THE MULTISCALE MECHANICS OF SURFACE MODIFICATION PROCESSES ON ASTEROIDS

by

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Abstract

The solar system is sprinkled with detritus known as asteroids, with tens of thousands of these rocky masses clustered in the asteroid belt that lies between Mars and Jupiter. These rocky bodies are debris from early solar system formation and have witnessed a long history of surface modification processes. The presence of regolith (a layer of fine-grain loose and consolidated rocks) on such airless bodies is attributed to the reaccumulation of impact ejecta and to the gradual breakdown of boulders by micrometeoritic impacts. However, ejecta velocities for small kilometer-sized asteroids typically exceed the gravitational escape velocity. This greatly limits the amount of retained debris following a high-velocity impact event and suggests that other mechanisms could also be involved in the regolith generation process. Recently, it has been observed that airless bodies in the solar system show signs of a thermally driven process. Cracks in Martian boulders exhibited preferential orientations pointing towards solar induced thermal stresses, and ponds on asteroid (433) Eros imply the existence of an active mechanism that is capable of breaking down rocks in-place, without causing high-velocity ejecta.
ABSTRACT

In this thesis, we develop techniques to bridge the varying timescales related to three surface evolution mechanisms on asteroids: thermal fatigue, mechanical disruption, and gravitational reaccumulation. The primary aim of this dissertation is to bridge the gap between material fragmentation experiments performed at lab scales, to material failure in the extreme thermomechanical environment of the solar system. The thermal fatigue mechanism is first considered, and a numerical model is formulated to capture the crack tip driving force throughout an asteroid’s diurnal cycle (a few hours) until the complete fracture of a surface rock occurs ($10^3$–$10^6$ years). The efficiency of thermal fatigue is demonstrated and compared to breakdown estimates from mechanical erosion by micrometeorite impacts. A simple analytical scaling model is derived that allows the prediction of rock breakdown rates by thermal fatigue for different airless bodies in the solar system.

Next, thermal cycling and mechanical characterization experiments are conducted on a meteorite sample to track the crack growth. The experiments revealed that cracks showed preferential extension along inclusion interfaces, which acted as stress concentration sites. Using an experimentally informed numerical model, insight into the thermally induced stress field in the meteorite is obtained. The heterogeneous mineral grains inside a meteorite are identified as key actors in the thermal fragmentation process. An examination of the interface bonding between the inclusion and matrix is then performed, showing that weaker interfaces typical of iron-rich meteorites could relax the thermal mismatch stresses and reduce the efficiency of thermal
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fatigue in small meteorites.

Finally, attention is shifted to the timescales related to hypervelocity impacts onto asteroids. Two regimes with drastically different timescales are considered: the material mechanical response (from a few microseconds up to some tens of seconds), and the gravity response (from hours to days). A hybrid framework is developed to capture the mechanical response during the first seconds after impact through a multiscale material model implemented in a Material Point Method code, which is then coupled with an $N$-body gravity code to examine the fate of the fragmented material as the ejecta interacts with the asteroid’s gravitational field during the subsequent hours. It is then shown that large asteroids (tens of km in diameter) may sustain higher impact energies than previously expected. This newly formulated hybrid approach is able to simulate a variety of asteroid impact events, such as angled impacts and spinning targets, from fragmentation to ejection and gravitational reaccumulation.

Thesis Advisor and Primary Reader: Professor KT Ramesh

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First and foremost, I express my deepest gratitude to my advisor, Professor KT Ramesh. I joined KT Ramesh’s group in the fall of 2011 with very little research experience, and soon found myself a member of a larger network of senior graduate students, postdocs, and researchers exploring cutting edge experimental and modeling techniques with applications ranging from biological tissues to metals and brittle materials. I was instantly captivated by KT’s manner of breaking down seemingly complex scientific problems into a set of well-defined research objectives. I was constantly amazed listening to KT discussing in details the research history of mild traumatic brain injury, the intricate details of a recent study on twinning in magnesium, and the fine interaction between fragmentation and amorphization in boron carbide. I soon learned how to draw similarities across those seemingly different fields by asking: what are the fundamental physical processes that dominate the material
behavior? With KT, the next question would almost inevitably be: how can we push the material to extreme conditions that activate the full range of these mechanisms? It is this multidisciplinary aspect that makes research with KT a fun challenge, and my research journey at Hopkins has been very enriching thanks to the variety of fields that I have learned about. KT’s guidance, perfectionism, and desire to produce well rounded scientists have been fundamental in shaping me into the scientist I am today.

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To my loving & supportive family
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6.1 Artistic representation of an asteroid surface where thermal fatigue was an active mechanism driving the surface evolution. Small rocks are preferentially depleted, while larger boulders have long through cracks and are surrounded by an apron of scabbed rocks. Image courtesy of Seth Izen. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 181
Artistic representation of asteroid regolith of different diameters (40cm, 10cm, 5cm, and 2cm; from left to right), illustrating the possible characteristic signatures of thermal fatigue and impact-generated regolith. The large 40 cm boulder shows prominent evidence of impacts (zap pits), while the intermediate 10cm boulder has a combination of both impacts and through cracks propagated by thermal fatigue. The 5 cm rock contains a through crack that was driven by thermal fatigue, whereas the 2cm rock shows less evidence of any thermal cycling. Advanced thermomechanical models could help in identifying the possible signatures and expected rock sizes as a consequence of each mechanism. Image courtesy of Seth Izen.
Chapter 1

Introduction and Motivation

Engineering practices and principles have found applications at the crossroads of multidisciplinary fields: from shaping our understanding of injuries in brain tissues, to predicting the outcomes of multi-megajoule impacts of objects in our solar system. The advances in characterization, experimentation, and computational modeling techniques have allowed us to obtain insights into the fundamental physical processes that influence a material’s response to dynamic loading. However, in order to build predictive capabilities, an understanding of the interaction of multiple mechanisms along with their characteristic timescales and length scales is required. Recent advances in computational capabilities, particularly in efficient code parallelization on multi-core clusters, have unlocked the potential to develop and execute sophisticated multi-physics models. Such models incorporate the essential material behavior as observed in a laboratory and extrapolate to those scales and applications beyond...
the limitations of a lab setting. In that light, the primary aim of this dissertation is to bridge the gap between material fragmentation experiments performed at lab scales, to material failure in the extreme thermomechanical environment of the solar system. Material fragmentation in these environments spans timescales ranging from microseconds to millions of years, and length scales from millimeters to tens of kilometers. In particular, three surface modification processes that drive the rock size-frequency distribution on the surface of airless bodies in the solar system are considered in this thesis: (1) rock fracture by thermal fatigue, (2) asteroid shattering and disruption by hypervelocity impacts, and (3) rock ejection and reaccumulation by gravitational interaction.

1.1 Surfaces of airless bodies in the solar system

Our solar system contains a vast number of “airless bodies”, such as asteroids, comets, planets, and some moons. Asteroids, often referred to as minor solar system bodies, are considered to be relatively pristine and unaltered objects that preserve clues from the earliest epochs of our solar system [1-3]. The exposure of asteroids to collisional evolution and the extreme space environment results in considerable surface modification. Surface properties influence the observable traits (optical and thermal properties, physical structure, chemical and mineralogical properties [4,5]).
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and as such, interpretation of the remotely collected data relies on assumptions about the key mechanisms involved in the formation of the surface layer. In addition to being the layer that is remotely sensed, the surface is also the layer that will first support any machinery (landers, rovers) before being excavated for samples. It is therefore mission-critical to properly understand the distributions of size, shape, and composition of the loose and unconsolidated rocks (also known as regolith) that may envelop the surface of asteroids of interest. It used to be thought that small (< 10 km) asteroids would develop a negligible regolith layer [1]. However, recent missions, such as the JAXA Hayabusa mission that visited the 350 m diameter near-Earth asteroid (25143) Itokawa tell a different story. In September 2005, JAXA’s Hayabusa mission recorded high-resolution close-up images (\sim 6 \text{ mm/pixel}) from altitudes of 80 to 63 m above the surface of asteroid (25143) Itokawa. It revealed (Fig. 1.1) a surface of blocky nature, lacking impact craters, and with fine regolith mostly present in two areas: the Muses Sea and Sagamihara. These smooth terrains coincide with low-gravity potentials and are generally homogeneous, featureless, and relatively flat, filled with particle sizes ranging from millimeters to centimeters [6].

In June 2011, the Hayabusa mission returned to Earth with approximately 1534 rocky particles from Itokawa [7]. Among the returned particles, over 70% (1087) were monomineralic, including 580 olivine particles, 126 low calcium (Ca) pyroxenes, 56 high-Ca pyroxenes, 186 feldspars, 113 troilites, 13 chromites, 10 Ca phosphates, and 3 Fe-Ni metal [8]. X-ray microtomography was used to understand particle texture,
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Figure 1.1: The Hayabusa mission captured close-up images of asteroid (25143) Itokawa, revealing a surface covered with numerous boulders and a rich regolith layer of rocks with varying sizes. Image reproduced from Saito et al. (2006).

mineralogy, and surface features in comparison with those evident in lunar soil. It was found that none of the particles contain melt products caused by meteorite impact, such as those seen in lunar soils [9]. Furthermore, noble gas content analysis of the grains showed that they were saturated with solar wind atoms, meaning that Itokawa’s regolith was rejuvenated and had been exposed to solar wind for less than 8 million years [10]. Consequently, a surface rejuvenation process that can maintain these small fragments ought to be active on Itokawa’s surface.

A related case of interest emerged when the European Space Agency’s Rosetta spacecraft entered a close orbit about the Jupiter family comet 67P/Churyumov-Gerasimenko on 6 August 2014. The Rosetta lander, Philae, was equipped with an optical spectroscopic and infrared remote imaging system (OSIRIS) and acquired images of the surface at scales of <0.8 meter per pixel [11,12]. The OSIRIS ob-
observations suggested that the surface features could be grouped into five categories: dust-covered terrains, brittle materials with pits and circular structures, large-scale depressions, smooth terrains, and exposed consolidated surfaces \[11\]. While it was stated that “the surface of comet 67P is almost devoid of recognizable impact craters,” several surprisingly large cracks have been observed, including a 500 m long crack in the Anuket region and a 200 m long fracture in the Aker region \[11\]. The lack of concrete evidence of shear displacement along the cracks and impact sites that might produce these cracks raises questions about the mechanisms that govern the surface evolution \[11\],[13\], especially given the large temperature variation and temporal gradients likely to be experienced by cometary \[11\] and small asteroid \[14\] surfaces over diurnal and orbital time scales.

In addition, surface images of Asteroid (433) Eros acquired by the NEAR-Shoemaker mission revealed more than 300 features that morphologically resemble ponds. These ponds (Fig. 1.2) appear to be smooth and fine-grained (sub-cm), and consist of aprons surrounding central boulders (Fig. 1.2b). These aprons imply that boulders on Eros are eroding in place through a thermally driven process.
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1.2 Thermal fatigue as a regolith evolution mechanism

Cracks are naturally present in geologic materials at virtually any lengthscale. These cracks may grow dynamically (following an impact event, for example), but could also grow in a sub-critical and stable manner (such as in fatigue). This sub-critical fatigue crack growth occurs in materials that experience a cyclic loading with a stress intensity factor that does not exceed the material’s critical toughness, but with stresses large enough to induce dissipative deformations near the crack tip. The cracks therefore slowly extend and propagate and could eventually fragment a material after a sufficiently large number of cycles. Small airless bodies lack a protective “atmosphere” layer and their surfaces are therefore directly exposed to the harsh space environment, and the temperature variations between an asteroid’s “day” and “night” can be extremely large. In addition, asteroids have a typical rotational period of less than 10 hours. The surface temperatures of asteroids then follow a diurnal cycle with considerable temperature changes as the Sun rises or sets. Thermal stresses arise from thermal gradients as well as from mismatches in the coefficients of thermal expansion of the minerals. These stresses can produce driving forces on cracks in the regolith material, leading to the opening and extension of microscopic cracks. Rock breakdown can then occur as cracks continue to grow gradually over a sufficiently large number of thermal cycles. This process is known as thermal fatigue, and
progressive thermal fatigue leads to thermal fragmentation (e.g. Kranz et al. [15]).

