

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\[
\beta_t = \frac{1}{1 + p_L^{t-1}(\frac{1}{\beta} - 1)}, \quad \forall t \geq 1
\] (2.5)

The lender’s belief of the probability that the entrepreneur is type \(H\) is increasing over time, so the lender gradually learns the entrepreneur’s type throughout the relationship. She uses the lender-borrower relationship as the collateral to continuously screen out the type \(L\) entrepreneurs. This mechanism is consistent with the traditional theory of collateral in the static context. As the type \(L\) entrepreneur gradually fades out, the lender knows that the entrepreneur is type \(H\) with probability one at the end of the day. Hence, when the transaction cost \(C\) is moderate, the collateral is an effective screening device. The probability of repaying \(\alpha_t\) is increasing over time as well. The more likely it is that the entrepreneur is type \(H\), the higher is the probability that he will repay.

Let \(V(t)\) be the lender’s equilibrium value function at time \(t\), then we have

\[
V(t) = \alpha_t(r_t + \delta V(t + 1)) + (1 - \alpha_t)(-1 + \frac{\delta}{1 - \delta \pi}), \quad \forall t \geq 1
\] (2.6)

The lender’s time \(t\) value function has two parts, based on the entrepreneur’s repay decision. If he repays, the lender earns the interest \(r_t\) and the relationship proceeds to the next period. Otherwise, she loses the principal, and starting from the next period, she takes her outside option forever. Then the no deviation condition for the lender is given by the following inequality.

\[\]
\[ V(t) \geq \frac{1}{1 - \delta} \pi, \quad \forall t \geq 1 \]  

(2.7)

The right hand side of inequality (2.7) is the payoff if she reneges. Then the lender will not deviate if doing so is suboptimal. Combining it with the entrepreneur’s no renego condition, we write the lender’s profit maximization problem as follows.

\[
\max_{\{r_t\}_{t=1}^{\infty}} V(1) \quad (2.8)
\]

such that (2.3) and (2.7) hold

We claim that the optimal loan contract has the fastest price property, by which we mean in equilibrium, the entrepreneur makes payments to the lender as early as possible. To see this, we need to realize that both parties value their payment streams differently over time. If we ignore the discounting, by inequality (2.3), the type \( H \) entrepreneur does not have any preference over the timing of the payments. In contrast with the type \( H \) entrepreneur, the lender strictly prefers early payments to later ones, because she does not want to share the value of the relationship with the type \( L \) entrepreneur, who will default eventually. Therefore, the lender charges as much as possible during the early stage of the relationship and stay at par thereafter.

We need the following two lemmas to completely characterize the optimal loan rate pattern.

**Lemma 2.3.** In equilibrium, if \( V(t) = \frac{1}{1 - \delta} \pi \), then \( V(t') = \frac{1}{1 - \delta} \pi \), \( \forall t' > t \).

**Lemma 2.4.** In equilibrium, if \( r_t < R \), then \( V(t + 1) = \frac{1}{1 - \delta} \pi \), \( \forall t \geq 1 \).
Lemma 2.3 describes the pattern of the lender’s value function in equilibrium. Since
the lender adopts the fastest price strategy, she cares today more than the future, so
her value function is decreasing over time. The lower bound of her value function is
\( \frac{1}{1-\delta \pi} \), given by inequality (2.7). Then the lender’s value function stays at this level
after hitting the lower bound. Lemma 2.4 links the lender’s value function to the
equilibrium loan rates. It tells us that the only reason the lender does not charge \( R \)
is that she is unable to do so. A high interest rate today must be compensated by
the low interest rates in the future, so that entrepreneur will not renege. However, if
the future interest rates are too low, the lender will renege. Therefore, the interest
rate asked by the lender today cannot make her future promise incredible.

By lemma 2.4, the lender sets the loan rate to be \( R \) in the first period and continues
doing so whenever possible. Since \( p_L(1 + R) \geq 1 + \hat{r} + C + (1 - p_L)K \), we have
\( 1 + R \geq 1 + \hat{r} + C \), so the lender will stop charging \( R \) after some finite periods. By
lemma 2.3 and lemma 2.4, starting from two periods after the stopping period, the
lender’s value function is equal to \( \frac{1}{1-\delta \pi} \). This gives the loan rates for the periods that
are two periods after the stopping time. The only rate that is left unknown is the
loan rate for the time that is one period after the stopping point. Given that all other
loan rates are known, it can be solved by looking at the no deviation condition for
the type \( H \) entrepreneur.

Formally, let \( r_t^+ = \frac{1+\pi}{\alpha_t} - 1 \), then \( r_t^+ \) solves \( V(t) = \alpha_t(r_t^+ + \delta V(t + 1)) + (1 - \alpha_t)(-1 + \frac{\delta}{1-\delta \pi}) \) when both \( V(t) \) and \( V(t + 1) \) are \( \frac{1}{1-\delta \pi} \). Hence, \( r_t^+ \) is the lender’s reservation
price when she stays at par at time $t$. Let $T$ be defined by the following equation.

$$T = \max\{t \in \mathbb{N} | \sum_{\tau=1}^{t} \delta^{\tau-1}(1 + R) + \sum_{\tau=t+1}^{\infty} \delta^{\tau-1}(1 + r^+_\tau) \leq \frac{\delta}{1 - \delta}(1 + \hat{r} + C)\} \quad (2.9)$$

Then $T$ is the last period that the lender charges $R$ in equilibrium. Based on the discussion above, starting from period $T + 2$, the loan rate is $r^+_t$, for every $t \geq T + 2$.

Finally, the time $T + 1$ loan rate $r^*_{T+1}$ solves the following equation.

$$\sum_{\tau=1}^{T} \delta^{\tau-1}(1 + R) + \sum_{\tau=T+2}^{\infty} \delta^{\tau-1}(1 + r^+_{\tau}) + \delta^T(1 + r^*_{T+1}) = \frac{\delta}{1 - \delta}(1 + \hat{r} + C) \quad (2.10)$$

We summarize the optimal loan rates and the lender’s belief updating process along the equilibrium path by the following proposition.

**Proposition 2.1.** When the transaction cost is moderate,

i) Under the optimal relational contract, the loan rates are $r^*_t = R$, if $t \leq T$; $r^*_t = r^+_{t}$, if $t \geq T + 2$ and $r^*_{T+1}$ is defined by equation (2.10).

ii) The lender uses the collateral (lender-borrower relationship) to discipline the entrepreneur and to screen out the type $L$ entrepreneur over time. When the transaction cost is moderately large, the lender gradually learns the entrepreneur’s type throughout the relationship. At the end of the day, the lender screens out the type $L$ entrepreneur with probability one.
Notice that the probability of repaying $\alpha_t$ is increasing over time in equilibrium, $r_t^+$ is decreasing in $t$. It is clear that the optimal loan rates are non-increasing over time, because $R = r_T^+ > r_{T+1}^+ > r_{T+2}^+$ and $r_t^+ = r_t^+$ after the time $T+2$. Figure 2.2 illustrates the equilibrium loan rates under the optimal relational contract. Then the optimal relational contract is in favor of the entrepreneur as the relationship advances.

Borrowing from the underground bank becomes cheaper as the relationship deepens, so the value of the collateral is increasing over time. The entrepreneur has more incentives to repay during the later stage of the relationship.

The lender wants to use the future loan rate discounts as the reward to induce the entrepreneur to repay. She knows the type $L$ entrepreneur will default eventually because the liquidation is too expensive, so she wants to minimize the reward to him. That is why the lender adopts the strategy satisfying the fastest price property.
Moreover, this strategy raises the value of the collateral over time and provides further incentives for the entrepreneur to repay. Unfortunately, due to the restriction on the transaction cost $C$, these incentives are not strong enough to convince the type $L$ entrepreneur to repay when the project fails. The following proposition states these results.

**Proposition 2.2.** When the transaction cost is moderate,

i) In equilibrium, the optimal relational contract becomes in favor of the entrepreneur as the relationship advances.

ii) In equilibrium, the value of the collateral is increasing over time, so the entrepreneur has more incentives to repay during the later stage of the relationship.

### 2.4.2 The Regular Bank is Very Inefficient

In this subsection, we explore the optimal relational contract when the regular bank’s loan service is extremely inefficient. Suppose the transaction cost $C$ is sufficiently large so that $R < K < \frac{\delta}{1-\delta}(1+\hat{r}+C) - \frac{1}{1-\delta}(1+\pi)$. Our previous analysis shows that if the transaction cost is moderate, in equilibrium, the type $L$ entrepreneur never repays when the project fails. Then it is natural to ask whether it is possible that in equilibrium, conditional on the project failure, the type $L$ entrepreneur ever repays. The answer is affirmative. The following proposition provides the sufficient condition under which the type $L$ entrepreneur may repay when the project fails.

**Proposition 2.3.** In equilibrium, if $K < \frac{\delta}{1-\delta}(1+\hat{r}+C) - \frac{1}{1-\delta}(1+\pi)$, then there
exists some $t$ such that at time $t$, the type $L$ entrepreneur repays when the project fails.

Proposition 2.3 says the type $L$ entrepreneur may repay when the project fails if the cost of repaying is less than the supremum of the value of the collateral. Notice that the value of the collateral is determined by the expected future transactions between the two agents, the collateral value varies as the position of the relationship changes. If for some collateral, its value is greater than the repaying cost, then the lender can use this collateral to discipline the type $L$ entrepreneur. As a result, when the relationship moves to the position where its corresponding collateral is this one, the type $L$ entrepreneur will repay when the project fails.

Formally, suppose the contrary is true, then the type $L$ entrepreneur never repays when the project fails. The value of the collateral $\sum_{\tau=1}^{\infty} (p_L \delta^{\tau-1} \delta (1 + \hat{r} + C + (1 - p_L)K - p_L(1 + r_{t+\tau})))$ is less than the cost of repaying $1 + r_t + K$ for every $t$. Hence, we have

$$1 + r_t + K > \sum_{\tau=1}^{\infty} (p_L \delta^{\tau-1} \delta (1 + \hat{r} + C + (1 - p_L)K - p_L(1 + r_{t+\tau})), \quad \forall t \geq 1$$

If the type $L$ entrepreneur never repays in equilibrium, then the lender’s profit maximization problem is exactly the same as that in the previous subsection. Therefore, we have $\lim_{t \to \infty} r_t = \lim_{t \to \infty} r_t^+ = \pi$. Now let $t$ go to infinity for both sides of the inequality above, we get the inequality (2.2) and arrive at a contradiction.