The ability of diurnal temperature cycling to cause a breakdown of surface rocks and boulders on Earth and other planetary bodies has been heavily debated for more than a century. Recently, studies based on field observations, laboratory experiments, and modeling have confirmed the effectiveness of such thermal weathering on Earth, Mars, and airless bodies [14, 16, 17]. In this thesis, we explore the relative efficiency of thermal fatigue in fragmenting small (cm-sized) rocks, and develop techniques to bridge the varying timescales of thermal fatigue: from temperature variations on the order of a few minutes, to fragmentation times on the order of millions of years.
1.3 Hypervelocity impacts: fracture, fragmentation, ejection, and reaccumulation

The early era of the solar system was heavily shaped by heavy bombardment \[18\], with impact fluxes substantially higher than over the past 3.5 billion years. Much of the history of large asteroid and planet formation and evolution can be traced back to a series of high-energy impact events. Hypervelocity impacts onto an asteroid can alter both the asteroid’s surface and its interior. Detailed information about asteroid shapes, composition, and orbital dynamics has been collected through remote sensing \[19\] and spacecraft missions \[20\]; however, the internal structure of those asteroids remains poorly constrained. Is a given asteroid a fractured monolith, or a rubble pile of gravitationally bound fragments? This distinction is difficult to make even for asteroids with known bulk densities. As pointed out in \[21\], the same data for asteroid 433 Eros could be interpreted to support a monolithic structure with impact-induced grooves \[22\], or to support a rubble pile \[23\]. Impact events probe the interior of asteroids through the propagation of stress waves through the target. The response of an asteroid to those stress waves would be different depending on its internal structure: a coherent monolithic target will transmit waves more efficiently than a rubble pile. The extent of damage (lineament formation, for example) and
Figure 1.3: In July of 2005, the Deep Impact spacecraft released a 370 kg impactor into the path of comet Tempel 1 at a closing speed of about 10 km/s. The impact event as imaged by the on-board High Resolution Instrument is shown in sequence at 0.84 s intervals (left). A plume of hot material is ejected outwards at a speed of 5–10 km/s. Look-back images (right) 45 minutes after the impact shows an ejecta plume that is still connected to the comet nucleus, indicating a gravity-dominated evolution of the ejected material. Images reproduced from A’Hearn et al. (2005).

Material ejection (size-frequency distributions of blocks away from an impact crater) can therefore be used as indicators of the nature of the target. For instance, the Deep Impact mission [24] probed the interior composition of comet 9P/Tempel by striking it with an impactor at 10 km/s. The ejection of a large amount of fine dust particles (Fig. 1.3) indicated that the comet may not be a mere aggregate of gravitationally bound material [25], despite being an extremely weak and highly porous target [26].

Since laboratory experiments alone can not sufficiently reproduce the scales and conditions within asteroid impacts, numerical modeling has become a valuable approach for providing insight into the internal structure of asteroids, asteroid surface modification, and the formation of asteroid families and satellites.

Studies of asteroid hazard mitigation, together with the prospect of kinetic impactor-based space experiments such as the DART mission [27], also call for detailed compu-
tational modeling of hypervelocity impacts onto small (sub-km) asteroids. A primary result of interest in asteroid mitigation studies is a measure of the momentum transfer characterized by the momentum enhancement factor \(^{28,29}\). By ejecting material from an impacted asteroid, the momentum of what remains of the target can be substantially increased. Predicting the behavior of a target following an impact event also has direct implications on the possible planetary defense techniques that could be employed to protect Earth from the threat of a potentially hazardous object.

Hypervelocity impacts activate a large number of mechanisms that occur at different scales. Material very near to the impact site may experience phase transformations (if the impact energies are high enough). In addition, the material is shocked as stress waves propagate with finite speeds, leading to stress states that can nucleate and grow cracks (also at finite speeds) that eventually lead to fragmentation. Fragmented material can flow and re-arrange to evolve the overall porosity of the material. All these mechanisms kick in during the seconds following initial contact.

Predicting the outcomes of hypervelocity impacts through computational modeling requires an understanding of not only the physical processes that occur, but also their interactions, which are typically hard to decouple in impact experiments. After shock waves have propagated through an asteroid target, the evolution of impact-generated debris is controlled by the gravitational interaction of the individual fragments, which is nearly impossible to replicate in experiments onto cm-sized samples. These gravitational forces induce changes in the velocity field of the ejected masses, which could
lead to reaccumulation onto the target’s surface or possibly the formation of asteroid families.

1.4 Organization of the thesis

This thesis covers a range of topics related to the fracture of brittle geologic materials in the solar system. The focus of this thesis is the development of predictive numerical and computational models that bridge various lengthscales and timescales while capturing the essential mechanisms of fracture. The work is presented in 6 chapters. In Chapter 1, the background and motivation behind this cross-disciplinary work is provided. The development of a numerical code that is capable of simulating the thermal fatigue breakdown of regolith is presented in Chapter 2. A simplified analytical scaling relation for fragmentation rates as a function of rock sizes is then developed. In Chapter 3, characterization experiments are performed on a meteorite sample, along with thermal cycling experiments to observe the fatigue growth of a natural pre-existing crack in the sample. A numerical model is then used to obtain insights onto the stress state within the sample during the experiment, and the effects of mismatches in expansion coefficients of the individual grains are highlighted. The model developed in Chapter 2 is then applied to predict the crack growth in a representative mesh of the actual microstructure of the sample. Realizing the importance

\footnote{Chapter 2 is in collaboration with Dr. Marco Delbo, Observatoire de la Côte d’Azur, Nice.}

\footnote{Chapter 3 is in collaboration with Dr. Kavan Hazeli, University of Alabama, Huntsville.}
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of the heterogeneous grains on the thermal fatigue fracture of regolith, a study of the grain-matrix interface is outlined\(^3\) in Chapter 4. The effect of interfacial bonding properties on the resulting thermal stresses and driving forces for crack growth are explored. In Chapter 5, attention is shifted to the multiscale modeling of impact processes. A hybrid numerical approach that is capable of bridging between multiple physical mechanisms occurring during a hypervelocity impact event is developed\(^4\).

The first few seconds following the contact of an impactor and target are simulated in a highly parallelizable multi-physics numerical platform. Then, a consistent hand-off technique is formulated to move onto an \(N\)-body gravity code that calculates the inter-particle gravitational forces and ejecta evolution. The key differences and significant advances from past work in the literature, and their implications with regards to ongoing space missions will be highlighted. Lastly, the summary of the work presented in this thesis and opportunities for future research directions are discussed in Chapter 6.

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\(^3\)Chapter 4 is in collaboration with Dr. Guy Libourel, Observatoire de la Côte d’Azur, Nice.

\(^4\)Chapter 5 is in collaboration with Dr. Derek C. Richardson, University of Maryland, College Park.
Chapter 2

The efficiency of thermal fatigue in regolith generation on small airless bodies

2.1 Introduction

Asteroids, comets, and small planets in the solar system are covered with a layer of loose and unconsolidated rocks called regolith. Surface imagery acquired by space probes has indicated that even the relatively small kilometer-sized asteroids carry a complex blanket of fine rocks smaller than a few centimeters. Images of asteroid (25143) Itokawa and comet 67P/Churyumov-Gerasimenko highlighted the existence of freshly exposed boulder surfaces, indicating that the evolution of these
boulders is driven by an active surface rejuvenation process that could differ from those manifested on the lunar surface [30].

It is generally accepted that the lunar regolith is the byproduct of a long history of mechanical disruption wherein large boulders are eroded by a series of micro- and macro-meteoritic impacts [31]. The abundance of impact-induced agglutinates points towards the dominant role that meteoritic impact had on the evolution of the lunar regolith. However, on small asteroids, the ejecta velocities from an impact event typically exceed the asteroid’s escape velocity, and most of the resulting small debris would be lost into space as opposed to being reaccumulated on the surface. Early models of impact-induced regolith on airless bodies indicated that small asteroids (10 km or smaller) should only retain a negligible regolith layer [1, 32, 33]. Consequently, asteroidal regolith generation remained, for the most part, a poorly understood mechanism [34].

Recently, thermal fatigue was suggested as a mechanism for in-place rock breakdown capable of generating fine-scaled regolith layers without any subsequent ejection. Observational evidence in favor of thermal fatigue in the solar system includes the work of Eppes et al. [16] who collected orientation measurements from more than 1,800 cracks visible in nearly 1,500 rocks photographed by the Spirit rover during its journey on the Martian surface. Their measurements showed that these cracks exhibit preferred orientations consistent with solar-induced thermal stresses. Dombard et al. [17] also showed that some boulders inside regolith “ponds” on 433 Eros
have debris aprons, which were interpreted as the evidence that these rocks erode in place, likely due to thermal cracking. In another work, Delbo et al. [14] examined fragmentation induced by the diurnal temperature variations on asteroids. Their results indicated that thermal fatigue could play an important role in the generation of fine-grain regolith on small asteroids, and that the process is several orders of magnitudes faster than mechanical impact. Delbo et al. also showed that thermal fatigue could break down larger rocks faster than smaller ones, owing to the greater temperature gradients in the bigger rocks—a trend that is in contrast to what is seen in the case of mechanical disruption by microimpacts. This latter result provides an explanation for the presence of fine regolith on small asteroids, but it is not clear how it would apply for larger boulders (the Delbo et al. calculations were for surface rocks with diameters between 1 and 10 cm). The presence of meter-sized rocks on several asteroids, such as those pictured on Itokawa [30, 35], indicates that thermal fatigue ought to behave differently at these larger sizes.

Here we demonstrate that thermal fatigue has an inherent lengthscale that characterizes the bounds on the rate of fragmentation for rocks in the range of 1–50 cm. We first develop a thermomechanical model that tracks the evolution of a crack due to the diurnal temperature variations in an asteroidal rock and efficiently bridges across the vastly different timescales of rotation and time to fracture. Using the model, we compute the trends in the time-to-fragmentation of sub-meter-sized rocks. We then develop a generalized analytical expression to predict the survival times of regolith
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particles on small airless bodies. Note that the “regolith” breakdown study performed
in this chapter is limited to the aforementioned range of 1–50 cm, and it will be shown
that thermal fatigue would be most efficient for rocks of such diameters.

2.2 Numerical Modeling of Thermal Fatigue

Thermal fatigue cracking of regolith results from the fluctuations in the temperature field that develop driving forces (which are not merely the stresses) on cracks within a rock. These cracks may then experience sub-critical fatigue growth until fragmentation. In this section, we present a numerical framework that calculates the crack tip driving forces due to thermal cycling and predicts crack growth over a large number of cycles.