From proposition 2.3, we know that the type $L$ entrepreneur may repay even when
the project fails. How does this relate to the evolution of the value of the collateral? Before answering this question, we have to characterize the optimal relational contract. According to the three properties mentioned before, we consider the relational contract that is in equilibrium, the type $L$ entrepreneur does not repay when the project fails until the relationship lasts for $\hat{t}$ periods. We call this relational contract self-enforcing if neither of the agent wants to deviate. The no renege condition for the type $H$ entrepreneur remains the same and is given by the inequality (2.4). The no renege condition for the type $L$ entrepreneur changes because the value of the collateral depends on the date, starting from which he always repays regardless of the project outcome. Let $U(t)$ be the value of the collateral for the type $L$ entrepreneur at time $t$. Then we have

$$U(t) = \delta(1 + \hat{r} + C + (1 - p_{t+1})K - p_{t+1}(1 + r_{t+1}) + p_{t+1}U(t + 1))$$

where $p_{t+1}$ is the probability of repaying for the type $L$ entrepreneur at time $t$ in equilibrium. It is clear that $p_{t+1} = p_L$, if $t < \hat{t}$ and $p_{t+1} = 1$, otherwise. Therefore, the no renege conditions for the type $L$ entrepreneur are

$$1 + r_t \leq U(t) \text{ and } 1 + r_t + K > U(t), \quad 1 \leq t < \hat{t}$$

$$1 + r_t + K \leq U(t), \quad \forall t \geq \hat{t}$$

Now we consider the no renege condition for the lender. Let $\hat{\alpha}_t$ be the probability of repaying at time $t$ and $\hat{\beta}_t$ be the lender’s time $t$ belief of the probability that the
entrepreneur is type $H$. Then

$$\hat{\alpha}_t = \begin{cases} 
\alpha_t, & \text{if } t < \hat{t} \\
1, & \text{otherwise}
\end{cases} \quad \text{and} \quad \hat{\beta}_t = \begin{cases} 
\beta_t, & \text{if } t \leq \hat{t} \\
\hat{\beta}_{t-1}, & \text{otherwise}
\end{cases}$$

where $\alpha_t$ and $\beta_t$ are defined in the previous subsection. In contrast to the lender’s belief updating process discussed in the previous case, the screening process stops after the type $L$ entrepreneur decides to always repay. By proposition 2.2, we can see the tension between the two roles of the collateral in the relationship. When the collateral performs well in screening, it does poorly on disciplining the entrepreneur. Because the only way to kick out the type $L$ entrepreneur is to make him refuse to repay. It is clear that when the transaction cost $C$ is large, the collateral is more effective in disciplining the entrepreneur and its ability as a screening device degenerates.

Let $\hat{V}(t)$ denote the lender’s value function at time $t$ based on the underlying relational contract. Then

$$\hat{V}(t) = \hat{\alpha}_t(r_t + \delta \hat{V}(t + 1)) + (1 - \hat{\alpha}_t)(-1 + \delta \frac{\delta}{1 - \delta \pi}), \quad \forall t \geq 1$$

The lender’s value function takes a similar form to the one we had before. The difference comes from the probability of repaying, since starting from time $\hat{t}$, the type $L$ entrepreneur always repays. The lender has no incentive to deviate if doing so is not beneficial. Hence, the no renege condition for the lender is

$$\hat{V}(t) \geq \frac{1}{1 - \delta \pi}, \quad \forall t \geq 1$$
We write the lender’s profit maximization problem as follows.

$$\max_{\{r_t\}_{t=1}^{\infty}} \hat{V}(1)$$ (2.11)

such that the relational contract is *self-enforcing*

By proposition 2.3, the relational contract under which the type \(L\) entrepreneur never repays when the project fails is not *self-enforcing*. Then in equilibrium, there exists some \(t^*\) such that the type \(L\) entrepreneur does not repay until the relationship lasts for \(t^*\) periods. We follow two steps to narrow down the set of candidates for the optimal relational contract. First, we use the following lemma to characterize the shape of the *self-enforcing* relational contract that maximizes the lender’s profit.

**Lemma 2.5.** In equilibrium, let \(t^*\) denote the time, starting from which the type \(L\) entrepreneur always repays. Then the sequence of loan rates under the optimal relational contract is equivalent to one of the following forms:

1. If \(t^* = 1\), then \(r_t = \delta(1 + \hat{r} + C) - (1 - \delta)K - 1\), for every \(t \geq 1\).
2. If \(t^* \geq 2\), then either a) \(r_t = R\), if \(t < t^*\) and \(r_t = \bar{r} > \pi\), otherwise; or b) \(r_t = R\), if \(t < t^* - 1\), \(r_t = \pi\), if \(t \geq t^*\) and \(r_{t^*-1}\) solves

$$\max_{r_{t^*-1} \leq R} r_{t^*-1}$$

such that the relational contract is *self-enforcing*

A quick inspection of lemma 2.5 tells that conditional on \(t^*\), the potential candidates for the optimal relational contract are finite. This lemma does not characterize the
behavior of \( t^* \) in equilibrium, which is the goal of our second step. If we can find an upper bound for \( t^* \), then we can narrow down the domain of the lender’s profit maximization problem to a finite set. We define \( T^*(C) \) by the following equation.

\[
T^*(C) = \min \{ t \in N \mid \sum_{\tau=1}^{t} \delta^{\tau-1}(1 + R) > \frac{\delta}{1 - \delta}(1 + \hat{r} + C) \}
\]

Then we must have \( t^* < T^*(C) \), otherwise the no renege condition for the type \( H \) entrepreneur is violated, so the relational contract is not self-enforcing. Now we know that when solving the lender’s profit maximization problem, we only need to focus on the set of self-enforcing relational contracts such that i) the \( t^* \) implied by the relational contract is less than \( T^*(C) \) and ii) the relational contract follows one of the formats specified in lemma 2.5. Let \( X(C) \) denote this set of self-enforcing relational contracts, then \( X(C) \) is a finite set. Then we can rewrite the lender’s profit maximization problem as the following.

\[
\max_{\{r_t\}_{i=1}^{\infty}} \hat{V}(1) \quad (2.12)
\]

such that the relational contract is self-enforcing \( \{r_t\}_{i=1}^{\infty} \) is induced by \((\sigma_L, \sigma_E)\) for some \((\sigma_L, \sigma_E) \in X(C)\)

It is clear that the optimal relational contract exists since the domain of this optimization problem is finite. Under the optimal relational contract, the type \( L \) entrepreneur always repays when the relationship goes beyond \( t^* \) periods. Hence, the lender stops updating her belief on the entrepreneur’s type after that as the probability of repaying...
goes up to unity. In the meanwhile, the type $L$ entrepreneur will never be screened out after time $t^*$. The lender uses collateral to discipline the entrepreneur and to screen out the type $L$ entrepreneur. However, there is a tension between the effectiveness of the collateral policy in achieving each goal. When the lender successfully disciplines the entrepreneur, she cannot learn anything about the entrepreneur’s type as no default occurs. Similarly, when the lender screens out the type $L$ entrepreneur, she fails to discipline him as he fails to repay. In contrast to the previous case when the transaction cost $C$ is moderate, the optimal relational contract when $C$ is large presents a different story of the evolution of this tension throughout the relationship. The effect of disciplining gradually grows and eventually dominates the effect of screening, so in equilibrium, conditional on the project failure, the type $L$ entrepreneur refuses to repay at the beginning but reverses this decision later on. The following proposition summarizes these results.

**Proposition 2.4.** i) Under the optimal relational contract, when $R < K < \frac{\delta}{1-\delta} (1 + \hat{r} + C) - \frac{1}{1-\delta} (1 + \pi)$, the loan rates are defined by the lender’s profit maximization problem shown in (2.12).

ii) There is a tension between the effectiveness of the collateral policy in disciplining the entrepreneur and screening out the type $L$ entrepreneur. When the transaction cost is large, the former gradually grows and eventually dominates the latter. As a result, the lender cannot completely screen out the type $L$ entrepreneur throughout the relationship.
Similar to the case when $C$ is moderate, the optimal sequence of loan rates satisfies the fastest price property. Figure 2.3 displays the equilibrium loan rates under the optimal relational contract. The lender requires the payments to be paid as soon as possible, so the loan rates are non-increasing over time. The lender has two incentives to do so. First, she uses future interest rate discounts as the reward to today’s repayment. Since the type $L$ entrepreneur may default during the early stage of the relationship, the lender wants to minimize the reward to him. Second, the value of the collateral increases rapidly as the entrepreneur’s obligation to the ongoing relationship diminishes. Then the time that the lender has to wait for until she can use the collateral to discipline the entrepreneur is greatly shortened. In summary, the optimal relational contract becomes in favor of the entrepreneur as the relationship advances because the lender offers loan rate discounts when the relationship deepens.
Moreover, in contrast to the previous case, the value of the collateral grows over time and eventually dominates the cost of repaying, so the type $L$ entrepreneur who may not repay at the beginning will be willing to repay as the relationship moves forward. Proposition 2.5 presents these results.

**Proposition 2.5.** When the transaction cost is sufficiently large,

i) In equilibrium, the optimal relational contract becomes in favor of the entrepreneur as the relationship advances.

ii) In equilibrium, the value of the collateral grows over time and eventually dominates the cost of repaying, so the type $L$ entrepreneur who may not repay during the early stage of the relationship will be willing to repay as the relationship moves forward.

## 2.5 Conclusion

This paper studies the optimal relational contract in the underground banking system of emerging markets. We compare the underground banking system with the formal banking system and embed their difference into our model of relationship lending. We examine the equilibrium loan rate patterns under various settings and show how the relational contract mitigates the problem caused by hidden information and moral hazard. We also link the enforcement mechanism of the relational contract to the theory of collateral, so that the static story of collateral can be applied in a dynamic context.

Our framework produces several interesting findings. First, the relational contract is
useless in the static setting, because the value of the collateral is zero, so the loan contract is not self-enforcing. Second, the optimal relational contract satisfies the fastest price property and moves in favor of the entrepreneur as the relationship advances. Third, the lender uses the collateral (lender-borrower relationship) to discipline the entrepreneur and to screen out the type $L$ entrepreneur throughout the relationship. The value of the collateral increases as the relationship moves forward, and will eventually surpass the cost of repaying if the regular bank’s loan service is extremely inefficient. Fourth, there is a tension between the effectiveness of the collateral policy in disciplining and screening. When the transaction cost is moderate, the collateral policy does well in screening, but performs poorly in disciplining the entrepreneur. The reverse holds true if the transaction cost is high. As a result, conditional on the project failure, the type $L$ entrepreneur never repays if borrowing from the regular bank is convenient, and becomes willing to repay as the relationship advances if the loan application at the regular bank is overwhelming.

Our model does not explicitly model the competition between an underground bank and a regular bank. In general, they operate independently of each other, but sometimes they do compete for customers in the loan market. Moreover, we only focus on the grim trigger strategy profile. There could be other strategy profiles that outperform the grim trigger one. More research is needed to explore the role of the relational contract in the underground banking system of the emerging markets.
Chapter 3

Buying Local, Consumer Benefit

and Social Welfare

3.1 Introduction

Local firms make persistent efforts to take actions that are socially responsible. It is common for local firms to donate part of their profits to charities to help the poor in the local communities and to improve education programs for local neighborhoods. When local firms source goods and services locally, job opportunities are created for local workers. Local firms often encourage their employees to volunteer in the local community. In this way, companies actively engage in the improvement of living conditions. Apparently, either directly or indirectly, consumers benefit from the socially responsible practices carried out by local firms. They value these practices and have positive preferences for the products produced by local firms as a result.
This preference is called the buying local preference, a type of altruism by consumers toward local firms. The buying local preference prevails in many markets. In procurement auctions, many states and cities design their auctions in a way that is advantageous to local bidders. They contract with a local firm if the bid of this local firm exceeds the lowest non-local bid no more than a certain amount. The 2009 Survey of State Government Purchasing Practices finds that 27 states have a preference for in-state bidders. In 2007, 26 states reportedly had this preference. (The National Association of State Procurement Officials, 2009) During holiday seasons, many buying local campaigns are initiated in order to persuade consumers to visit stores in their neighborhoods and to shop locally. (Leyden, 2012, November 28) These campaigns aim at linking the local culture to the consumer’s spending behaviors, so that the local firms, which contribute to the local culture substantially, can increase their profitability.