2.2.1 Thermal Model

We consider an uncoupled solution that allows us to solve the heat equation (solar radiation and conduction) first, and use the resultant temperature field as an input to the thermomechanical deformation problem. Effectively, we are assuming that the stress state does not affect the thermal properties of the regolith and that the cracks have negligible effects on the temperature field.
The temperature profile in surface rocks is calculated using a diffusion model based on the asteroid thermal model developed by Spencer et al. [36]. This model calculates the temperature $T(t, z)$ as function of time and depth in a regolith at the equator of an asteroid by solving the heat equation:

$$\rho C_p \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2},$$

(2.1)

where $\kappa$ is the thermal conductivity, $C_p$ the heat capacity, $\rho$ the density and the coordinate $z$ increases from zero at the surface downwards. It is well documented that thermal conductivity varies with temperature, in particular in the case of meteorites [37, 38]. Recent astronomical observations showed this effect also in case of asteroids (Rozitis et al. [39]; with some potential hints also observed on the asteroid 6 Hebe by Marsset et al. [40]). For the case treated here, however, a temperature dependent thermal conductivity would only slightly enhance the maximum diurnal temperature variations by some degrees (as shown in [41]). Therefore, in this formulation we assume that the thermal properties do not depend on depth and temperature. As a consequence, the thermal cracking effects hereafter estimated are somewhat conservative. Given the uncertainties in all other thermo-mechanical parameters, we prefer to use the simpler model of temperature independent conductivity, which gives results that can be compared with other literature studies.

The heat diffusion equation is solved with the surface boundary condition of
where: $\sigma_{rad}$ is the Stefan-Boltzman constant; $\epsilon_{ir}$ the infrared emissivity, which is assumed to be equal 0.9 as common in studies of asteroid surfaces (see [41] and references therein); $A$ is the bolometric Bond albedo, which is typically a few percent for the generally dark asteroid surfaces, $r$ is the heliocentric distance of the asteroid in au; and $S_\odot$ is the solar constant at 1 au of 1370 W m$^{-2}$. The parameter $\mu$ is related to the direction cosine between the normal to the surface element and the direction to the sun $\theta$, with $\mu(t) = \cos [\theta(t)]$ for $-\pi/2 < \theta < \pi/2$ and $\mu(t) = 0$ otherwise. The angle $\theta$ represents the local day time of the surface element, i.e., $\theta = 2\pi/P$ where $P$ is the rotation period of the asteroid. This boundary condition implies that the regolith is treated as a flat surface. It is known that surface roughness affects the observed temperatures of airless bodies [42,43] ; however, this is a global effect on large surfaces. Locally, roughness can enhance the dayside temperature of rocks on the surface due to the mutual heating. This happens when a rock has a non-zero view factor towards warm surface elements (i.e.: the rock experiences local radiation due to the local topolgy), resulting in slightly larger temperature excursion between day and night. The flat surface thermal model we consider here is thus a conservative case.

At a depth of 5 meters, the additional boundary condition of $\frac{\partial T}{\partial z}|_{z=5\text{m}} = 0$ implies that there is no heat flow towards (or from) deeper into the body. These are similar to
Figure 2.1: Temperature as a function of depth during a 6 hour day/night cycle, plotted at 15 min increments. This temperature profile is typical of diurnal thermal cycling. The heat wave attenuates quickly beyond the skin depth, and temperature variations become negligible.

...the conditions imposed by classical asteroid thermal modeling (see [41] for a review).

The resulting time-dependent and depth-dependent temperature fields for rocks on the surface of a small carbonaceous chondritic near-Earth asteroid having a rotational period of 6 hours are shown in Fig. 2.1. This is used as the input to the computational model in Section 2.2.4.

In Fig. 2.1, the temperature as a function of depth during a 6 hour day/night cycle is plotted at 15 min increments (changing hues from red to blue). The regolith surface experiences the largest temperature changes, which are quickly attenuated and disappear below a depth of 20 cm centimeters. Notice that the temperature
variations as a function of depth are highly time-dependent, with the maxima being experienced at different times for different depths. This highlights the need for an efficient thermomechanical model that can solve for the stress fields at small increments of times, while being capable of tracking the crack growth for many years.

2.2.2 Thermomechanical Crack Growth Model

In this section, we develop a thermomechanical model that predicts the fatigue crack growth in a rock as a result of the spatiotemporally varying temperature field calculated in the thermal model (Section 2.2.1). To do this, we must compute the driving force on a crack tip in the rock. This driving force is called the stress intensity factor or SIF. It is important to recognize that the stress field computed within an uncracked body does not identify whether or not a crack will grow [44], and indeed, an accurate analysis of the stress field within an elastic rock containing a crack will always show an infinite stress at the crack tip. Understanding crack growth requires the use of the concepts of fracture mechanics. In general, cracks grow in one of three modes, each of which has an associated stress intensity factor: Mode I or tension, which results from tension perpendicular to the crack faces, with stress intensity factor $K_I$; Mode II or shear, which results from shear stresses acting parallel to the crack faces and causing crack sliding, with stress intensity factor $K_{II}$; and Mode III or antiplane shear, which results from shear displacements acting parallel to the crack front, with stress intensity factor $K_{III}$. These stress intensity factors fully describe
the stress, strain and displacement functions at the crack tip, and crack growth laws (crack direction, crack length and crack speed) are typically written in terms of these quantities [45,46]. Having multiple modes active at one time is said to define a mixed-mode problem, and the thermal fatigue problem turns out to be mixed mode. So, to model the thermal fatigue fragmentation caused by the growth of a crack in a rock, we must first compute the stress intensity factors as the temperature field is varying.

2.2.2.1 Background

Stress intensity factors depend on both the loading configuration as well as the geometry and crack size [47]. Closed-form analytical solutions only exist for some simple idealized cases such as remote tension on a cracked plate. Some “universal” weight function approaches have also been developed [48] but their application remains limited, especially with increasing complexity in loading profiles and crack configurations. Consequently, numerical approaches such as the Finite Element Method (or its extensions) are often needed. While traditional finite element method formulations are not capable of capturing the discontinuity in the displacement field that is caused by the presence of a crack, some measure of the singularity at the crack tip can still be achieved by explicitly meshing a sharp notch as part of the shape model. Post-processing techniques can then be used to estimate the stress intensity factor either by fitting the displacement or stress field solutions to the near-crack analytical solution, or by means of an energy integral approach [49]. However, the post-processing
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approach requires an increasingly fine mesh near the notch to obtain satisfactory results, which increases the computational cost and data preparation effort for each simulation.

As a consequence, numerical approaches that extend the traditional finite element method have been developed for modeling cracked bodies. These are often achieved by formulating special types of elements that include a singularity in their shape functions \([50, 52]\) and using them near the notch to reduce the required amount of fine-meshing. Other methods include the quarter-point finite element \([53, 54]\), the boundary collocation method \([55, 56]\), and the boundary elements method \([57, 58]\). These techniques can reasonably capture the singularity for a stationary crack tip. However, any crack propagation would then require the regeneration of a new mesh that conforms with the new crack geometry. This “remeshing” step is often the most computationally expensive step in an analysis, making these methods less suitable for crack evolution simulations.

One technique to avoid this remeshing step is the so-called eXtended Finite Element Method (XFEM) \([59, 60]\). Belytschko and Black (1999) \([59]\) first introduced XFEM by discretizing a crack in the mesh through the enrichment (provision of additional degrees of freedom) of elements near the crack tip and crack faces. The enrichments exploit the partition of unity property inherent in finite elements \([61]\) to include the asymptotic displacement field resulting from the crack without modifying the existing mesh. Moes et al. \([62]\) then introduced the generalized Heaviside step
function to characterize the discontinuous field across the crack faces away from the crack tip, and [63] detailed a procedure for multiple branched cracks through the use of a junction function.

In XFEM, cracks can be incorporated into an existing finite element mesh by first identifying the elements that contain the crack, and labeling their respective nodes as face-enriched nodes related to elements that are split by a crack, or tip-enriched nodes related to elements containing a crack tip (see Fig. 2.2). Cracks can then move freely within the mesh without requiring a reconstruction of the mesh itself, but rather a simple identification of the elements to be “enriched” with additional degrees of freedom. The displacement field can then be computed using the standard finite element solvers [64]. The stress intensity factor is then calculated in the post-processing stage using an interaction integral method [52][65].

2.2.2.2 Model Details

Here, we develop a custom XFEM implementation (based on Bordas et al. [66]) that solves for the displacement field in cracked rocks subjected to cyclic thermal loading. Since the near-surface temperature variations change quickly with time (see Fig. 2.1), the model is optimized to efficiently bridge between the relatively small timesteps within a single period of rotation (using 15-minute increments) and the longer timescale of fragmentation ($10^3 - 10^6$ years).

Consider a cracked rock that is thermally cycled. We idealize the rocks as plane
Figure 2.2: Mesh of a circular rock containing a surface crack. The gray-shaded elements are directly affected by the crack and will include additional shape functions (degrees of freedom). The nodes identified with a red square belong to elements that are “split” by the crack and will contain step-function enriched degrees of freedom. The nodes labeled with a green circle belong to the element that contains the crack tip and will be enriched with asymptotic functions to capture the singularity at the crack tip. Each element contains a sub-scale distribution of heterogeneities (inset on the right), which are assumed to be circular and distributed in a regular array. These inclusions will contribute to the global stress tensor. The left inset presents the crack-tip coordinate system and the domains for the interaction contour integral used to calculate the stress intensity factor. On the mesh, these contours span 3–5 elements around the crack tip. Symbols are defined in the subsequent pages of the main text.
strain cylindrical 2D bodies. First, a representative mesh of the geometry (circular in our case) is generated irrespective of the cracks. Then, cracks are overlaid on the mesh, and the elements that are split by a crack and those that contain crack tips are identified (Fig. 2.2). These elements are “enriched” with the additional degrees of freedom, so that the total displacement field \( u \) at a point \( x \) is written in terms of the regular FEM displacement (without a crack) and an XFEM displacement (related to the crack enrichments):

\[
  u(x) = \sum_i N_i(x) \hat{u}_i + \sum_j N_j(x) \psi(x) a_j
\]

where \( \hat{u} \) is the nodal displacement from traditional finite element formulation, \( N \) the finite element shape function, the \( a_j \)’s are the additional degrees of freedom related to the enrichment \( \psi \) that is asymptotic for crack tip enrichment \([67]\) and a Heaviside step-function for crack face enrichment.

The temperature field induces thermal strains, which are captured through additional nodal forces given for a unit thickness as:

\[
  f^{th} = \int_A BC \epsilon^{th} dA = \int_A BC (\alpha \Delta T) I dA
\]

with \( B \) being the displacement differentiation matrix, \( C \) the stiffness tensor, \( \epsilon^{th} \) the thermal part of the total strain, \( \alpha \) the thermal expansion coefficients, and \( I \) the identity tensor.
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Thereafter, the global stiffness matrix and the force vectors are constructed in the same manner as the traditional finite element method \[68\]. The nodal displacements are then obtained using an implicit solver, and the gradients of the displacement field gives the total strain tensor $\epsilon$. The Cauchy stress tensor ($\sigma$) is computed from the mechanical strain $\epsilon^m$, using the linear constitutive relation:

$$\sigma = C : \epsilon^m = C : (\epsilon - \epsilon^{th}) \quad (2.5)$$

Asteroidal rocks contain inclusions with varying thermomechanical properties. The mismatch in thermal expansion coefficients between inclusion and matrix generates additional internal stresses as the matrix constrains the grain’s expansion. An analogous behavior is often observed in composite materials subjected to high temperature variations \[69,71\]. Our model is also capable of treating inclusions as heterogeneities that are part of the material mesh, but such analysis will not be addressed in this chapter (see Chapter 3 for the role of inclusions). In this work, the inclusions are treated as sub-scale (Fig. 2.2) to our mesh resolution. The effective contribution of the thermal expansion mismatch stress caused by a volume fraction $f$ of inclusions is accounted for using a representative volume element approach following a modified Eshelby formulation \[72\] as:

$$\sigma^{TM} = \left( \frac{1 - 2\nu_{inc}}{E_{inc}} + \frac{1}{1 - f} \frac{1 + \nu_m}{2E_m} + \frac{f}{1 - f} \frac{1 - 2\nu_m}{E_m} \right)^{-1} (\alpha_m - \alpha_{inc}) \Delta T I \quad (2.6)$$
where \( E \) and \( \nu \) are the Young’s modulus and Poisson ratio, respectively. The subscript \( m \) refers to the matrix (or bulk) material properties, and \( inc \) refers to properties of the average inclusion (such as a chondrule).

Now that we have the displacements, strains, and stresses, we can compute the stress intensity factor at the crack tip. Typically, the energy release rate at the crack tip (which is related to the stress intensity factor) can be extracted by means of a J-integral approach \[73\]. The J-integral is a path-independent line integral that is equal to zero for a simply connected closed loop around a region containing no singularities. When the integral path contains a crack, the J-integral is equal to the energy release rate for the crack growth, which is related to the stress intensity factor at the crack tip. Note that the J-integral alone does not provide enough information to extract the individual measures of \( K_I \) and \( K_{II} \) in a mixed-mode crack problem, since it depends on their combined effects:

\[
J = \frac{(K_I^2 + K_{II}^2)}{E^*} \tag{2.7}
\]

where \( E^* \) is equal to the Young’s modulus at the crack tip \( E \) for plane stress, and \( E^* = E/(1 - \nu^2) \) for plane strain, with \( \nu \) the Poisson’s ratio at the crack tip. Note that \( K_{III} \) is not relevant for these 2D problems.

In this work, we use the interaction integral approach \[59,74\], which decouples the individual contributions of \( K_I \) and \( K_{II} \) by superposing the solution of assumed “auxiliary” fields (note that the linearity of the elastic problem implies that the su-
perposition of two fields at equilibrium should lead to another equilibrium state).

Proper choices of these auxiliary states allow for decoupling the $K_I$ and $K_{II}$ contributions. The J-integral for the superposed equilibrium state (superscript $sup$ below) would contain contributions from the auxiliary state (superscript $aux$ below) and the original state in XFEM (no superscript). That is:

$$J^{sup} = \frac{1}{E_{\text{tip}}} \left( (K_I + K_I^{aux})^2 + (K_{II} + K_{II}^{aux})^2 \right) = J + J^{aux} + I^{sup} \quad (2.8)$$

The $I^{sup}$ term is an interaction term that contains the coupling between the XFEM solution and the auxiliary field solution. It can be shown that $I^{sup}$ is a line integral of the form:

$$I^{sup} = \lim_{\Gamma_1 \to 0} \oint_{\Gamma_1} \left( \sigma_{ij}^{aux} u_{j,1} + \sigma_{ij} u_{j,1}^{aux} - \sigma_{jk}^{aux} \epsilon_{jk} \delta_{1i} \right) m_i q d\Gamma_1 \quad (2.9)$$

where $m_i$ are the components of the unit outward normal vector to the integrating contour $\Gamma_1$ (Fig. 2.2).

Recall that the fundamental property of the J-integral is that it equates to zero over a closed path away from the crack. However, Wilson et al. [65] showed that, in general, the integral is non-zero when thermal stresses are present, meaning that for thermal stress crack problems the crack tip stress intensity factors cannot be determined directly from such a line integral calculation. The alternate formulation
of [65] is used in our model after converting the line-integral into an equivalent domain integral (EDI) written in crack-tip coordinates [75, 76], as this is better suited for implementation in finite element codes since it is written in terms of quantities readily available from the analysis step:

\[
\begin{align*}
I^{\text{sup}} &= \int_A \left( \sigma_{ij}^{\text{aux}} u_{j,1} + \sigma_{ij} u_{j,1}^{\text{aux}} - \sigma_{jk} \epsilon_{jk}^{\text{aux}} \delta_{1i} \right) q_i dA \\
&\hspace{1cm}+ \int_A \left( \sigma_{ij}^{\text{aux}} \alpha(\Delta T)_{1j} \delta_{ij} \right) q dA \\
&\hspace{1cm}+ \int_A \left( \sigma_{ij}^{\text{aux}} \alpha(\Delta T)_{1j} \delta_{ij} \right) q dA
\end{align*}
\]  

(2.10)

Here, \( q \) is a weight function with values varying smoothly from 1 on \( \Gamma_1 \) to 0 on \( \Gamma_0 \) (Fig. 2.2). The second integral comes from the thermal contribution to the J-integral. The different stress intensity modes can now be individually extracted through the proper choice of auxiliary fields [74]. It follows from Eq. (2.8) that the mixed-mode stress intensity factors are related to the interaction integral as:

\[
I^{\text{sup}} = \int_A \left( \sigma_{ij}^{\text{aux}} u_{j,1} + \sigma_{ij} u_{j,1}^{\text{aux}} - \sigma_{jk} \epsilon_{jk}^{\text{aux}} \delta_{1i} \right) q_i dA
\]

Then, by equating Eqs. (2.11) and (2.10) and using the fundamental solution [77] for a crack in pure mode-I \((K_I^{\text{aux}} = 1, K_{II}^{\text{aux}} = 0)\), the \( K_{II} \) terms are dropped and we are left with \( K_I \) as the only unknown. Similarly, setting the auxiliary field to the pure mode-II solution \((K_I^{\text{aux}} = 0, K_{II}^{\text{aux}} = 1)\) gives \( K_{II} \).

Now that the mixed-mode stress intensity factors are known, we use an appropriate
fatigue crack growth law to describe the rate of crack propagation after a thermal excursion cycle. We here use the so-called Paris’ Law [46], which relates the crack length, \( a \), in a given number of cycles, \( N \), to the excursion in the effective stress intensity factor, \( \Delta K_{eff} \):

\[
d\frac{da}{dN} = C (\Delta K_{eff})^n
\] (2.12)

where \( C \) and \( n \) are material parameters fit to fatigue experiments. Note that these parameters generally depend on the material, environment, frequency, temperature, and loading. We use the energy release rate model [78] to define the effective stress intensity factor, combining the \( K_I \) and \( K_{II} \) contributions:

\[
K_{eff} = \sqrt{K_I^2 + K_{II}^2}
\] (2.13)

The crack is allowed to grow arbitrarily in the mesh, with an orientation determined by the maximum tangential stress criterion given by [79]:

\[
\theta = 2 \arctan \left( \frac{1}{4} \frac{K_I}{K_{II}} \pm \frac{1}{4} \sqrt{\left( \frac{K_I}{K_{II}} \right)^2 + 8} \right)
\] (2.14)
Figure 2.3: Flowchart of the thermomechanical model developed in this work. The pre-processing stage sets up the rock geometry, thermomechanical properties, and initial conditions. The processing stage runs in parallel on multiple computational nodes to solve different timesteps within a single period of rotation. The excursion in stress intensity factor is calculated in the post-processing stage and used to advance cracks until fragmentation.
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With these equations implemented into our model, we are able to perform high-resolution simulations of thermal fatigue crack growth over multiple cycles (see Fig. 2.3). We further optimize our algorithm to make use of multiple processors, making it suitable to be run on high-performance computational clusters. The computational “bottlenecks” of the XFEM algorithm in this case arise from: a) assembling the global stiffness matrix and force vector, b) solving the set of coupled linear equations, and c) detecting the nodes for enrichment after cracks propagate. Therefore, we first parallelize the stiffness matrix and force vector assembly process through an algorithm that maps the element-wise contributions to their position in the global matrices. This allows for resolving elements from different parts of the mesh onto separate computational cores and populating their contribution to the global stiffness matrix and force vector using the map. The computational speedup resulting from this matrix mapping is shown in Fig. 2.4. In addition, the temperature variation in our thermal fatigue problem has no effect on the global stiffness matrix during a single cycle (when the current crack has not yet propagated), and only affects the global force vector (following Eq. (2.4)). So during a single step, we only build the global stiffness matrix once and send the force vector for each time increment onto a separate node for parallel computation (that is, all time increments can be solved simultaneously on different computational nodes; Fig. 2.3) using an efficient parallel implicit matrix solver [80]. When the crack propagates at the end of a thermal cycle, we detect affected elements using a sub-grid search algorithm that starts from
elements surrounding the crack tip (instead of looping over all elements). This typi-
ically reduces the search algorithm to a much smaller loop since the fatigue crack
growth distance in a single cycle rarely exceeds a few elements from the original po-
sition. These features allow us to efficiently bridge across a large number of cycles
and makes our thermomechanical model effective for modeling the thermal fatigue
breakdown of rocks on airless bodies in the solar system.
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Figure 2.4: Benchmarking tests for building the global stiffness matrix, $K$, in a mesh constituting of 14,400 nodes and containing a single crack. $K$ is a $28924 \times 28924$ matrix when the additional degrees of freedom related to the crack are added. The blue line shows the speedup when building the stiffness matrix for the first time at the beginning of the simulation, and the red line shows the speedup when the stiffness matrix is re-evaluated after a crack propagates. A "worker" consists of a single core with two hyper-threaded sub-processors. The speedup using 8 workers (16 threads) reached $6 \times$ the baseline (single processor, no parallelization). Tests were performed on a local workstation having a single Intel(R) Xeon(R) CPU E5-2680 v3 @ 2.50GHz processor.
2.2.3 Stresses in cracked rocks under thermal cycling

We first discuss the stress fields that arise in a cracked rock as a result of thermal cycling. As a typical example, Fig. 2.5 presents the computed stress field in a 40 cm diameter rock containing a 15 cm crack. The figure shows six snapshots (the columns) from the computations, one every hour over the 6 hour period. The top row shows the temperature field in the rock at that time, and the bottom row shows the corresponding stress field in the cracked rock. For comparison, the middle row shows the stress field that would exist in the uncracked rock. The cracked rock has an entirely different stress field than the uncracked rock, with different maximum stresses, and with these maxima located in different parts of the rock. These differences highlight the importance of using fracture-mechanics solutions to accurately capture the singularity at the crack tip. Note that answering the question of how much a crack will grow in such a thermal cycle can only be done by using the stress intensity factor as a measure. Further, stresses alone cannot identify the extent of crack growth. Thus the peak stresses obtained from an uncracked rock alone are not reasonable indicators of the thermal fatigue lifespan of cracked rocks.
Figure 2.5: The temperature field in a 40 cm rock is plotted in the first row at 1 hour increments for a 6 hour thermal cycle. The stress field perpendicular to the crack faces (stresses in the crack opening direction) is shown on the second and third rows. The second row is for a rock without any crack, while the third row shows the stress field in a rock of the same size that has a 15 cm surface crack. Notice that the stress at the crack tip is singular and that the crack relaxes the stress field at its faces.
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2.2.4 Rate of rock breakdown by thermal fatigue

We now use the thermal and thermomechanical models presented so far to determine the time required to fracture rocks of varying diameters through the growth of a crack by thermal fatigue. We consider the case of a near-Earth asteroid with a 6-hour period of rotation, which is typical of many small asteroids [81]. We model circular rocks of different diameters, all containing an initial surface crack (see the mesh in Fig. 2.2 and Fig. 8 in [14]). We use the thermomechanical properties for carbonaceous chondrite (Table 2.1) and fatigue parameters of Carrara marble [82]. These are the same material properties and initial conditions as in [14]. The stress intensity factor is recorded at time increments of 15 minutes. At the end of a complete cycle, the excursion between the maximum and the minimum stress intensity factor is determined and the equivalent stress intensity factor excursion is calculated using Eq. (2.13). Then, Eqs. (2.12) and (2.14) are used to calculate the crack length increment and direction.