Given the prevalence of the buying local preference, it is important to explore how a socially responsible local firm survives in the market in which consumers are buying local. Indeed, the contributions of the local firm to its community grant it an advantageous position when dealing with its customers. When the local firm is a monopoly, it is interesting to see whether it will abuse its trading advantages in the transactions. For example, will it sell a low-quality product at a high price? On the other hand, when the local firm faces competition from an outside firm, it is natural to ask whether it will use the buying local preference to fight against its business
rival which potentially is more efficient. Moreover, it is interesting to compare the local firm’s behavior and the consumer (social) welfare between these two cases. We would like to know how market structure shapes the local firm’s incentive in product competition. This paper addresses all these questions.

We build a stylized model of a product market in which consumers are heterogeneous in the buying local preference. The buying local consumers receive additional utility when purchasing from the local firm. This extra utility consists of two parts, the utility from the profit contribution and the utility from the social spillover. We model these two effects separately when characterizing consumer behavior. Firms decide their prices and product qualities. Following the literature, we assume that quality is costly to produce and employ the production efficiency to control its cost.

We start by analyzing the market equilibrium in a monopoly market, from which we derive the implications of the buying local preference on the local firm’s strategy. Then we consider the case that the local firm’s production efficiency is the local firm’s private information. We examine how information asymmetry distorts the local firm’s equilibrium strategy. In particular, we explore how the local firm chooses its optimal signaling device in the costly separating equilibrium. Finally, we investigate the equilibrium of a duopoly market where the local firm competes with an online firm which is more efficient. We allow the horizontal differentiation and normalize the disutility yielded to a consumer when the product she buys is different from her ideal. We study the implications of the buying local preference for both firms. Moreover,
we compare the local firm’s strategy and consumer (social) welfare across different market structures.

We find that, in the monopoly market, the buying local preference induces the local firm to charge a higher price and to improve product quality. In fact, both parts of the buying local preference lead to a higher price but only the preference for the social spillover is the engine for the quality enhancement. We find a non-monotonic relationship between the buying local preference and consumer welfare. The local monopoly switches from full coverage to partial coverage when the buying local preference is sufficiently strong. Consequently, consumer surplus is fully taxed by the monopoly. For a similar reason, the relationship between the population of buying local consumers and social welfare is also non-monotonic.

We show that only the separating equilibrium survives the intuitive criterion after introducing the asymmetric information to our model of the monopoly market. In equilibrium, the local firm’s choice of its optimal signaling device depends on the population of the buying local consumers. When the size of the buying local consumer segment is moderate, the local firm uses the market coverage to signal. Then both its price and quality are independent of the buying local preference. On the other hand, if the size of the buying local consumer segment is large, the local firm prefers to use the product quality to signal. Then the buying local preference influences the price and the quality of the local firm. Contrary to traditional wisdom, we find that consumers (society) may be better-off when information friction is present. In fact, when the
local firm uses market coverage to signal, it produces at the social optimal level and covers the whole market. This will not happen in the case of perfect information. We find that, when the local firm is threatened by competition from an online firm, it relies on the buying local preference to mitigate the threat. In equilibrium, the buying local preference boosts the local firm’s price and product quality, and encourages the local firm to manufacture super quality products. It is possible that the local firm has higher product quality even though the online firm is more efficient in quality production. Meanwhile, the buying local preference puts pressure on the online firm to reduce its price, so that doing businesses in this market is less profitable. Clearly, the buying local preference makes the local firm more aggressive in determining its market strategy. We derive a non-monotonic relationship between the local firm’s price and the population of the buying local consumers. It comes from the fact that the local firm only sells to buying local consumers if the population of the buying local consumers is large. This coverage change generates a welfare trap, which means that consumers (society) may not always be better-off when more customers are buying local.

The remainder of this paper is organized as follows. Section 3.2 reviews related literature. Section 3.3 presents our benchmark model of the monopoly market. Section 3.4 introduces asymmetric information to our benchmark. Section 3.5 studies the duopoly competition. Section 3.6 concludes. All proofs are in the appendix.
3.2 Related Literature

Our work contributes to the literature of social preference which studies behavior when factors besides an agent’s own profits affect her utility. Becker (1974) studies behavior when altruism and envy enter an agent’s preferences. Fehr and Schmidt (1999) study the impact of fairness concerns on an agent’s behavior. When people suffer from unequal allocations, the market equilibrium is determined by the initial preference distribution. Similarly, Bolton and Ockenfels (2000) study the behavioral implications of inequality aversion but puts it in the incomplete information context. Goolsbee and Klenow (2002) find that individuals are more likely to buy home computers in areas with a high proportion of computer ownership. Amaldoss and Jain (2005) investigate the market outcome when social preferences, such as exclusivity and conformity, influence a consumer’s decision to purchase a conspicuous product. Consumers’ choices and behaviors are affected by their local neighborhoods or the geographic location. Consumers perceive that brands originated locally have better quality levels than those originated far away and such local dominance can explain the geographical difference in market shares for national brands of many consumer package goods. (Bronnenberg, Dhar, & Dubé, 2007) Bell and Song (2007) show consumers’ new trials of an Internet retailer are positively related to prior trials from proximate geographic regions. Cui, Raju, and Zhang (2007) explore how channel cooperation is affected when channel members care about fairness. T. Ho and Su (2009) study peer-induced fairness where social comparison brings economic agents
a reference point to compare their welfare across their peers. Leszczyc and Rothkopf (2010) investigate the bidding behavior in charity auctions when bidders have charitable motives. Iyer and Soberman (2013) study how the social preference of consumers affects product design under various market structures. Rotemberg (2013) studies the brand extension when firm altruism affects the consumer’s preference. Jiang, Ni, and Srinivasan (2014) study how a firm’s ethics affects market outcome in a credence good market where information asymmetry is present. We contribute to this active and growing literature by looking at a product market in which some consumers have altruism towards the local firm. We examine how uncertainty about the consumers’ social preference as well as competition affects the market outcome and consumer (social) welfare.

Our work also contributes to the literature of product design. Shaked and Sutton (1982) study the market equilibrium under monopolistic competition where both price and quality are endogenous. K. Moorthy (1984) investigates the product line design in a monopoly market and Katz (1984) considers a similar problem in a duopoly context. Motta (1993) studies the quality decision in a vertical differentiation model where the cost of quality production is either fixed or variable. Lehmann-Grube (1997) considers a model of vertical differentiation where the production cost of quality is convex and is independent of output and shows that the firm with higher quality enjoys a greater profit. Desai (2001) studies the product design in a market in which consumers have different valuations of product quality. Choudhary, Ghose, Mukhopadhyay, and
Rajan (2005) study the quality differentiation when personalized pricing is possible. Thomadsen (2007) explores the product positioning in the fast food industry and shows that the effect of the price competition on product positioning depends on the level of symmetry across firms. Amaldoss and Shin (2011) study how the size of a low-end market affects the quality decision of the pioneer firm and shows that it is possible for the pioneer firm to produce low quality goods and earn higher profits. Shi, Liu, and Petruzzi (2013) consider how product quality is influenced by the different distribution channels under various types of consumer heterogeneity. Our model differs from past works by focusing on the effect of the buying local preference on the local firm’s product design. Contrary to conventional wisdom, we show that when the buying local preference is strong enough, the local firm offers higher quality than the outside competitor does even though the former is less cost efficient than the latter. Our work is related to the literature of signaling as well. It is well known that there are two types of separating equilibrium, the costless separating equilibrium and the costly separating equilibrium (Spence, 1973; Rothschild & Stiglitz, 1976; Milgrom & Roberts, 1986; Welch, 1989). In the costly separating equilibrium, signaling is not free so information asymmetry devours part of the surplus which would have been enjoyed by the agents under perfect information (Desai & Srinivasan, 1995; S. Moorthy & Srinivasan, 1995; Desai, 2000; Mayzlin, 2006; Kalra & Li, 2008). Different from the past works of costly signaling, we consider an alternative signaling device that is costly to the local firm but is beneficial to the local economy. In the monopoly market of
imperfect information, using this signaling device, the local firm (monopoly) produces at the social optimal level and does not exercise market power. This is different from what would have happened in the case of perfect information.

3.3 Monopoly

We begin by considering a monopoly market in which a local firm sells a product of quality $q$ at price $p$ to consumers. Manufacturing high quality products is expensive because it requires the local firm to hire better raw materials and to employ highly skilled workers. Production process management is crucial in determining the local firm’s production cost as well. Being able to maximize raw materials utilization and to allocate employees efficiently can help the local firm to reduce its production cost substantially. Hence, we assume that the local firm’s marginal cost of producing the product of quality $q$ is $c(q) = kq^2$, where $k > 0$ measures the production inefficiency of the local firm. Production takes time, so we assume the local firm chooses its product quality $q$ before deciding its price $p$.

There are two consumer segments in the market, the buying local consumer segment and the non-buying-local consumer segment. We normalize the size of market to the unity and denote the fraction of consumers that are buying local by $\delta \in (0, 1)$. We assume each consumer has unit demand for the product and zero reservation utility. The utility received by a non-buying-local consumer when purchasing is the utility of consuming the product offered by the local monopoly. Let $\theta > 0$ be the consumers’
preference for product quality, we write this utility as \( u_{no} = \theta q - p \). Buying local consumers enjoy additional surplus when buying from the local firm because they feel that their purchases encourage small businesses and contribute to the local economy. We separate this surplus into two parts, based on the nature of how these purchases generate local economic activities. Every time a purchase occurs, the product price is paid to the local firm, part of which becomes the local firm’s profit while the rest (i.e. cost of production) goes to other local business entities if the local firm sources goods and services locally. The former is the local firm’s profit while the latter represents the revenue of business entities that support the local firm’s business. Thus they are different by definition. Moreover, different businesses contribute to the local economy in different manners. It is reasonable to identify each part separately. Let \( \alpha \in (0, 1) \) be the preference for supporting the local firm from which buying local consumers purchase and let \( \beta \in (0, 1) \) be the preference for the social spillover created by the local firm. The utility enjoyed by a buying local consumer when buying from the local firm is

\[
 u_b = \theta q - p + \alpha (p - kq^2) + \beta kq^2
\]

utility from utility from utility from
product consumption direct profit contribution social spillover

There are three stages in this game. First, the local firm decides its product quality \( q \). Then it chooses its price \( p \). After observing both product quality and price, consumers
Lemma 3.1. If \((1 - \alpha)(1 - \beta) < \delta < 1\), then the monopoly (local firm) only covers the buying local consumer segment. The equilibrium price \(p^*\) is \(\frac{(2 - \alpha - \beta)\theta^2}{4k(1 - \alpha)(1 - \beta)}\) and the equilibrium quality \(q^*\) is \(\frac{\theta}{2k(1 - \beta)}\). If \(\delta \leq (1 - \alpha)(1 - \beta) < 1\), then the monopoly (local firm) covers the whole market. The equilibrium price \(p^*\) is \(\frac{\theta^2}{2k}\) and the equilibrium quality \(q^*\) is \(\frac{\theta}{2k}\).