Although our modeling approach is able to solve for arbitrarily growing cracks, recall that the temperature field computed from the thermal model is one-dimensional (that is, the temperature at a given time is a function of only one direction) and that the geometry and crack configuration are symmetric. This symmetry in geometry and loading implies that no mode-II fragmentation will occur ($\Delta K_{II} = 0$ in every cycle), and consequently the crack will propagate over a straight path without any kinking ($\theta = 0$). Given these idealizations, we consider the rock to be broken when
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Table 2.1: Values and base dimensions assumed for the physical properties of the carbonaceous chondrite material considered in this study. The base dimensions are expressed in the MLTθ (Mass-Length-Time-Temperature) system.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
<th>Dimensions</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paris exponent</td>
<td>n</td>
<td>-</td>
<td>3.84</td>
<td>[82]</td>
</tr>
<tr>
<td>Paris pre-factor</td>
<td>C</td>
<td>m (MPa√(m))^{-n}</td>
<td>3 × 10^{-4}</td>
<td>[82]</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>E</td>
<td>GPa</td>
<td>45</td>
<td>[14]</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>ν</td>
<td>-</td>
<td>0.24</td>
<td>[11]</td>
</tr>
<tr>
<td>Bulk expansion coefficient</td>
<td>αm</td>
<td>K^{-1}θ^{-1}</td>
<td>8.5 × 10^{-6}</td>
<td>[83]</td>
</tr>
<tr>
<td>Chondrule expansion coefficient</td>
<td>αinc</td>
<td>K^{-1}θ^{-1}</td>
<td>10.4 × 10^{-6}</td>
<td>[84]</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>κ</td>
<td>W m^{-1} K^{-1}</td>
<td>0.5</td>
<td>[37]</td>
</tr>
<tr>
<td>Specific Heat Capacity</td>
<td>C_p</td>
<td>J kg^{-1} K^{-1}</td>
<td>500</td>
<td>[37]</td>
</tr>
<tr>
<td>Bulk density</td>
<td>ρ</td>
<td>kg m^{-3}</td>
<td>1662</td>
<td>[37]</td>
</tr>
<tr>
<td>Thermal inertia</td>
<td>Γ</td>
<td>J m^{-2} s^{0.5} K^{-1}</td>
<td>640</td>
<td>[37]</td>
</tr>
<tr>
<td>Rotation period</td>
<td>τ</td>
<td>h T</td>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>

the initial surface crack length becomes nearly equal to the rock diameter; that is, when the crack extends to the bottom-most element in the mesh. Note that a break in symmetry could occur when using a 2-D temperature field, or including multiple initial cracks, or an initially inclined surface crack. In these cases, fragments could also be produced by the coalescence of multiple cracks or by flaking surface material. While such scenarios can be captured reasonably through our numerical model, they are not considered in this work in the interest of first extracting some key physical ideas that can be simply applied for different asteroids in the solar system.

We have performed thermal fatigue simulations for such carbonaceous chondrites of diameters between 1 cm and 50 cm, and our results on the thermal fragmentation of these rocks are presented in Fig. 2.6. The figure shows the time taken (in years) to fragment a rock of a given size (the blue data points are obtained from our simulations). The bars around each data point represent a variation of 30% in
the Paris coefficient $C$. Note that the computed survival times can be as high as $10^8$ years, while the thermal cycling period is 6 hours, demonstrating the need for an efficient computational scheme to handle this wide range of timescales. For rocks smaller than 7 cm, our model predicts rock survival times (Fig. 2.6, in blue) that are consistent with those reported in Delbo et al. (2014) [14] (Fig. 2.6, in red) using lower-accuracy computations. We predict longer fragmentation times than [14], but our more accurate results are still within their reported error margins. Our simulations also show that rocks small rocks will take longer to fracture through thermal fatigue than larger rocks (up to approximately 7 cm). The trend captured here and in Delbo et al. (2014 [14] has also been stipulated by Molaro et al. [85] for lunar rocks (albeit from purely stress-based arguments). However, Fig. 2.6 shows that beyond approximately 7 cm, this trend is reversed, in that larger rocks require increasingly longer times to fragment. This is primarily because although larger thermal gradients may cause larger stresses, however the largest temperature fluctuations are limited to the near-surface (Figs. 2.1 and 2.5), and temperature changes become negligible far from the skin depth. When cracks reach a size larger than the skin depth, the driving force at the crack tip gradually decreases, and the rate of growth of the crack is slowed down.

For comparison, Fig. 2.6 also shows the survival times of the rocks as a consequence of the micrometeorite impact mechanism, as calculated using the results from [86], with the modifications applicable to asteroid surfaces made in [14]. In Delbo et
al. [14], the mechanical disruption estimates were calculated for the 99% probability of survival. However, Horz et al. [86] notes that the maximum survival times could be up to 25% shorter than this upper bound, depending on the exact particle abrasion rates and the continuously decreasing effective cross-section caused by the abrasion.

To provide a more conservative comparison between thermal fatigue and mechanical disruption, we consider here the 50% probability of survival of the rocks by mechanical impacts (dashed line in Fig. 2.6). Comparison of the thermal fatigue and impact breakdown mechanisms thus shows that the impact process may become at least as efficient a breakdown process as thermal fatigue for rocks as large as 50 cm, and is likely to be more efficient than thermal fatigue for larger rocks. Note again that this computation is for carbonaceous chondrites at 1 AU and with a diurnal cycle of 6 hours.

The driving force for such sub-critical fatigue crack growth is the excursion in stress intensity factor $\Delta K$, as shown in the Paris law (Eq. (2.12)). For all crack sizes shown here, the maximum stress intensity factor is always smaller than the fracture toughness $K_{IC}$, and the crack grows only in fatigue and no dynamic fracture occurs.

In order to obtain a better understanding of the non-monotonic effect of thermal fatigue with respect to rock size, Fig. 2.7 shows the computed excursion in stress intensity factor (normalized by the material’s fracture toughness) as a function of crack length (normalized by the rock diameter) for rocks of a range of sizes. Note that when the normalized crack length reaches 1, the rock is completely broken.
Figure 2.6: Survival time of cm-sized (diameter) carbonaceous chondrites at 1 au and a diurnal cycle of 6 hours. The data points in red are from Delbo et al., while the blue data points are obtained from the model presented in this work. Error bars represent uncertainties (30% variation) in the Paris’ parameters. The dashed line shows the times at which 50% of these same rocks are broken by micrometeoroid impacts, using the Horz et al. model with the appropriate modifications for asteroids. The survival times of rocks smaller than the diurnal skin depth (approx. 7 cm) follows the same trend as reported in Delbo et al.: larger rocks are fragmented faster than smaller rocks. However, as rock diameters become larger than the skin depth, a reversal in the trend is observed. That is, the crack growth speed of thermal fatigue driven surface crack is greatly reduced in these larger rocks.
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Looking first at the rocks that are less than 10 cm in diameter, we see that there is an increasing trend in $\Delta K$ with increasing crack length. This means that for these rocks, the driving force for crack growth increases as the crack grows (so that for the same thermal cycles, the rock is increasingly likely to break as the initial crack increases in length as a result of prior thermal cycles). Thus the crack growth accelerates as the crack grows. However, this is only true for rocks that are smaller than the diurnal skin depth. For the larger rocks, with diameters $> 10$ cm in this case, we see from Fig. 2.7 that the crack tip driving force $\Delta K$ decreases as the crack grows, leading to a decrease in the crack growth rate, and thus these larger rocks take an increasingly long time to break. This is true even though these larger rocks do sustain large thermal gradients. The decrease in the driving force is greater for larger rocks in this size domain, and thus larger rocks take a longer time to fragment through thermal fatigue. Essentially our detailed simulations show that for rock sizes significantly larger than the diurnal skin depth, thermal fatigue can efficiently grow cracks up to a certain size (roughly $60\%$ of the diameter), but after that crack growth is greatly slowed down.

Note that this observation is largely independent of the chosen crack growth law. The crack tip driving force that we compute here in Fig. 2.7 depends on the current crack length rather than the crack growth history, and so this change in crack tip driving force will be observed regardless of the specific growth law. Here we assume a simple Paris-type relationship for fatigue growth, which may not properly describe the crack tip speed in thermal fatigue. Further experimental work is required to determine
the applicability and the parameters of the Paris’ law for thermal fatigue crack growth in different meteoritic materials. For a different growth law, the computed crack tip driving force will essentially be the same, but the computed rate of growth of the crack would be different (i.e., how long it takes the crack to cross the rock will change, but the trends with respect to the size effects will be the same).

The results of Fig. 2.7 indicate that there is a characteristic lengthscale that separates two distinct regimes in thermal fatigue: one that is dominated by larger temperature fluctuations happening near the crack tip, and another where the temperature changes become far-field relative to the crack tip location. This characteristic lengthscale is, to first order, determined by the diurnal skin depth (Fig. 2.1). In addition, the two trends described here and shown in Fig. 2.6 both appear to follow a power law-type dependence. That is, they may be fit to two distinct slopes on a log-log curve (in the same manner as the mechanical impact scaling). In the next section, we will use the concepts of this lengthscale and size-dependence of the excursion in stress intensity factor to derive a simplified relationship for the survival rate of rocks as a function of rock diameter.

### 2.3 Scaling Analysis

Our fracture mechanics model computes the stress intensity factor excursion during each asteroid rotation until the fragmentation of the rock. The time to fracture of
Figure 2.7: Normalized excursion in stress intensity factor as a function of crack length normalized by the rock diameter. In small (less than 10 cm) rocks, $\Delta K/K_{cr}$ generally increases with increasing crack size, especially between $a/d$ of 0.1 and 0.7. Rocks much larger than the skin depth see a different trend, wherein the excursion in stress intensity factor decreases as the crack size increases. The implications of these trends on the survival rates of asteroid rocks are discussed in Section 2.3.
the rock is determined essentially by the number of thermal cycles that occur before the crack length becomes equal to the rock diameter. However, the amount of crack growth in a given cycle is not constant, so that the crack could (for example) spend a very long time (a large number of cycles) growing very slowly, and then accelerate rapidly and cover the remaining length in a very few cycles. Thus, in order to obtain a reasonable analytical estimate of the time it takes to crack a given rock, we primarily need to know the conditions under which the crack spends the largest fraction of its time, when it is growing very slowly.

Now, the fatigue crack propagation law adopted in this work is the classical Paris law, relating the excursion in stress intensity factor at the crack tip to an incremental displacement of the crack front. In principle, it is therefore sufficient to integrate the Paris law to obtain an estimate for the predicted time to fracture. As Fig. 2.7 shows, the excursion in the stress intensity factor is itself a fairly nonlinear function of crack length, especially in the largest rocks. However, in terms of estimating the time to fracture, all we need is to know how $\Delta K$ depends on crack length during the times when the crack is growing very slowly, since this dominates the total time to fracture.

Our simulations show that for “small rocks,” i.e., rocks smaller than the skin depth, $\Delta K$ increases approximately linearly with crack length for at least 80% of the total time before fracture (this occurs until the crack length reaches about 70% of the rock’s diameter). The excursion in stress intensity for rocks smaller than the skin depth with normalized crack lengths $a/d$ between 0.1 and 0.7 can be fit to a linear
function \( \Delta K(a) \approx \bar{A}a + \bar{B} \) with a Pearson correlation coefficient \( r = 0.988 \). This linear approximation greatly simplifies the integration of the Paris law, Eq. (2.12).

In Section 2.4, we will discuss some of the consequences of this approximation.