Market coverage is determined by the relative importance of each consumer segment. Since a buying local consumer is more valuable to the local firm than a non-buying-local consumer, the local firm only serves the buying local consumers if its population is sufficiently large. The equilibrium quality is higher under partial coverage because the buying local consumers care about the extra economic activities generated by the local firm’s business, which is measured by its production cost. Manufacturing locally is rewarding, so the local firm increases its product quality when its production process is highly local. The equilibrium price is also higher under partial coverage. The buying local preference grants the local firm the ability to charge more. Indeed, the buying local consumers are willing to pay more if either the local firm makes more profits or the product quality is higher. The former encourages the local firm to raise its price because its profit margin increases by doing so. The latter requires the local firm to price more as the production cost is higher.

Interestingly, as mentioned before, the latter effect induces the local firm to manufacture better products. A detailed inspection over the buying local preference tells that
for the part of the buying local preference governed by $\alpha$, the interest of buying local consumers is aligned with that of the local firm. Hence, the local firm has no incentive to adjust its quality to accommodate this part of the buying local preference. However, for the part that is ruled by $\beta$, the interest of both parties diverge which provides the local firm with incentives to promote its product quality. Therefore, both parts affect the equilibrium price while only the latter influences the equilibrium quality.

**Corollary 3.1.** In equilibrium, the local firm’s price is increasing in both parts of the buying local preference while its quality is only increasing in the preference for the social spillover.

Corollary 3.1 displays the direction of the effects that buying local preference casts on the equilibrium prices and qualities. Both parts of the buying local preference inflate the product price while only the part related to the social spillover enhances the product quality.

**Lemma 3.2.** If $(1 - \alpha)(1 - \beta) < \delta < 1$, then the equilibrium profit $\pi^*$ is $\frac{\delta \theta^2}{4k(1-\alpha)(1-\beta)}$ and the consumer surplus $CS^*$ is $0$. If $\delta \leq (1 - \alpha)(1 - \beta)$, then the equilibrium profit $\pi^*$ is $\frac{\theta^2}{4k}$ and the consumer surplus $CS^*$ is $\frac{(\alpha + \beta) \delta \theta^2}{4k}$.

Lemma 3.2 describes the local firm’s profit and the consumer welfare in equilibrium. The local firm faces the trade-off between the profit margin and the market share when deciding its optimal coverage. When the total willingness to pay of buying local consumers is sufficiently large, the local firm only targets on the buying local consumer segment and extracts all surplus of the deals. Therefore, consumers are left with
nothing after purchasing. On the other hand, when only few consumers are buying local, the high profit margin from these people cannot compensate the loss from giving up the rest of the market. Hence, the local firm sells to everyone. Notice that non-buying-local consumers are just break-even, the buying local consumers benefit in this case because they enjoy additional utilities from the buying local preference. Then consumer surplus is positive in this situation.

**Proposition 3.1.** *The relationship between the population of buying local consumers and consumer welfare is non-monotonic.*

The non-monotonicity comes from the fact that the local firm changes its market coverage as buying local preference grows. When the buying local preference is weak and the population of buying local consumers is small, the local firm serves the whole market and the consumer welfare is \( \frac{(\alpha + \beta)\theta^2}{4k} \). As long as the full coverage maintains, consumer surplus is increasing in the buying local preference. Once the buying local preference becomes too strong, the local firm only sells to the buying local consumers and absorbs every surplus from them. As a result, the consumer surplus drops to zero. Hence, a mild buying local preference favors consumers overall.

**Proposition 3.2.** *The local firm’s profit is increasing in the buying local preference while the relationship between the social welfare and the population of buying local consumers is non-monotonic.*

The positive relationship between the buying local preference and the equilibrium profit is obvious. The buying local preference generates additional willingness to
pay among buying local consumers which is completely captured by the local firm under partial coverage. The non-monotonic relationship between the buying local preference and the social welfare is a result of proposition 3.1. Figure 3.1 illustrates this non-monotonicity.

![Figure 3.1: The effect of $\delta$ on social welfare.](image)

When the buying local preference is strong, the local firm chooses partial coverage even the size of buying local consumer segment is small. A discrete loss in social welfare incurs for this coverage adjustment.

### 3.4 Asymmetric Information

Our previous analysis shows that in a monopoly market of perfect information, the local firm benefits from its social contributions and exercises the market power when designing the product. In the reality, however, there may be information asymmetry...
between the local firm and consumers. For example, the local firm’s production cost may not be perfectly observed by the public. Consumers usually do not have perfect information on the contracts under which the local firm sources goods and services. The local firm’s management strategy and production technology are its business secrets, which will not be released to the public in general. Most small businesses are not publically traded, so they are not required to disclose their operation details in financial statements. Therefore, we would like to know how our results under perfect information change when there is information asymmetry in the market. In particular, we are interested in the local firm’s strategy in this signaling game and the impact of the information asymmetry on consumer (social) welfare.

Following our example, we consider the situation where consumers do not observe $k$, but know its prior distribution. Suppose $k$ takes two values $k_L$ and $k_H$, where $0 < k_L < k_H$. Let $\rho \in (0,1)$ denote the probability that $k$ is $k_L$. Recall that non-buying-local consumers receive utility only from product consumption, they do not care whether the local firm is an efficient manufacturer. Their utility function is the same as that in the monopoly case. Buying local consumers, however, have incentives to learn the production inefficiency level $k$, because it affects the additional surplus from the buying local preference. They infer $k$ from the observed price and product quality offered by the local firm. We write the utility for a buying local consumer when purchasing from the local firm as
There are four stages in this game. The first two stages are the same as before. In the third stage, the buying local consumers form expectations on $k$ based on observed price $p$ and quality $q$. Finally, all consumers make their purchase decisions. We use intuitive criteria (Cho & Kreps, 1989) to refine our equilibrium.

**Lemma 3.3.** *No pooling equilibrium survives the intuitive criteria.*

Lemma 3.3 follows from the fact that the utility function of buying local consumers satisfies single crossing property. Hence, we only need to focus on separating equilibrium. We assume $\alpha > \beta$, so the inefficient type ($k = k_H$) has incentives to mimic the efficient one ($k = k_L$). This assumption holds if buying local consumers value the local firm’s profit more than its cost. It happens when the local firm denotes a substantial fraction of its profit to help local neighborhoods. Let $\gamma = \frac{k_L}{k_H} \in (0, 1)$ denote the easiness that the inefficient type imitates the efficient type. When $\gamma$ is small, the production efficiency between two types differs greatly. It is hard for the inefficient type to mimic the efficient type on the behaviors of price and product quality.

**Lemma 3.4.** *If $0 < \gamma \leq \frac{1 - \alpha}{1 - \beta}$, then there is a costless separating equilibrium.* When $(1 - \alpha)(1 - \beta) < \delta < 1$, the inefficient type local firm’s price $p_H^*$ is $\frac{(2 - \alpha - \beta)\theta^2}{4k_H(1 - \alpha)(1 - \beta)^2}$ and
quality $q^*_H$ is \( \frac{\theta}{2k_H(1-\beta)} \); the efficient type local firm's price $p^*_L$ is \( \frac{(2-\alpha-\beta)\theta^2}{4\gamma k_H(1-\alpha)(1-\beta)^2} \) and quality $q^*_L$ is \( \frac{\theta}{2\gamma k_H(1-\beta \alpha)} \). When $\delta \leq (1-\alpha)(1-\beta) < 1$, the inefficient type local firm's price $p^*_H$ is \( \frac{\theta^2}{2k_H} \) and quality $q^*_H$ is \( \frac{\theta}{2k_H} \); the efficient type local firm's price $p^*_L$ is \( \frac{\theta^2}{2\gamma k_H} \) and quality $q^*_L$ is \( \frac{\theta}{2\gamma k_H} \).

Lemma 3.4 presents the costless separating equilibrium. In this equilibrium, both types behave exactly the same as what they would have under perfect information. The efficient type does not have to provide extra quality to signal its type. Indeed, when $\gamma$ is small, the nature of production cost endows the efficient type a great advantage in producing high quality goods. The inefficient type wants to manufacture high quality goods too, however, its costly production process prevents it from doing so. Signaling occurs when partial coverage is optimal, because under full coverage, the marginal consumer is non-buying-local.

The buying local preference determines the region of the costless separating equilibrium. When $\alpha$ is large, buying local consumers benefit greatly from the local firm’s profit contribution. Since the efficient type’s profit is higher in equilibrium, this increases the temptation of imitation from the inefficient type and narrows the region of the costless separating equilibrium. On the other hand, when $\beta$ is large, buying local consumers value the local firm’s social spillover highly. It is cheaper for the inefficient type to generate social spillover, so a large $\beta$ reduces the incentive of the inefficient type to mimic. Hence, the costless separating equilibrium is more likely to exist if $\beta$ is large.
When both types do not differ very much, the inefficient type may find it profitable to improve its quality and mimic the efficient type. Then costless separating equilibrium no longer exists. However, the separating equilibrium still exists. It requires the efficient type to exchange part of its profit for a convincing signal.

Lemma 3.5. If \((1 - \alpha)/(1 - \beta) < \gamma < 1\), then there is a costly separating equilibrium. When \((1 - \alpha)(1 - \beta) < \delta < 1\), the inefficient type local firm’s price \(p_H^*\) is \(\frac{(2-\alpha-\beta)\theta^2}{4k_H(1-\alpha)(1-\beta)^2}\) and quality \(q_H^*\) is \(\frac{\theta}{2k_H(1-\beta)}\); if the efficient type chooses partial coverage, the equilibrium price \(p_L^*\) is \(\frac{\theta^2((2-\beta(2-\gamma)-\alpha\gamma)-2\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))}}{4k_H((1-\alpha)(1-\beta)-\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))^2}}\) and the quality \(q_L^*\) is \(\frac{\theta}{2k_H(1-\beta)-\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))}}\); if the efficient type chooses full coverage, the equilibrium price \(p_L^*\) is \(\frac{\theta^2}{2\gamma k_H}\) and the quality \(q_L^*\) is \(\frac{\theta}{2\gamma k_H}\). When \(\delta \leq (1 - \alpha)(1 - \beta) < 1\), the inefficient type local firm’s price \(p_H^*\) is \(\frac{\theta^2}{2k_H}\) and quality \(q_H^*\) is \(\frac{\theta}{2k_H}\); the efficient type local firm’s price \(p_L^*\) is \(\frac{\theta^2}{2\gamma k_H}\) and quality \(q_L^*\) is \(\frac{\theta}{2\gamma k_H}\).

Lemma 3.5 presents the costly separating equilibrium. The cutoff condition for the efficient type’s optimal coverage when \((1 - \alpha)(1 - \beta) < \delta < 1\) will be displayed in lemma 3.6.

When \(\gamma\) is close to unity, the cost of mimicking is small. Hence, the efficient type has to spend some of its profit to build a barrier against possible mimicking. Similar to the situation of costless separating equilibrium, signaling takes place when partial coverage is optimal under perfect information. Contrary to conventional wisdom, the efficient type can use either quality or coverage to signal its type. When quality signaling is optimal, the efficient type significantly boosts its product quality so that
deviation is not beneficial for the inefficient type. On the other hand, when coverage signaling is optimal, the efficient type serves the whole market at its best price and product quality. In this case, the inefficient type never deviates. Because choosing full coverage is suboptimal and mimicking is even worse than it.

The efficient type’s product quality may depend on $\alpha$ in the costly separating equilibrium. In this case, the efficient type produces super quality products to distinguish itself from the inefficient type. This gives a constraint on the behavior of the efficient type. Price is no longer sufficient for the efficient type to capture the utility from the direct profit contribution while satisfying the no deviation constraint. As a result, the product quality is also used to extract the utility from the direct profit contribution, which depends on $\alpha$.

**Corollary 3.2.** *In the costly separating equilibrium, when the efficient type uses product quality to signal, both the local firm’s price and quality are increasing in the buying local preference.*

Corollary 3.2 demonstrates the comparative statics of the costly separating equilibrium. Both equilibrium price and product quality for the efficient type positively depend on $\alpha$ and $\beta$. Intuitively, when $\alpha$ and $\beta$ are large, buying local consumers are willing to pay more for the product. Given $\alpha > \beta$, buying local consumers prefer efficient production process. The benefit of mimicking is therefore increasing in the buying local preference. To hamper the potential deviation, the efficient type has to enhance its product quality correspondingly. Price goes up too, to compensate for
the higher quality.

Lemma 3.6. In the costly separating equilibrium, when the efficient type uses product quality to signal, the equilibrium profit $\pi^*_L$ is $\frac{\delta \theta^2 ((1-\beta)(2-\gamma)-2\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma)})}{4k_H(1-\alpha)(1-\beta)-\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))}$; when the efficient type uses coverage to signal, the equilibrium profit $\pi^*_L$ is $\frac{\theta^2}{4\gamma k_H}$. Hence, the efficient type uses quality to signal if $\delta > \frac{(1-\alpha)(1-\beta)-\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))}}{\gamma((1-\beta)(2-\gamma)-2\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))}}$.

Lemma 3.6 shows the condition when the efficient type switches its signaling device. When there are many buying local consumers in the market, full coverage is suboptimal for the local firm. Because the forgone profit from these valuable customers dominates the benefit recouped from the non-buying-local consumers. Therefore, quality signaling is favorable to the local firm when $\delta$ is large.

The following figure illustrates the conditions under which three types of signaling

Figure 3.2: The types of separating equilibrium.
occur. In region I, there is no cost associated with the signaling. Since signaling is free if either $\gamma$ or $\delta$ is small, we expected region I to be L-shaped. The slim area next to the region I is the region II, the place where coverage signaling occurs. Coverage signaling is expensive when $\delta$ is large, so region II contains all $(\delta, \gamma)$ pairs that $\delta$ is moderate and $\gamma$ is large. Region III is the area in which the efficient type uses quality to signal its type. When both $\delta$ and $\gamma$ are large, signaling is costly and coverage signaling is not preferred.

The equilibrium price and quality in the costless separating equilibrium is the same as what it would have been if there was no information asymmetry. Consumer welfare does not change in the region I compared with the monopoly case. In region II, however, consumers may be better-off. In the monopoly case, the efficient type only serves buying local consumers while here it covers the whole market. Signaling induced by the information asymmetry makes consumers better-off. This welfare improving signaling fails to sustain in the region III. Indeed, similar to what would happen in the monopoly case, the efficient type extracts every surplus from the buying local consumers in equilibrium. Customers are merely able to break even when quality signaling is selected. The following proposition summarizes this result.

**Proposition 3.3.** When $(1 - \alpha)(1 - \beta) < \delta < \frac{(1-\alpha)(1-\beta)-\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma))}}{\gamma((1-\beta)(2-\gamma)-2\sqrt{(\alpha-\beta)(1-\beta)(1-\gamma)})}$, the efficient type uses market coverage to signal its type, then information asymmetry benefits consumers and the economy overall.
Figure 3.3 presents an example of this proposition. It displays the change in consumer welfare after information asymmetry is introduced and the local firm is the efficient type. This welfare change is positive for some moderate $\delta$, where the local firm uses
coverage to signal its type, and is zero otherwise.

Although information asymmetry may favor consumers, it may not benefit the whole society. Figure 3.4 shows the effect of information asymmetry on the change in social welfare when the local firm is the efficient type.

As discussed in the previous analysis, signaling is costless when the population of buying local consumers is small. Hence, the social welfare stays the same after information asymmetry comes in. When $\delta$ moves to the region where coverage signaling occurs, consumers are better-off, and this welfare gain dominates the loss of the local firm’s profit. In this case, information asymmetry benefits the economy overall. Such gain in consumer welfare disappears when the size of buying local consumer segment is large. However, the cost of signaling does not evaporate. Therefore, the effect of information asymmetry on the social welfare becomes negative.

3.5 Duopoly

Our analysis of the monopoly case shows that the local firm recoups the benefits of its socially responsible activities in the product market. The buying local preference induces the local firm to improve its product quality, but the resulting consumer benefit is offset by the simultaneous price increase. Hence, the business strategy adopted by the local firm is not friendly to the consumers in the product market.

Then it is natural to ask whether the local firm will become friendly to consumers if competition comes in. Will the local firm further enhance its product quality under
competition? Is it possible that the local firm asks for less when the population of the buying local consumers is large? In general, it is interesting to see whether competition can turn the local firm into a “socially responsible” firm in the product market. This section tries to answer these questions.

We consider a situation in which an outside firm enters the local market and competes with the local firm. This outside firm neither manufactures products locally nor donates its profit to improve the welfare of neighborhoods. Buying local consumers do not receive additional utility when buying from it because their purchases do not contribute to the local economy. Such outside firm can be a foreign online retailer for example. In reality, online retailers are the major competitors of local businesses. The adoption of new distribution technologies and the utilization of scale economy in production grant the online retailer a significant cost advantage over the local firm. Both aggressive pricing strategies and convenient online shopping services challenge the effect of the buying local preference in competition and help the online retailer to steal business from the local firm.

Now, we introduce a foreign online firm to the market described in the monopoly case, so there are two firms in the market. Let the local firm be firm 1 and the foreign online firm be firm 2. Both firms decide their own price and quality. Let $p_i$ and $q_i$ be firm $i$’s price and quality respectively. The utility received by a consumer when buying at firm 1 is the same as that in the monopoly case. Let $u_1$ denote this utility. Since no consumer enjoys the additional surplus from the buying local preference
when shopping at the foreign online firm, consumer's utility of choosing firm 2 is the
utility of product consumption as well as the value of online shopping services.

\[ u_2 = \theta q_2 - p_2 + x \]

utility from product consumption + utility from online shopping

The utility from shopping online, \( x \), describes the consumer's taste of e-shopping.

We assume \( x \) is uniformly distributed over the interval \([-\epsilon, \epsilon]\) and is independently identically distributed across consumers. The bound \( \epsilon > 0 \) measures the taste dispersion and captures the market competition intensity. When \( \epsilon \) is small, online shopping services alone cannot distinguish the online firm. Hence, its deal has to be very attractive in order to secure its market shares in both market segments.

The cost structure of both firms are the same as that in the monopoly case. Firm \( i \)'s marginal cost of production is \( c_i(q_i) = k_i q_i^2 \), where \( k_i > 0 \) is firm \( i \)'s production inefficiency. As mentioned above, the online retailer is more cost efficient than the local firm. We assume \( 0 < k_2 < k_1 \).

There are three stages in this game. First, both firms choose their product quality simultaneously. After learning the rival's quality decision, both firms simultaneously set their price. Finally, consumers observe the price and the product quality of both firms and make their purchase decisions. A complete discussion of this game is beyond the purpose of this section, we only focus on the situation where the market equilibrium is an interior solution.
There are two types of equilibrium in this game. One is under which the local firm only sells to buying local consumers and the other one is under which it serves both market segments. It is possible that the local firm prefers the former while the online firm desires the latter. To resolve this coordination problem, we assume the market equilibrium favors the local firm. Because it is easier for the local firm to update the needs of local people, and the local firm is more flexible in responding to the shocks of them.

Lemma 3.7. When the local firm only sells to buying local consumers (partial coverage), the market equilibrium is $p_1^* = \frac{(6-\alpha(1-\delta)-\beta(2-\delta)-2\delta)\theta^2 k_2 - (1-\beta)^2 k_1 (\theta^2 - 12\epsilon k_2)}{4(1-\alpha)(1-\beta)^2 (4-\delta) k_1 k_2}$, $q_1^* = \frac{\theta}{2(1-\beta) k_1}$; $p_2^* = \frac{-\delta\theta^2 k_2 + 2(1-\beta) k_1 ((3-\delta)\theta^2 + 2(2+\delta) \epsilon k_2)}{4(1-\beta)(4-\delta) k_1 k_2}$, $q_2^* = \frac{\theta}{2k_2}$. The equilibrium profits are $\pi_1^* = \frac{\delta((2-\delta)\theta^2 k_2 - (1-\beta) k_1 (\theta^2 - 12\epsilon k_2))^2}{32(1-\alpha)(1-\beta)^2 (4-\delta)^2 \epsilon k_1^2 k_2^2}$ and $\pi_2^* = \frac{(\delta\theta^2 k_2 - (1-\beta) k_1 ((2-\delta)\theta^2 + 4(2+\delta) \epsilon k_2))^2}{32(1-\beta)^2 (4-\delta)^2 \epsilon k_1^2 k_2^2}$. When the local firm sells to all consumers (full coverage), the market equilibrium is $p_1^{**} = \frac{(4-3\alpha\delta - \beta\delta)\theta^2 k_2 - (1-\beta)^2 k_1 (\theta^2 - 12\epsilon k_2)}{12(1-\alpha\delta)(1-\beta)^2 k_1 k_2}$, $q_1^{**} = \frac{\theta}{2(1-\beta) k_1}$; $p_2^{**} = \frac{-\theta^2 k_2 + 2(1-\beta) k_1 ((2-\delta)\theta^2 + 3\epsilon k_2)}{12(1-\beta) k_1 k_2}$, $q_2^{**} = \frac{\theta}{2k_2}$. The equilibrium profits are $\pi_1^{**} = \frac{(\theta^2 k_2 - (1-\beta) k_1 (\theta^2 - 12\epsilon k_2))^2}{288(1-\alpha\delta)(1-\beta)^2 \epsilon k_1^2 k_2^2}$ and $\pi_2^{**} = \frac{(\theta^2 k_2 - (1-\beta) k_1 (\theta^2 - 12\epsilon k_2))^2}{288(1-\alpha\delta)(1-\beta)^2 \epsilon k_1^2 k_2^2}$.

Lemma 7 states the equilibrium prices, product qualities and profits for both types of market equilibrium. When $\pi_1^* > \pi_1^{**}$, partial coverage is optimal for the local firm. Hence, the former type is the market equilibrium. When $\pi_1^* \leq \pi_1^{**}$, full coverage is better for the local firm. Then the latter one becomes the market equilibrium.

The preference for direct profit contribution, $\alpha$, does not affect the online firm’s behavior. The local firm exploits this preference completely via its profit margin,
so there is no further bonus of this preference for the local firm on the demand competition. The online firm only cares about the demand competition with the local firm. Hence, the effect of the preference for direct profit contribution on the online firm’s behavior is neutral.