Now, using the assumption that \( \Delta K(a) \approx \bar{A}a + \bar{B} \), with \( \bar{A} \) and \( \bar{B} \) the linearization parameters, we can integrate the Paris fatigue crack growth law (Eq. (2.12)) from the initial crack length \( a_0 \) at time \( t = 0 \) to a final crack length \( a_f = d \) at the approximated failure time \( t_f \). Now, recognizing that \( dN = 1/\tau dt \) where \( \tau \) is the period of rotation, we get:

\[
\frac{t_f}{\tau} \sim \frac{1}{CA(1-n)} \cdot (\bar{A}d + \bar{B})^{1-n}
\]  

(2.15)

Note that here we only seek to understand the power-law dependence of \( t_f \) on \( d \). We do not calculate the exact value of the integral since the linearization parameters \( \bar{A} \) and \( \bar{B} \) are different for different rock diameters. After some analytical simplifications to Eq. (2.15), we can express the time to fracture as a function of rock diameter as:

\[
d \sim \left( \frac{t_f}{\tau} \right)^{\frac{1}{1-n}} \left[ \bar{A}^nC(n-1) \right]^{\frac{1}{1-n}}
\]  

(2.16a)

\[
\frac{t_f}{\tau} \sim (d)^{1-n} \frac{1}{\bar{A}^nC(n-1)}
\]  

(2.16b)

Eq. (2.16a) and Eq. (2.16b) give us relatively simple approximate relationships between the time to fracture and the size of the rock (for rocks smaller than the diurnal
skin depth), and shows that this relationship is dominated by the power $\frac{1}{1-n}$ in the former case and $1 - n$ in the latter case. Notice that $t_f/\tau$ is dimensionless, and $A^n C$ has base units of $L^{1-n}$, making the right-hand side of Eq. (2.16b) also dimensionless.

For the case of rock diameters larger than the diurnal skin depth, the crack grows quickly to a normalized length of $a/d = 0.2$ in less than 10% of the total time to fracture. The bulk of the time to fracture occurs as the crack is growing progressively slower until $a/d \sim 0.95$. For this domain, a linear fit does not capture adequately the shape of the excursion in stress intensity factors, and so the integration cannot be simplified in the same manner as for the smaller rocks. Instead, we directly fit to the final fragmentation times in Fig. 2.6 and obtain that $d \propto t^{\frac{1}{1-n}}$. We will show that these two slopes provide a good description of the thermal fatigue lifetime of rocks on asteroids with different composition, heliocentric distances, and rotation periods.

In order to understand the contributions of the thermophysical and mechanical variables to the final fragmentation time, we make use of the Buckingham $\pi$ theorem. The different $\pi$-groups are identified by first constructing the dimensional matrix of all variables relevant to the problem (Table 2.1). Mathematically, the $\pi$-groups represent the null-space of the dimensional matrix that are obtained from the matrix’s kernel vector. In this analysis, we have the following $\pi$-groups:
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\[ \pi_1 = \frac{d^2 \cdot \rho \cdot C_p}{\kappa \cdot \tau} \]  

(2.17)

\[ \pi_2 = \frac{d^4 \cdot \rho \cdot \alpha}{\kappa \cdot \tau^3} \]  

(2.18)

\[ \pi_3 = \frac{\kappa \cdot \tau^3 \cdot \Delta T}{d^4 \cdot \rho} \]  

(2.19)

\[ \pi_4 = \frac{\tau^2 \cdot E}{d^2 \cdot \rho} \]  

(2.20)

\[ \pi_5 = \frac{t_f}{\tau} \]  

(2.21)

\[ \pi_6 = \frac{t_f}{\tau} \]  

(2.22)

One must recognize, of course, that this set is not unique as any combination of the dimensionless groups also yields a dimensionless group. However, this particular set has some useful attributes. \( \pi_1 \) is effectively a measure of the diameter normalized by the skin depth, \( \delta \). Eq. (2.16b) can be re-expressed in dimensionless form after normalizing the diameters by the skin depth as:

\[
\frac{d}{\delta} = \Lambda_m \left( \frac{t}{\tau} \right)^m;
\]

\[
\begin{cases} 
m = \frac{1}{1-n}, & \frac{d}{\delta} \leq 1 \\

m = \frac{1}{n-1}, & \frac{d}{\delta} > 1
\end{cases}
\]

(2.23)

where \( \Lambda_m \) is a scaling parameter that is equal to \( N_{\delta f}^{-m} \), where \( N_{\delta f} \) is the number of cycles needed to fragment a rock with diameter equal to the skin depth. The advantage of this scaling relation is that the survival time of rocks by thermal fatigue
can be estimated by performing a single XFEM simulation for a rock of size equal to the skin depth, and extrapolating to rocks with different sizes using Eq. (2.23). This scaling analysis thus provides us with a simple way to understand the effects of thermal fatigue in terms of the time to fracture and rock size.

Using this simple scaling, figure Fig. 2.8 shows a normalized representation of Eq. (2.23) and the survival rates of rocks on small asteroids with the same rotational period but with different thermomechanical properties (both ordinary and carbonaceous chondrites) and heliocentric distances (both near-Earth asteroids and main-belt asteroids). In addition, we plot a normalized version of the data presented in Ravaji et al. (2018) [90]. Ravaji et al. calculated the predicted thermal fatigue lifetime of a 10 cm diameter ordinary chondrite-like rock on a near-Earth asteroid, as a function of the asteroid’s rotation period using the same model of [14]. They indicate that extrapolation of rock lifetimes for different rotational periods is not linear, but did not indicate how the scaling should be performed. In Fig. 2.8 we show that by scaling the rock diameter by the skin depth, which itself is a function of rotational period, the survival times of [90] collapse well onto the slopes identified in this work. Despite using a different thermomechanical model, these estimates fit well onto our nondimensional plot, especially for those rocks that are smaller than ∼ 4 skin depths. Note that beyond around 5 skin depths, the efficiency of thermal fatigue is greatly reduced, and the scaling arguments overestimate the breakdown rates by an order of magnitude.
Figure 2.8: Scaled survival times of carbonaceous chondritic and ordinary chondritic rocks on near-Earth asteroids and in the main belt. The $x$-axis is a measure of the survival rates normalized by the number of cycles for fragmenting a rock with diameter equal to the skin depth. It is defined from Eq. (2.23) as $\bar{t} = \Lambda^m t/\tau$. The $y$-axis is the rock diameter normalized by the skin depth. With this dimensionless representation, the two slopes intersect at (1, 1). The circle and square symbols are results from the numerical model. The solid lines represent the slopes identified in Eq. (2.16a) and the shaded region represents confidence intervals related to the Paris’ exponent $n$. 

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What remains is to obtain an understanding of the meaning of the scaling parameter $\Lambda_m$. We do this by using a parametric analysis with our simulations to extract the dependence of $\Lambda_m$ on $C$, $E$, $\alpha$, and $\Delta T$. This allows us to re-write Eq. (2.23) as :

$$
\frac{d}{\delta} \sim C^{-1.25} E^{-1} \rho^{-0.5} C_p^{-0.5} (\alpha \Delta T)^{-1.5} \left( \frac{t}{\tau} \right)^n
$$

This is a powerful relationship, because we can now estimate the survival rates of rocks for the same material and similar rotational periods with different positions in the solar system (that is, with different $\Delta T$).

As an example, by scaling the results from Figs. 2.6 and 2.8 using the scaling factors in Eq. (2.24), we present in Fig. 2.9 the predicted time to break down (defined by the colored contours) a 10 cm rock on a C-type asteroid as a function of the period of rotation and the heliocentric distance. The dark blue contours are of the order of 10,000 years, and the yellow corresponds to a million years (as shown in the key on the right of the figure). Note that in Fig. 2.9 the temperature excursion changes with heliocentric distance, and the skin depth changes with rotation period. We assume a solid, competent bedrock with negligible regolith shielding the temperature excursions. The presence of a layered surface or initial fine-grained regolith blanket would greatly change the estimated skin depth (see discussion in Section 2.4), and would lead to larger times for disruption. These results therefore are a lower-bound on how fast thermal fatigue could fragment a single rock on the surface of asteroids.

The contours in Fig. 2.9 imply that asteroids at different orbital positions and with
different rotational periods may exhibit similar degrees of thermal fatigue on their surfaces. That is, if the thermomechanical properties of rocks on 162173 Ryugu and 101955 Bennu are similar, then they should exhibit similar thermal fatigue breakdown behavior of 10 cm rocks despite having different rotation periods. 162173 Ryugu is being visited by Hayabusa2 at the time of this writing, and 101955 Bennu is the target of the OSIRIS-REx mission, which will begin detailed observations of the asteroid by the end of 2018. We hope that our results will therefore help in interpreting the observations from sample characterization for both missions.

We also note the existence of an “intermediate” range in rotational periods (4 − 15 h) where our model predicts that thermal fatigue would be most efficient. Very fast rotators have small skin depths, and larger rocks would require increasingly larger times to fragment. Slow rotators have larger skin depths and rocks much smaller than that skin depth would also take increasingly longer times to fragment (see discussion for Fig. 2.6), as well as requiring a longer time to complete a single period.

2.4 Discussion

For cm-sized rocks, we predict that thermal fatigue can be several orders of magnitude faster than micrometeorite impacts in fragmenting such small rocks on small airless bodies with a ∼ 6 h period of rotation, in-line with the results of Delbo et al. (2014) [14]. In [14], thermal fatigue appeared to occur faster in larger rocks, owing
Figure 2.9: Predicted time (in years) to break down a 10 cm diameter rock on a C-type asteroid as a function of period of rotation and heliocentric distance, using the scaling factors in Eq. (2.24). Some C-type asteroids are highlighted on the plot using their semi-major axis and rotation period as reference. These were not modeled directly, and discrepancies from Table 2.1 in their particular surface and material composition would lead to different breakdown rates than those predicted in this figure. These results indicate the existence of an “intermediate” range in rotational periods where thermal fatigue would be more efficient. Very fast rotators have small skin depths, and larger rocks would require increasingly larger times to fragment. Slow rotators have larger skin depths and take longer time to complete a single period, and rocks much smaller than that skin depth would also take increasingly longer times to fragment (see discussion for Fig. 2.6).
Figure 2.10: Predicted time (in years) to break down a 10 cm rock on an S-type asteroid as a function of period of rotation and heliocentric distance, using the scaling factors in Eq. (2.24). Some S-type asteroids are highlighted on the plot using their semi-major axis and rotation period as reference. These were not modeled directly, and discrepancies from Table 2.1 in their particular surface and material composition would lead to different breakdown rates than those predicted in this figure.
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to the larger temperature gradients they experience. Their results for $\leq 10$ cm rocks indicated a speed-up of thermal fatigue as rock sizes increase, implying that much larger rocks should be relatively scarce on the surface of asteroids. Similarly, Molaro et al. (2017) [85] reported an increase in peak stresses for large boulders ($\geq 1$ m) on the lunar surface as opposed to $\leq 30$ cm rocks. They concluded, using purely stress-based arguments, that this could suggest a lack of very large boulders (diameters of several skin depths) on airless bodies. However, we have shown here that thermal fatigue is characterized by a signature lengthscale related to the thermal skin depth of the asteroid. While peak stresses due to thermal gradients can increase in larger rocks, they are mostly limited to the near-surface and can be efficiently relaxed with the introduction of a crack. Peak stresses derived from purely elastic solutions are therefore poor indicators of the fatigue behavior. As cracks grow to lengths much larger than the skin depth, the driving force is greatly reduced, leading to a slowdown in thermal fatigue crack advance. Hence, our results imply the existence of a size domain over which thermal fatigue is mostly active.