The intensity of market competition affects the price of the local firm more than the price of the online firm. It is clear that $\frac{\partial}{\partial \epsilon} p_1^* = \frac{3}{(1-\alpha)(4-\delta)} > \frac{2+\delta}{4-\delta} = \frac{\partial}{\partial \epsilon} p_2^*$ and $\frac{\partial}{\partial \epsilon} p_1^{**} = \frac{1}{1-\alpha \delta} > 1 = \frac{\partial}{\partial \epsilon} p_2^{**}$. Recall that the local firm’s profit matters to buying local consumers and the profit dissipates when the market is competitive, the deal offered by the local firm becomes less appealing when $\epsilon$ falls. Then the local firm has to further cut its price to stop losing clients to the online firm. Conversely, form the same rationale, when $\epsilon$ increases, the local firm can charge more than the online firm does. Full coverage magnifies the effect of market competition on prices. We have $\frac{\partial}{\partial \epsilon} p_1^* < \frac{\partial}{\partial \epsilon} p_1^{**}$ and $\frac{\partial}{\partial \epsilon} p_2^* < \frac{\partial}{\partial \epsilon} p_2^{**}$. Indeed, serving both consumer segments makes the local firm have full contact with the online retailer. The competition between them is the fiercest in this case as a price reduction can steal the business from its rival in both consumer segments. Thus, each of them has incentives to drop its price more than what would have been otherwise.

Departing from the monopoly case, the local firm’s product quality may depend on the size of the buying-local-consumer segment as well. After the online firm enters the market, extracting every surplus from the marginal consumers is no longer optimal for the local firm. The trade-off between profits from both consumer segments appears.
To balance the benefits of two market segments, the local firm takes the size of buying local consumers into account in its product design.

The local firm is less cost efficient than the online firm. However, due to the buying local preference, its product quality could be greater than that of the latter. Since the preference for social spillover encourages the local firm to manufacture high quality goods and the size of buying local consumers reinforces this effect, it is possible that the product of the local firm has better quality. The following proposition summarizes this result.

**Proposition 3.4.** In equilibrium, if the buying local preference is sufficiently strong and prevalent \((\beta \delta > 1 - \frac{k_2}{k_1})\), the local firm’s product quality is always higher than the online firm’s product quality.

A detailed inspection of the market equilibrium tells that the buying local preference inflates the local firm’s price and raises its product quality. The same rationale in the monopoly case applies. On the other hand, this preference makes the online firm’s price more competitive. As the buying local preference becomes stronger, the buying local consumers favor the local firm more. A price drop can help the online firm restore its demand. Interestingly, the product quality of the online retailer is independent of this preference. Buying local does not reward the online firm for making high quality products. Since the price is determined after the product quality is chosen, price alone is sufficient for the online firm to compete for demand. Corollary 3.3 states these results.
Corollary 3.3. In equilibrium, i) market competition magnifies the effect of the buying local preference on the local firm’s quality; ii) the online firm’s price is decreasing in the preference for social spillover.

The size of buying local consumers, however, influences the local firm in a different manner. When the market equilibrium is the partial coverage type, the local firm’s price increases in the population of buying local consumers. Such effect turns to negative after the market equilibrium induces the local firm to serve both market segments. To understand this non-monotonic relationship, consider the change of the local firm’s product quality when the type of market equilibrium switches. Fix all parameters except the size of buying local consumers $\delta$. Under full coverage, both price and product quality of the local firm are affected by $\delta$. In other words, the local firm uses both instruments in demand competition. The local firm’s product quality is increasing in $\delta$ under full coverage, which extends its capacity of raising prices. When the market equilibrium is partial coverage type, the local firm’s product quality is independent of $\delta$. Now, the local firm only uses price in demand competition. Based on the similar argument in corollary 3.3, the online firm reduces its price as $\delta$ increases. In response to the rival’s price cut, the local firm drops its price as $\delta$ increases. The following proposition presents this result.

Proposition 3.5. In equilibrium, the local firm’s price increases in the population of the buying local consumers under full coverage and decreases in it under partial coverage. The online firm’s price always decreases in this population.
Similar to the results in the monopoly case, the buying local preference boosts the local firm’s profit. Increasing the size of buying-local-consumer segment also benefits the local firm, because these consumers are willing to pay more to shop at the local store. For the online firm, all these effects become negative. Competing at a disadvantageous position means one has to exert more effort. Here, it means the online firm has to sacrifice part of its profit to make its offer more appealing to the public. Corollary 3.4 gives these results.

**Corollary 3.4.** *In equilibrium, the buying local preference benefits the local firms and hurts the online firm.*

In the monopoly case, consumers get no rents under partial coverage because the local firm exercises its market power by asking for every penny that consumers are willing to pay. With competition, this result no longer holds. In fact, consumers are able to grasp the extra utility of buying local as the online firm prices aggressively. The following figure illustrates how the population of buying local consumers affects consumer surplus.

When $\delta$ is small, the local firm serves both market segments. Consumers are better-off overall as more of them are buying local. Such a sweet situation disappears when the local firm adjusts its market coverage. Under partial coverage, competition becomes less intensive, so both firms can charge more for its product. As a result, there is a discrete welfare loss for consumers at the point where the type of market equilibrium changes. After getting over this trap and moving to the new market coverage, the
previous mechanism applies again and consumer surplus increases as the population of buying local consumers grows. Similar results hold for social welfare, and are not repeated here. Figure 3.6 shows the relationship between the size of buying-local-consumer segment and social welfare.

![Figure 3.5: The effect of $\delta$ on consumer welfare.](image)

Going back to the policy campaign mentioned at the beginning of this paper, it seems reasonable for the local government to push citizens to visit local stores by endowing them with the buying local preference. Most of the time, both consumers and local economy are overall better-off as more consumers are buying local. However, a welfare trap occurs when the local firm is spoiled and switches its market coverage. Therefore, a detailed study of the local market is necessary to ensure the favorable outcome of such campaigns.
3.6 Conclusion

This paper studies the market implications of the buying local preference in a product market. When consumers are buying local, they have altruism toward the local firm and enjoy additional surplus from buying from the local firm. We separate this surplus into two parts, utility from profit contribution to the local firm and the utility from generating social spillover. In general, the former grants the local firm more market power and the latter induces the local firm to improve its product quality. Intuitively, the former aligns the interest of consumers and the local firm while the latter does the converse.

We examine the buying local preference under various market settings and show several interesting findings. In a monopoly market where consumers are heterogeneous
in the buying local preference, the relationship between consumer (social) welfare and the buying local preference is non-monotonic. The local firm refuses the serve the whole market when selling to buying local consumers only is sufficiently profitable. The exercise of market power is weakened as information asymmetry comes in. When consumers do not know the local firm’s efficiency of quality production, the more efficient type may use market coverage rather than product quality to signal its type. In this case, both consumer welfare and social welfare achieve the first best which is better than what would have been under perfect information. In the duopoly market where the local firm competes with the online firm, buying local preference endows the local firm an advantage in product competition. We show that when the buying local preference is strong, the local firm offers higher quality than the online firm even though the latter is more efficient in producing quality. Contrasting that with the monopoly case, we find a non-monotonic relationship between the local firm’s price and the population of buying local consumers. The non-monotonicity comes from the fact that the local firm only targets at buying local consumers when the population of buying local consumers is large. In fact, this coverage change creates the welfare trap that greatly dampens consumer (social) welfare.

Our work on buying local preference highlights the impact of social preference on economic and social behavior. In our model, the social preference is exogenous so it will be interesting to examine this impact when the social preference itself is endogenous. Buying local consumers may become more buying local if they find that
this preference helps the local firm win the competition with the online firm. On the other hand, buying local consumers may feel disappointed when they realize that the buying local preference only leads to a rise in the local firm’s price. A dynamic framework of product competition is needed to explore the evolvement of the buying local preference and its impact on economic behavior. Will the evolution of the buying local preference be stable? How do consumer welfare and social welfare evolve over time? These questions are left for future research.
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Appendix

A1 Missing Proofs for Chapter 2

Lemma 2.2: The right hand side of inequality 2.3 is the suboptimal future benefit for the type L entrepreneur if no deviation today. Since it is greater than the repaying cost for the type H entrepreneur, which is same as that for the type L entrepreneur, we know the type L entrepreneur will not deviate when the project is successful.

Lemma 2.3: Suppose the contrary is true, then the lender can increase her profit by reducing the time $t'$ interest rate by $\epsilon$ and increasing the time $t$ interest rate by $\delta t' - t \epsilon$. It is clear that such adjustment does not change the self-enforcement of the underlying relational contract.

Lemma 2.4: Suppose the contrary is true, then the lender can increase her profit by reducing the time $t + 1$ interest rate by $\epsilon$ and increasing the time $t$ interest rate by $\delta \epsilon$. It is clear that such adjustment does not change the self-enforcement of the underlying relational contract.

Lemma 2.5: i) This is straightforward. ii) There are two cases, either $V(t^*) > \frac{1}{1 - \delta} \Pi$
or \( V(t^*) = \frac{1}{1-\delta} \). a) If \( V(t^*) > \frac{1}{1-\delta} \), then \( r_t = R \) if \( t < t^* - 1 \). Otherwise, the lender can increase her profit by reducing \( r_{t^*} \) by \( \epsilon \) and increasing \( r_{t'} \) by \( \delta_{t^* - t'} \epsilon \) where \( r_{t'} < R \).

For the \( r_{t^*-1} \), we only need to realize that the lender is indifferent between \( r_{t^*} \) and \( V(t^*) \). b) If \( V(t^*) = \frac{1}{1-\delta} \), following the similar argument, we must have \( r_t = R \) if \( t < t^* - 1 \). The optimality of \( r_{t^*-1} \) immediately follows from its definition.

**Proposition 2.1:** i) It is shown in the paper. ii) By equation 2.5, we have \( \beta_t \) is increasing in \( t \) and \( \lim_{t \to \infty} \beta_t = 1 \).

**Proposition 2.2:** i) In equilibrium, the loan rates are non-increasing, so the contract becomes in favor of entrepreneur as the relationship advances. ii) The value of the collateral is decreasing the future loan rates, so when the equilibrium loan rates are non-increasing, the value of the collateral grows over time.

**Proposition 2.4:** i) It is shown in the paper. ii) In equilibrium, we have \( \hat{\alpha}_t = \alpha_t < 1 \), if \( t < t^* \) and \( \hat{\alpha}_t = 1 \), if \( t \geq t^* \), so the collateral policy becomes more effectively in disciplining the entrepreneur as the relationship moves forward. Notice that \( \hat{\beta}_t = \beta_t < 1 \), if \( t \leq t^* \) and \( \hat{\beta}_t = \hat{\beta}_{t-1} \), if \( t > t^* \), the lender is never certain about the entrepreneur’s type since starting from time \( t^* \), the type \( L \) entrepreneur always repays.

**Proposition 2.5:** i) The same as the first part of the proposition 2.2. ii) It comes from the no renege condition for the optimal relational contract which is self-enforcing.
A2 Missing Proofs for Chapter 3

Lemma 3.1: Market demand $D(p,q) = \delta$, if $\theta q - p < 0$ and $\theta q - p + \alpha(p - kq^2) + \beta kq^2 \geq 0$; $D(p,q) = 1$, if $\theta q - p \geq 0$; $D(p,q) = 0$, otherwise. The local firm’s profit maximization problem is $\max_p \max_q (p - kq^2) D(p,q)$. Hence, $D(p,q) = \delta$ if and only if $\theta q - p + \alpha(p - kq^2) + \beta kq^2 = 0$ and $D(p,q) = 1$ if and only if $\theta q - p = 0$. The equilibrium price and quality under partial coverage is $p^* = \dfrac{(2 - \alpha - \beta)\theta^2}{4k(1 - \alpha)(1 - \beta)}$ and $q^* = \dfrac{\theta}{2k}$. Then the equilibrium profit under partial coverage and full coverage are $\delta \theta^2 \dfrac{\beta^2}{4k(1 - \alpha)(1 - \beta)}$ and $\dfrac{\beta^2}{4k}$.