These newfound size domain for thermal fatigue could help in determining the abundance and distribution of cm-sized rocks, which will be the main excavation targets in asteroid mining and sampling missions. For instance, it has been shown that asteroid (433) Eros has a complex regolith [30,91] whose size-frequency distributions [92] suggested that the mechanisms for depleting 20 cm diameter rocks seemed to be very different than those for 100 m blocks. Their observation is in-line with the
lengthscales determined in this study, which may imply that thermal fatigue could
be a contributing mechanism for the disaggregation of these relatively small surface
rocks. In addition, the recently launched OSIRIS-REx mission that is targeting the
asteroid (101955) Bennu \cite{93} will attempt to sample and return a minimum of 60 g
from the asteroid’s bulk regolith. Bennu is a carbonaceous asteroid with a 4.3 h
rotation period and an estimated thermal skin depth of $\sim 5$ cm. In this work, we
predict that for such a case, most cm-sized rocks should have experienced some degree
of thermal fatigue, and we expect that the sampled rocks returned by OSIRIS-REx
should show evidence of an active thermal fatigue mechanism.

Furthermore, the effects of thermal fatigue could possibly be identifiable in the
thermal inertia measurements. It has been shown that the thermal inertia of surfaces
covered with small particles (sand-like) is smaller than those covered by rocky frag-
ments (of $\sim 10$ cm diameter) \cite{94}. In that sense, a small fast-rotating asteroid with
a large surface temperature variation would generate an increased number of small
fragments, which could lead to a decrease in the measured thermal inertia. Therefore,
in the cases where thermal fatigue is driving the regolith evolution, thermal inertia
(and skin depth) could be direct indicators of the evolution history of the rock sizes
that blanket an asteroid’s surface.

For sufficiently large rocks, our model suggests that thermal fatigue would be more
efficient in flaking off material from the surfaces as opposed to a complete fracture,
which results in an apron of fine-grained regolith surrounding larger boulders. This
behavior has been observed on (433) Eros [95], where the aprons provided evidence that boulders are eroding in place, giving a probable explanation for the nearly 300 pond-like features on Eros [17]. Such cases may be investigated through our model by solving for the 2D temperature field and tracking the growth of multiple crack networks in the thermomechanical XFEM model. The results derived here rely on a 1D thermal solution of an idealized spherical asteroid. The temperature profile does not treat surface roughness, shadowing effects, or changes in thermal conduction introduced by cracks or rock boundaries. A more sophisticated thermal model may lead to a different temperature profile than that in Fig. 2.1, which would give a different crack tip driving force (Fig. 2.7). As an additional caveat, an estimation of thermal fatigue in particular rocks with local shadowing may not be a straightforward extrapolation of our results and would require using the numerical model with the appropriate temperature inputs.

While our new thermomechanical model constitutes a significant advance from previous semi-analytical models, it is still important to realize the simplifications and inherent assumptions that we have made. For instance, we are now using a two-dimensional mechanical model with thermal inputs from a one-dimensional heat diffusion model. We do not consider the cases where a crack deviates from a straight path, which may cause scabbing of the larger rocks by gradually chipping off material from the surface. A break of the symmetry could occur when using a 2D temperature field, or with the interaction of multiple initial cracks. In these cases, fragments
could be produced by crack coalescence or by flaking surface material. Given that an idealized case of a single thermal fatigue crack growth model has demonstrated the importance of this mechanism, it is worthwhile to make future efforts to include the more complex behavior of a network of cracks, as well as the coupling between thermal fatigue and mechanical impact. It is unlikely that rocks on airless bodies would have only a single active surface crack that drives the thermal fragmentation.

Natural geological materials have a large distribution of flaws with different sizes and orientations. These flaws can evolve by thermal fatigue, but are also modified through non-catastrophic micrometeoritic impacts. Impacts by micrometeorites nucleate and propagate cracks, which can then be grown through fatigue. In that sense, these two mechanisms are collaborating processes in asteroidal regolith generation: micro-meteoritic impacts nucleate cracks in rocks that will propagate due to thermal fatigue, which in turn weakens the body making a subsequent impact more effective.

The numerical model developed in this work has the capability to calculate the crack-tip driving forces resulting from the interaction of a large network of cracks. However, it is unclear if the Paris law would still be an appropriate representation of the crack growth rates. The Paris law parameters are empirical fits to fatigue experiments on single crack growth. In addition, uncertainty quantification through surrogate modeling of fatigue growth [96] showed a major contribution of the Paris’ $C$ parameter to the errors in predicted fatigue cycles. These parameter uncertainties contribute to large error bars in the fatigue life prediction. [97] demonstrated that
different forms of the Paris law could yield essentially identical predicted growth rates for laboratory-scale experiments. In geophysical applications, however, we need to extrapolate beyond the bounds of laboratory scales, and different forms of the Paris law diverge substantially, with the form chosen in this work yielding the most conservative estimates. In our simulations, the crack always grew in fatigue mode. There may be a limit for how small the excursion in stress intensity factors should be for any growth to occur (typically known as the stress intensity threshold). So far, there has not been any clear experimental evidence of the existence of a subcritical crack growth limit or threshold for crack growth in geological material. Indeed, fatigue experiments performed by [98] on granite showed crack speeds as low as $10^{-11}$–$10^{-12}$ m/s without encountering a crack arrest limit. The reliable prediction of thermal fatigue lifetime of regolith is therefore limited by the absence of a significant database fatigue experiments on planetary materials and in environments comparable to airless bodies (for example, thermal cycling experiments conducted in a vacuum chamber). Regardless of the exact growth law being used (assuming cracks are growing sub-critically in fatigue), we do not expect that the size-dependence we identify in this work to be affected. The crack tip driving forces (shown in Fig. 2.7) are insensitive to the precise expression of the crack growth law. The rate of breakdown as a function of rock diameter, however, would be greatly affected.

We note that several stress relaxation mechanisms such as creep could occur during these long loading cycles that may modify the stress response and crack growth
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behavior. These relaxation effects have been reported in sedimentary and hard rocks [99,101], and can relax peak stresses by 10–15% in roughly 13 hours. It is not inconceivable, then, that microstructure evolution through creep would relax the induced diurnal thermal stresses over the course of several thousands of years, leading to a slower thermal fatigue cracking.

Mismatches in expansion coefficients between the individual grains could play an important role in driving the thermal fatigue fragmentation of regolith with sizes much smaller than the skin depth. In this work, we consider an averaged contribution to the stress tensor from inclusions by means of a sub-scale representative volume element approach. With this homogenization approach, it is possible that the stresses arising from thermal mismatch alone could be underrepresented, especially in the smaller rocks. These cases will be explored in Chapters 3 and 4.

The simplified analytical integration of the Paris law presented in Section 2.3 showed a good fit to the numerical model results despite using a linear approximation of the excursion stress intensity factors. The derived analytical expressions for the breakdown time can therefore be used for a first-order estimate of thermal fatigue fragmentation for rocks that are of the same order as the skin depth. Rocks with diameters that were much larger than the skin depth showed weak response to thermal cycling, suggesting that the thermal fatigue response would be very weak beyond such sizes. We note also that thermal fatigue scaling is highly nonlinear with changes in rotation period. This is in a small part due to the longer times associated with each
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fatigue cycle, but is more dependent on the resulting surface temperature variations and its spatiotemporal evolution. Fast rotators experience more thermal cycles for the same amount of time as slower rotators. However, a fast rotator experiences smaller temperature variations per cycle, and therefore smaller driving forces on the crack. This suggests that thermal fatigue would be most efficient on “intermediate” rotators (periods of rotation $\sim 6$ hours), and the scaling arguments developed in this work would be best applied to such asteroids. In addition, a layered subsurface in asteroids would greatly affect the temperature profile (and gradients) from that shown in Fig. 2.1. [102] showed that the magnitude of temperature oscillation can be reduced in the subsurface by a factor of 5 in a layered subsurface. Similarly, [85] showed that the regolith provides a strong insulation to buried rocks on the lunar surface and reduces their temperature excursion by a factor of almost 18. For these cases, which in our study would be analogous to a reduction in both the skin depth and the temperature excursion, we would not expect to see significant thermal fatigue crack growth in those buried rocks.

Modeling thermal fatigue is a computationally intensive process that requires careful tracking of the crack front over a large number of cycles. The computational framework needs to be able to bridge efficiently between the relatively fast rotational periods (several hours), to the final fragmentation time of a rock (thousands to millions of years). The numerical model presented here is designed to efficiently model the fatigue growth of cracks in regolith over a several thousands of years, while us-
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ing timesteps as small as 15 minutes. More sophisticated fracture codes, such as the mapped finite element method [103], have been developed recently. The mapped finite element method provides a high-order approximation of problems with cracks and yields optimal convergence rates without additional degrees of freedom or special shape functions. Such models may provide improved accuracy at reduced computational costs and their implementation in the code developed in this work could be potential paths for improving the numerical algorithm. This advance in numerical modeling capabilities further emphasizes the need for careful baseline thermal fatigue experiments that constrain the material parameters and crack growth laws that are used in such predictive models.

2.5 Conclusions

We find that the rate of rock breakdown by thermal fatigue has two domains that are described by distinct slopes (Fig. 2.6). Using an approximate integration of the Paris law based on the results of the detailed fracture simulations, we derive measures for these two slopes. Normalized plots of computed fragmentation times for different rock diameters seem to align well under these analytically derived curves. Our analysis demonstrates that rocks with diameters much larger than the diurnal skin depth will exhibit poor thermal fatigue cracking. In such rocks, cracks can grow quickly by thermal fatigue up to a certain length, after which their progress is slowed.
down greatly (or even arrested completely).

The identification of this critical lengthscale provides bounds on the areas where thermal fatigue is likely to be the dominant mechanism in rock disaggregation. We expect that large boulders would not be fragmented completely by thermal fatigue, but would have long straight cracks that grow in the direction of maximum circumferential thermal stress, just as in the Martian boulders shown in [16]. We expect that this critical lengthscale may be an important indicator for the expected regolith size distribution. The thermal fatigue contribution may thus be reflected as deviations in the regolith particle size-frequency distribution as a result of preferentially depleting rocks that are of dimensions comparable to the skin depth.
Chapter 3

Thermomechanical characterization of meteorite samples: experiments and numerical simulations

3.1 Introduction and Background

Asteroids, often referred to as minor planets, are considered to be relatively pristine and unaltered objects that preserve clues from the earliest epochs of our solar system [1–3]. However, the exposure of asteroids to collisional evolution and the ex-

†The present chapter is largely based on a collaborative work published by Hazeli et al. [104]. The contribution of C.M. to this publication was significant as noted in [104].
treme space environment results in considerable surface modification. Surface properties influence the observable traits (optical and thermal properties, physical structure, chemical and mineralogical properties [4, 5]), and as such, interpretation of the remotely collected data relies on assumptions about the key mechanisms involved in the formation of what can currently be observed. However, many of the major surface modification agents and their associated effective rates are yet to be understood. Recent missions, such as the JAXA Hayabusa mission that visited the near-Earth asteroid (25143) Itokawa and ESA’s Rosetta mission to a cometary nucleus, namely 67P/CG, provide actual ground-collected information on the physical and chemical properties of the regolith covering the targeted bodies’ surfaces, allowing us to better understand and constrain the evolution history of these rocks.