Comparing these two profits, the cutoff condition for coverage changes follows.

Lemma 3.2: The local firm’s equilibrium profit is $(p - kq^2) D(p,q)$. The equilibrium price and quality are given by lemma 3.1. When $(1 - \alpha)(1 - \beta) < \delta < 1$, we have $\theta q^* - p^* + \alpha(p^* - kq^2) + \beta kq^2 = 0$ and $\theta q^* - p^* < 0$. Then we have $CS^* = 0$. When $\delta \leq (1 - \alpha)(1 - \beta)$, we have $\theta q^* - p^* = 0$ and $\theta q^* - p^* + \alpha(p^* - kq^2) + \beta kq^2 = \dfrac{(\alpha + \beta)\theta^2}{4k}$. Then we have $CS^* = \delta \dfrac{(\alpha + \beta)\theta^2}{4k}$.

Lemma 3.3: Suppose the contrary is true, let $(p^*, q^*)$ be the equilibrium price and quality. Let $\tilde{k} = \rho k_L + (1 - \rho) k_L$, then we have $\theta q^* - p^* + \alpha(p^* - k_Lq^2) + \beta k_Lq^2 > \theta q^* - p^* + \alpha(p^* - \tilde{k}q^2) + \beta \tilde{k}q^2 \geq 0$. Now take $k' : k_L < k' < \tilde{k} < k_H$ and some small $\epsilon > 0$. Let $q' = q^* + \epsilon$ and $p' = p^* + k'q'^2 - k_Lq^2$. Since $\epsilon$ can be arbitrarily small, we have $\theta q' - p' + \alpha(p' - k_Lq'^2) + \beta k_Lq'^2 > 0$. Then under this new price and quality, when consumers believe that the firm is efficient, the demand stays the same for both types, but the profit margin increases for the efficient type and decreases for
the inefficient type.

**Lemma 3.4:** Equilibrium prices and qualities follows from lemma 3.1. When \( \delta \leq (1 - \alpha)(1 - \beta) < 1 \), full coverage is optimal so costless separating equilibrium automatically holds. When \( (1 - \alpha)(1 - \beta) < \delta < 1 \), the low type \((k_H)\) deviation profit is \(\frac{(1+\alpha+(2-\alpha-\beta)\gamma)\delta \theta^2}{4(1-\alpha)(1-\beta)^2\gamma^2k_H} \). Its non-deviation profit is \(\frac{\delta \theta^2}{4k_H(1-\alpha)(1-\beta)} \). Comparing these two profits, the cutoff condition follows.

**Lemma 3.5:** Under quality signaling (partial coverage), the high type’s \((k = k_L)\) price and quality are defined by the consumer’s individual rationality constraint and the low type’s \((k = k_H)\) incentive compatibility constraint. The former is \(\theta q - p + \alpha(p - k_L q)^2 + \beta k_L q^2 = 0\) and the latter is \(\delta(p - k_H q)^2 = \frac{\delta \theta^2}{4k_H(1-\alpha)(1-\beta)} \). Under coverage signaling (full coverage), the equilibrium price and quality follows from lemma 3.1.

**Lemma 3.6:** Using the equilibrium prices and qualities given by lemma 3.5, the equilibrium profits are immediate. The efficient type uses quality to signal if doing so yields a higher profit. This is given by the last inequality in this lemma.

**Lemma 3.7:** When the local firm chooses full coverage, the indifferent consumer in both consumer segments are defined by \(\theta q_1 - p_1 + \alpha(p_1 - k_1 q_1^2) + \beta k_1 q_1^2 = \theta q_2 - p_2 + x_{buy} \) and \(\theta q_1 - p_1 = \theta q_2 - p_2 + x_{no}\). The local firm’s demand is \(D_1(p_1, q_1) = \delta \frac{x_{buy} + \epsilon}{2\epsilon} + (1 - \delta) \frac{x_{no} + \epsilon}{2\epsilon}\) and the online firm’s demand is \(D_2(p_2, q_2) = 1 - D_1(p_1, q_1)\).

Price game gives firm \(i\)’s first order condition \(\frac{\partial}{\partial p_i} D_i(p_i - k_i q_i^2) = 0\). Then \(p_1 = \frac{3\epsilon + \theta q_1 + 2k_1 q_1^2 - 3\alpha \delta k_1 q_1^2 + \beta k_1 q_1^2 - \theta q_2 + k_2 q_2^2}{3(1-\alpha \delta)}\) and \(p_2 = \frac{1}{3}(3\epsilon - \theta q_1 + k_1 q_1^2 - \beta \delta k_1 q_1^2 + \theta q_2 + 2k_2 q_2^2)\).

Quality game gives firm \(i\)’s first order condition \(\frac{\partial}{\partial q_i} D_i(p_i - k_i q_i^2) = 0\). Then the
equilibrium prices and qualities follow. When the local firm chooses partial cov-
ference, the indifferent consumer in both consumer segments are defined by \( \theta q_1 - p_1 + \alpha(p_1 - k_1 q_1^2) + \beta k_1 q_1^2 = \theta q_2 - p_2 + x_{buy} \) and \( 0 = \theta q_2 - p_2 + x_{no} \). The local firm’s demand is \( D_1(p_1, q_1) = \delta x_{buy} + \epsilon_2 \) and the online firm’s demand is \( D_2(p_2, q_2) = \delta x_{no} + \epsilon \). Follow the similar steps, solving the price game gives

\[
p_1 = \frac{\delta - \delta^2 - 2 \theta q_1 + 2 \delta q_1 - 2 \alpha k_1 q_1^2 + \beta \delta k_1 q_1^2 - \alpha \delta^2 k_1 q_1^2 + \theta q_2 - k_2 q_2^2}{2 \delta - \alpha \delta^2 - 2 \theta q_1 + 2 \delta q_1 - 2 \alpha k_1 q_1^2 + \beta \delta k_1 q_1^2 - \alpha \delta^2 k_1 q_1^2 + \theta q_2 - k_2 q_2^2}
\]

\[
p_2 = \frac{2 \epsilon + \delta \epsilon - \delta \theta q_1 + \delta^2 \theta q_1 - \beta \delta \theta q_1 + 2 \theta q_2 - \delta \theta q_2 + 2 k_2 q_2^2}{4 - \delta}
\]

The equilibrium prices and qualities follow from the first order conditions of the quality game.

**Proposition 3.1:** When \( \delta \leq (1 - \alpha)(1 - \beta) \), we have \( \frac{\partial}{\partial \delta} CS^* = \frac{(\alpha + \beta)\theta^2}{4k} > 0 \). Otherwise, we have \( \frac{\partial}{\partial \delta} CS^* = 0 \).

**Proposition 3.2:** The first part immediately follows from lemma 3.2. The social welfare is \( \frac{\theta^2}{4k} + \frac{(\alpha + \beta)\delta \theta^2}{4k} \) when \( \delta \leq (1 - \alpha)(1 - \beta) \) and is \( \frac{\delta \theta^2}{4k(1 - \alpha)(1 - \beta)} \) otherwise. It is clear that social welfare is increasing in each region. Moreover, we have

\[
\lim_{\delta \to (1 - \alpha)(1 - \beta)^+} \frac{\delta \theta^2}{4k(1 - \alpha)(1 - \beta)} = \frac{\theta^2}{4k} < \frac{\theta^2}{4k} + \frac{(\alpha + \beta)\delta \theta^2}{4k}. \]

The non-monotonicity follows.

**Proposition 3.3:** The first part of inequality follows from the fact that signaling occurs when partial coverage is optimal. The second part of inequality comes from lemma 3.6. Consumer welfare and social welfare reach their first best under coverage signaling. Under perfect information, however, the local firm only serves buying local consumers so neither consumer welfare nor social welfare is not maximized.

**Proposition 3.4:** Under partial coverage, we have \( q_1^* = \frac{\theta}{2(1 - \beta)k_1} \) and \( q_2^* = \frac{\theta}{2k_2} \). Then \( q_1^* > q_2^* \) implies \( \beta > 1 - \frac{k_2}{k_1} \). Under full coverage, we have \( q_1^{**} = \frac{\theta}{2(1 - \beta \delta)k_1} \) and \( q_2^{**} = \frac{\theta}{2k_2} \).
Then \( q_1^{**} > q_2^{**} \) implies \( \beta \delta > 1 - \frac{k_2}{k_1} \). Combining, we have \( \beta \delta > 1 - \frac{k_2}{k_1} \).

**Proposition 3.5:** Under full coverage, we have
\[
\frac{\partial}{\partial \beta} p_1^* = \frac{(\alpha + 7\beta - \beta(15\alpha + \beta)\delta + 2\alpha\beta(3\alpha + \beta)\delta^2)k_2 + \alpha(-1 + \beta)\delta k_1(\theta^2 - 12\epsilon k_2)}{12(1 - \alpha \delta)^2(1 - \beta \delta)^3 k_1 k_2}.
\]
It is suffice to show the numerator is positive. In the non-buying-local consumer segment, the indifferent consumer’s location is \( \frac{\epsilon}{\alpha, \beta, \delta} < 0 \). Under full coverage, we have \( \beta \delta > 0 \).

Notice that the underlying numerator is increasing in \( \epsilon \), take \( \epsilon = \frac{1}{12} \theta^2 \left( \frac{1}{k_2} - \frac{1}{k_1} \right) > 0 \), we have the numerator \( > \beta(7 + \delta(-\beta + 6\alpha^2\delta + \alpha(-4 + \beta\delta)(3 + \beta\delta))\theta^2 k_2 \). When \( 0 < \alpha, \beta, \delta < 1 \), \( \beta(7 + \delta(-\beta + 6\alpha^2\delta + \alpha(-4 + \beta\delta)(3 + \beta\delta)) > 0 \). Hence, the numerator is positive.

Under partial coverage, we have \( \frac{\partial}{\partial \beta} p_1^* = -\frac{2\theta^2 k_2 + (1 - \beta)k_1(\theta^2 - 12\epsilon k_2)}{4(1 - \alpha \delta)^2 k_1 k_2} \). In the buying local consumer segment, the utility received by the indifferent consumer is \( \theta q_1^* - p_1^* + \alpha(p_1^* - k_1q_1^*) + \beta k_1 q_1^2 = \frac{2\theta^2 k_2 - (1 + \beta)k_1(\theta^2 - 12\epsilon k_2)}{4(1 - \alpha \delta)^2(1 - \beta \delta)^2 k_1 k_2} > 0 \). Hence, we have \( 2\theta^2 k_2 - (1 + \beta)k_1(\theta^2 - 12\epsilon k_2) > 0 \). It is clear that \( \frac{\partial}{\partial \beta} p_1^* < 0 \).

For the online firm, we have \( \frac{\partial}{\partial \beta} p_2^* = -\frac{\beta\theta^2}{12(1 - \beta \delta)^2 k_1} < 0 \) and \( \frac{\partial}{\partial \beta} p_2^* = \frac{2\theta^2 k_2 - (1 + \beta)k_1(\theta^2 - 12\epsilon k_2)}{2(1 - \beta \delta)^2 k_1 k_2} < 0 \).

**Corollary 3.1:** We have \( \frac{\partial}{\partial \alpha} p^* = \frac{\theta^2}{4k(1 - \alpha)^2(1 - \beta)} > 0 \), \( \frac{\partial}{\partial \gamma} p^* = \frac{(3 - 2\alpha - \beta)\theta^2}{4k(1 - \alpha)(1 - \beta)^3} > 0 \) and \( \frac{\partial}{\partial \beta} q^* = \frac{\theta}{2k(1 - \beta)^2} > 0 \).

**Corollary 3.2:** We have
\[
\frac{\partial}{\partial \alpha} p_L^* = \frac{\theta^4 k_H((1 - \beta)(1 - 4\beta - 3\alpha(-1 + \gamma) + 3\beta\gamma)\theta k_H - (\theta^2 - 12\epsilon k_2)(\alpha - \beta)^2 k_H)}{4(1 - \alpha)^2 k_H(1 - \beta)^2(1 - \gamma)^2 k_H^3}.
\]

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When $0 < \gamma < 1$ and $0 < \beta < \alpha < 1$, $\frac{\partial}{\partial \beta} p^*_L > 0$. We have

$$\frac{\partial}{\partial \beta} p^*_L = \frac{\theta^5 k_H^2((1+\gamma)(1+3\alpha-4\beta)(-1+\beta)+(\alpha-\beta)^2\gamma)\theta k_H + (1+3\alpha-4\beta+2\alpha-3\beta^2\gamma+\alpha(1-\gamma))\sqrt{(\alpha-\beta)(1-\gamma)(1-\gamma)\theta^2 k_H^3}}{4(1-\alpha)\sqrt{(\alpha-\beta)(1-\gamma)(1-\gamma)\theta^2 k_H^3}}.$$ 

When $\frac{1-\alpha}{1-\beta} < \gamma < 1$ and $0 < \beta < \alpha < 1$, $\frac{\partial}{\partial \beta} p^*_L > 0$.

We have $\frac{\partial}{\partial \alpha} q^*_L = \frac{\theta^2(\alpha-\beta)(1-\gamma)\theta k_H + (1+\alpha-2\beta)(-1+\gamma)\theta^2 k_H^2}{4(\alpha-\beta)(1-\gamma)(1-\gamma)\theta^2 k_H^2} > 0$.

We have $\frac{\partial}{\partial \beta} q^*_L = \frac{\theta^2(\alpha-\beta)(1-\gamma)\theta k_H + (1+\alpha-2\beta)(-1+\gamma)\theta^2 k_H^2}{4(\alpha-\beta)(1-\gamma)(1-\gamma)\theta^2 k_H^2} > 0$. When $\frac{1-\alpha}{1-\beta} < \gamma < 1$ and $0 < \beta < \alpha < 1$, $\frac{\partial}{\partial \beta} q^*_L > 0$.

**Corollary 3.3:** i) *partial coverage:* we have $\frac{\partial}{\partial \alpha} p^*_1 = \frac{(2-\delta)\theta^2 k_2(1+3\alpha-4\beta)(1-\gamma)}{4(1-\alpha)(1-\beta)(1-\gamma)k_1 k_2}$.

In the buying local consumer segment, the indifferent consumer’s location is

$$\frac{1}{4} + \frac{\theta^2((1+3\alpha-4\beta)(1-\gamma))}{(1-\epsilon)k_1}.$$  It must be greater than $-\epsilon$, so we have $\epsilon > \frac{1}{12} \frac{\theta^2}{k_1} - \frac{2-\delta}{(1-\beta)k_1}$.

Notice that $\frac{\partial}{\partial \alpha} p^*_1$ is increasing in $\epsilon$, use the constraint on $\epsilon$ derived above, we have $\frac{\partial}{\partial \alpha} p^*_1 > 0$.

We have $\frac{\partial}{\partial \beta} p^*_1 = \frac{(10+2\alpha(1-4\beta)+\alpha(2-2\beta)-3\delta)\theta^2}{4(1-\alpha)(1-\beta)(1-\gamma)k_1 k_1}$. When $0 < \alpha, \beta, \delta < 1$, the numerator is positive, so $\frac{\partial}{\partial \beta} p^*_1 > 0$.

We have $\frac{\partial}{\partial \beta} p^*_2 = \frac{\delta \theta^2}{4(1-\beta)^2(1-\beta)k_1} < 0$.

ii) *full coverage:* we have $\frac{\partial}{\partial \alpha} p^*_1 = \frac{\delta \theta^2 k_2(1+3\alpha-4\beta)(1-\gamma)}{12(1-\alpha)(1-\beta)(1-\gamma)k_1 k_2}$. By proposition 3.5, $\epsilon > \frac{1}{12} \frac{\theta^2}{k_1} - \frac{1}{k_1}$.

We have $\frac{\partial}{\partial \beta} p^*_1 = \frac{\delta (7-6\alpha-3\beta)\theta^2}{12(1-\alpha)(1-\beta)(1-\gamma)k_1} > 0$.

We have $\frac{\partial}{\partial \beta} p^*_2 = \frac{-\delta \theta^2}{12(1-\beta)^2 k_1} < 0$.

The comparative statics for equilibrium quality are straightforward and therefore, are omitted.

**Corollary 3.4:** i) *partial coverage:* we have $\frac{\partial}{\partial \alpha} q^*_1 = \frac{\delta ((-2+3\alpha-4\beta)k_2(1+3\alpha-4\beta)(1-\gamma))}{32(1-\alpha)^2(1-\beta)^2(1-\gamma)(1-\gamma)k_1 k_2} > \frac{\delta (2-2\beta)k_2(1+3\alpha-4\beta)(1-\gamma)}{32(1-\alpha)^2(1-\beta)^2(1-\gamma)(1-\gamma)k_1 k_2}$.
We have $\frac{\partial}{\partial \beta} \pi_1^* = \frac{(2-\delta)\delta^2((2-\delta)\theta^2 k_2 + (-1+\beta)k_1(\theta^2 - 12\epsilon k_2))}{16(1-\alpha)[1-\beta]^4(4-\delta)^4 k_2^4 k_1^2 k_2^2}$. By corollary 3.3, $(2-\delta)\theta^2 k_2 + (-1 + \beta)k_1(\theta^2 - 12\epsilon k_2) > 0$, so $\frac{\partial}{\partial \beta} \pi_1^* > 0$.

We have $\frac{\partial}{\partial \beta} \pi_1^* = \frac{((2-\delta)\theta^2 k_2 + (-1+\beta)k_1((2-\delta)\theta^2 k_2 + (-1+\beta)(4+\delta)k_1(\theta^2 - 12\epsilon k_2))}{32(1-\alpha)[1-\beta]^2(4-\delta)^4 k_2^4 k_1^2 k_2^2}$. In the non-buying-local consumer segment, the indifferent consumer’s location is $\frac{-(2+\delta)\beta + \delta^2}{1-\beta} + \frac{\epsilon^2}{\beta - \epsilon}$. It must be greater than $\epsilon$, so we have $\epsilon > \sqrt[4]{2}(\frac{\delta}{2(1-\beta)k_1} + \frac{1}{k_2})$. Use this inequality, we have $(8 + (-10 + \delta)\delta)\theta^2 k_2 + (-1 + \beta)(4 + \delta)k_1(\theta^2 - 12\epsilon k_2) > 0$. Hence, $\frac{\partial}{\partial \beta} \pi_1^* > 0$.

We have $\frac{\partial}{\partial \beta} \pi_2^* = \frac{\delta^2(1-\delta)k_1((2+\beta)k_1(-2+\delta)\theta^2 - 4(2+\delta)\epsilon k_2))}{16(1-\beta)^4(4-\delta)^2 k_2^4 k_1^2 k_2^2}$. In the buying local consumer segment, the indifferent consumer’s location is $\frac{(1-\delta)\epsilon + \frac{1}{4}\theta^2}{\beta - \theta} + \frac{1}{k_2}$. It must be smaller than $\epsilon$, so $\epsilon > \frac{\theta^2((1-\beta)k_1 - (-2+\delta)k_2)}{4(1-\beta)((5+20)k_1 k_2)}$. Use this inequality, we have $-\delta^2 k_2 + (-1 + \beta)k_1((-2 + \delta)\theta^2 - 4(2 + \delta)k_2) > 0$. Hence, we have $\frac{\partial}{\partial \beta} \pi_2^* < 0$.

We have $\frac{\partial}{\partial \beta} \pi_1^* = \frac{\theta^2 k_2 + (-1 + \beta)k_1((2-\delta)k_2 + (-1 + \beta)k_1((-2+\delta)\theta^2 - 4(2+\delta)\epsilon k_2))}{8(1-\beta)^2(4-\delta)^3 k_2^4 k_1^2 k_2^2}$. By corollary 3.3, we have $2\theta^2 k_2 + (-1 + \beta)k_1(\theta^2 - 12\epsilon k_2) > 0$. Thus, we know $\frac{\partial}{\partial \beta} \pi_2^* < 0$.

**ii) full coverage:** we have $\frac{\partial}{\partial \epsilon} \pi_1^* = \frac{\delta^2}{2\epsilon^2(1-\beta)^2(4-\delta)^4 k_2^4 k_1^2 k_2^2}$. By corollary 3.3, $\theta^2 k_2 + (-1 + \beta)k_1(\theta^2 - 12\epsilon k_2) > 0$. Hence, we have $\frac{\partial}{\partial \epsilon} \pi_1^* > 0$.

We have $\frac{\partial}{\partial \beta} \pi_1^* = \frac{(\theta^2 k_2 + (-1 + \beta)k_1((2-\delta)k_2 + (-1 + \beta)k_1(\theta^2 - 12\epsilon k_2))}{288(1-\alpha)^2(1-\beta)^4 k_2^4 k_1^2 k_2^2}$. Recall that $\theta^2 k_2 + (-1 + \beta)k_1(\theta^2 - 12\epsilon k_2) > 0$, we have $(-2 + \alpha(-1 + 3\beta \delta))\theta^2 k_2 + \alpha(1 - \beta \delta)^2 k_1(\theta^2 - 12\epsilon k_2) + [\theta^2 k_2 + (-1 + \beta)k_1(\theta^2 - 12\epsilon k_2)](1 + \beta \delta) = 2\beta(-1 + \alpha \delta)\theta^2 k_2 < 0$. Then we know $(-2 + \alpha(-1 + 3\beta \delta))\theta^2 k_2 + \alpha(1 - \beta \delta)^2 k_1(\theta^2 - 12\epsilon k_2) < 0$. Therefore,
we have \( \frac{\partial}{\partial \delta} \pi_1^* > 0 \).

To see \( \frac{\partial}{\partial \beta} \pi^*_{2**} < 0 \) and \( \frac{\partial}{\partial \delta} \pi^*_{2**} < 0 \), we only need to show the online firm’s demand is deceasing in both \( \beta \) and \( \delta \). Because its price and quality are non-increasing in both \( \beta \) and \( \delta \) as shown in corollary 3.3. Under full coverage, the online firm’s demand is

\[
\frac{1}{2} - \frac{1}{2 \xi} \left( \frac{1}{12} \beta^2 \left( \frac{1}{(1-\beta) \kappa_1} - \frac{1}{\kappa_2} \right) \right).
\]

It is clear that the demand is deceasing in both \( \beta \) and \( \delta \).
Curriculum Vitae

Weining Bao was born in Duyun, China on August 26, 1986. He obtained a B.A. in Economics and Finance from University of Hong Kong, Hong Kong in 2008. He entered the Ph.D. program in Economics at Johns Hopkins University in 2008.