In September 2005, JAXA’s Hayabusa mission recorded high resolution close-up images (6 mm/pixel) from altitudes of 80 to 63 m above the surface of asteroid (25143) Itokawa. It revealed a surface of blocky nature, lacking impact craters, and with the fine regolith mostly present in two areas: the Muses Sea and Sagamihara. These smooth terrains coincide with low-gravitational potentials and are generally homogeneous, featureless, and relatively flat, filled with particle sizes ranging from millimeters to centimeters [6]. In June 2011, the Hayabusa mission returned to Earth with approximately 1534 rocky particles from Itokawa [7]. Among the returned particles, over 70% (1087) were monomineralic, including 580 olivine particles, 126 low calcium (Ca) pyroxenes, 56 high-Ca pyroxenes, 186 feldspars, 113 troilites, 13 chromites,
CHAPTER 3. THERMOMECHANICAL CHARACTERIZATION OF METEORITE SAMPLES: EXPERIMENTS AND NUMERICAL SIMULATIONS

10 Ca phosphates, and 3 Fe-Ni metal [8]. X-ray microtomography was used to understand particle texture, mineralogy, and surface features in comparison with those evident in lunar soil. It was found that none of the particles contain melt products caused by meteorite impact, such as those seen in lunar soils [9]. The asteroid Itokawa is spectroscopically [105] and mineralogically [8] linked to LL chondrite meteorites. Furthermore, noble gas content analysis of the grains showed that they were saturated with solar wind atoms, meaning that Itokawa’s regolith was rejuvenated and had been exposed to solar wind for less than 8 million years [10]. Consequently, a surface rejuvenation process that can maintain these small fragments, ought to be active on Itokawa’s surface. In addition, laboratory-impact-generated fragments tend to be much more angular or faceted with cleavage surfaces, while the Hayabusa fragments are relatively smooth at similar lengthscales [106]. These facts appear to be inconsistent with the commonly made assumption that the mm to cm-sized rocky particles are predominantly formed by impact processes on the surface of airless bodies similar to Itokawa [32, 107]. The Hayabusa team attributed the roundness of the particles to abrasion during regolith migration [9]. There are a number of other features (angular or faceted with cleavage surfaces) observed on asteroidal regolith that are not fully consistent with the impact generation mechanism (e.g. [14, 17, 95, 108]). Therefore, the presented work explores the role of thermal fatigue in chondrites as an effective rock disaggregation mechanism, in particular for objects larger than a few mm.

The ability of diurnal temperature cycling to cause a breakdown of surface rocks
and boulders on Earth and other planetary bodies has been heavily debated for more than a century. Recently, studies based on field observations, laboratory experiments and modeling have confirmed the effectiveness of such thermal weathering on Earth, Mars, and airless bodies.\cite{14,16,17} The surface temperature of asteroids follows a diurnal cycle with considerable temperature changes as the Sun rises or sets. Thermal stresses then arise from through-the-depth thermal gradients as well as from mismatches in the coefficients of thermal expansion of the minerals. These stresses can produce driving forces on cracks in the regolith material, leading to the opening and extension of microscopic cracks. As cracks continue to gradually grow, they can lead to rock breakdown when the number of temperature cycles becomes sufficiently large. This process is known as thermal fatigue, and progressive thermal fatigue leads to thermal fragmentation e.g.\cite{15}. Note that the magnitude of the stresses themselves is no indication of whether fracture will occur, since the fracture process is controlled by the asymptotic stress intensity factor at the crack tip rather than the stresses\cite{109}. In previous work, a semi-analytical framework was developed to examine the stress intensity factor at a crack tip in thermal fatigue. The method was applied to examine the efficiency of the process in the fragmentation of centimeter-sized surface rocks as a result of a thermal-fatigue driven crack growth\cite{14}. In this context, recent modeling efforts using artificially generated microstructures\cite{110} calculated grain-scale stresses induced by diurnal temperature variations in a mock microstructure consisting of hexagonal “grains”. It was shown that strongly heterogeneous stress
distributions can arise as a result of thermal loading, which may play a role in the thermally driven breakdown of rocks on some airless bodies. The work presented here also addresses the diurnal thermomechanical processes that may lead to asteroidal rock cracking, but does so by examining a real chondrite microstructure with locally measured mechanical properties, using both experiments and simulations, and thus providing ground truth to help quantify the stress distributions that develop.

A related case of interest emerged when the European Space Agency’s Rosetta spacecraft entered a close orbit about the Jupiter family comet 67P/Churyumov-Gerasimenko on 6 August 2014. The Rosetta spacecraft was equipped with an optical spectroscopic and infrared remote imaging system (OSIRIS) and acquired images of the surface at scales of <0.8 meter per pixel \([11, 12]\). The OSIRIS observations suggested that the surface features could be grouped into five categories: dust-covered terrains, brittle materials with pits and circular structures, large scale depressions, smooth terrains, and exposed consolidated surfaces. While it is stated that “the surface of comet 67P is almost devoid of recognizable impact craters”, several surprisingly large cracks have been observed, including the 500 m long crack in the Anuket region and the 200 m long fracture in the Aker region \([11]\). The lack of concrete evidence of shear displacement along the cracks and impact sites that might produce these cracks raises the question surrounding the mechanisms that govern the surface evolution \([11, 13]\), especially given the large temperature variation and temporal gradients likely to be experienced by cometary \([11]\) and small asteroid \([14]\) surfaces over diurnal
and orbital time scales.

In this chapter the viability of the thermal fatigue mechanism for the breakdown of small (cm-sized) rocks as a result of the thermal expansion mismatch of the individual grains is examined through experiments that capture the evolution of the full-field strains on the surface of a chondrite as a function of the cycling temperature and microstructure. The experimental results presented here quantify the strain fields developed within a real chondritic microstructure during thermal cycling, and also reveal the preferential crack path within that microstructure (we focus here on crack path rather than crack growth rate). These observations are then explained in the context of a real-microstructure-based finite element model that is compared to the experimental results.

3.2 Asteroid surface evolution and regolith formation

The collisional generation of superficial regoliths is a common process that modifies pristine asteroidal material. Further surface modification is also attributed to space weathering processes from solar wind sputtering to micrometeorite bombardment [108-111]. Impacts evolve the surface by creating craters, gardening, and ejecting the excavated material. Impact not only influences the size distribution of surface materials, but also results in spectral alteration by impact melt formation products.
(agglutinates) [112]. Other space weathering processes include irradiation, implantation, and sputtering from solar wind particles that can change physical properties such as optical spectra [4] or microstructural features such as rock porosity [113]. Finally, thermal fragmentation due to thermal fatigue has been recently suggested as a contributing, and sometimes even dominant, mechanism for the origin of regolith on small asteroids [11, 14, 17, 95, 110]. The relative contributions of these processes are important in determining the likely nature of the regolith on asteroids. Simulations of these processes suggest at least two very different regolith formation timescales: a thermal fatigue timescale of $10^4$-$10^6$ years under specific temperature excursion rates [14], and a micro-meteorite impact rate of $10^8$-$10^9$ years for a given meteorite flux and impact probability. These timescales are particularly important for determining the age and projected life expectancy of the surfaces of asteroids. A thorough understanding of regolith in terms of size-frequency distribution, shape distribution, and mechanical properties may explain the observed asteroidal surface features.

Rocks on the surface of asteroids experience thermal cycles, typically with a period of a few hours. The consequences of such a temperature change are two-fold: First, any sufficiently large rock will sustain a temperature gradient, resulting in an internal stress field that is spatiotemporally varying. Second, any rock heterogeneity will develop local stress concentrations as a result of the mismatch in the coefficients of thermal expansion (CTE) for the rock’s individual phases e.g. [13, 110]. We note
that these effects on real and simulated microstructures have been studied extensively for many years in the engineering literature, (e.g. [114] and references therein), using tools such as OOF2 that are only recently being adopted within the planetary science community. We can learn much from these earlier approaches (particularly with respect to the size of the representative volume elements that must be modeled to obtain robust computational results). Estimating the magnitude of the resultant stress heterogeneities requires knowledge of the thermomechanical properties of each individual phase. It has been previously shown (see Appendix A) that these heterogeneities play an important role in determining the resultant fragment sizes during high-velocity impacts. The current work aims at exploring the role that material heterogeneities may have in the development of internal material stress concentrations, and their implications on the thermal fatigue growth of cracks.

3.3 Experimental Procedure

Thermal fatigue characterization was performed on an L6 ordinary chondrite (GRO 85209) that was found in the Grosvenor Mountains, Antarctica, and provided by the Smithsonian Institution. The experimental procedure combined state-of-the-art techniques for chemical, mechanical, and thermomechanical characterization of materials. Fig. 3.1 demonstrates the hybrid experimental setup which was used to examine the microstructure and its global and local response to the thermal cycles.
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The setup is composed of an energy-dispersive X-ray spectrometer to quantify major constituent phases and their surface area fractions prior to the thermal fatigue experiment, a nanoindentation technique to measure the mechanical properties of the targeted phases, and a digital image correlation technique coupled with an IR camera to simultaneously record local and global strain development during thermal cycles. X-ray computed tomography (XCT) was used to visualize the internal features within the non-transparent material and to gain insight into the volumetric grain size and shape distribution of the chondrite constituents. Fig. 3.1 also illustrates the character of the results that can be found from each technique used in this investigation. A detailed description of the techniques and the corresponding results are presented in this section. Combining several characterization techniques with computational modeling was necessary to understand the mechanisms associated with developing internal stresses and crack preferential path under thermal cycling. In addition to constructing a reliable thermomechanical model to simulate the response of the sample used in the experiments, it was important to identify some key variables and model parameters, including the sample’s microstructural components, their volume fractions, and their associated mechanical and thermal properties.
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Figure 3.1: Hybrid experimental set up composed of: a) ex-situ and b) in-situ measurement of the microstructure and its global and local response to the thermal cycles. (a) We used an energy dispersive X-ray spectrometer to quantify major constituent phases and their surface area fractions prior to the thermal fatigue. The nanoindentation technique was implemented to measure mechanical properties of the targeted phases. (b) We integrated the Digital Image Correlation (DIC) technique with an IR camera to simultaneously record local and global strain development during thermal cycles. This approach allowed us to successfully correlate strain quantification to the corresponding phases at a given time and temperature.
3.3.1 Computed Tomographic measurements

Computed Tomographic (CT) measurements were used to visualize the internal features within the meteorite sample (which of course is visually opaque), giving insight into the volumetric grain size and shape distribution of the underlying microstructure. These measurements were important because it ensures that what we see and measure on the surface is a good representation of the material. X-ray Computed Tomography is a technique for noninvasively characterizing the internal three-dimensional structure of a volume of material [115]. The method, analogous to that used extensively in the medical imaging community, is an extension of classical X-ray radiography and uses a large number of radiographs obtained while the sample is rotated within a high-flux X-ray beam. Once a sufficiently large number of 2D images have been acquired, computational reconstruction of the sample volume can be performed through algorithms such as filtered back-projection and parallel beam geometry, using the variation of X-ray attenuation in the sample volume [115–117]. Analysis of 14 samples (5mm$^3$ each) with synchrotron X-ray microtomography ($\mu$CT) was completed using the GSECARS 13-BMD beamline at the Advanced Photon Source (APS) at the Argonne National Laboratory. An imaging setup similar to that described in [118] and techniques similar to those employed in [119] were both utilized. Monochromatic X-rays of 46 keV energy were used during the imaging experiments, and the system provided a spatial resolution of 6.12 $\mu$m/voxel edge. These tomographic measurements were used to identify and assess
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the distributions and configurations of phases in the L6 chondrite throughout the
volume. Selected CT image slices are shown in Fig. 3.2 starting from the top and
moving towards the bottom of the specimen to visualize the spatial distribution of
the microstructural features, including Fe-Ni and chondrules. These CT measure-
ments reveal that the through-thickness variations in the area fraction distributions
of the constituent phases are small, and thus the features observed on the surface are
reasonably representative of the underlying microstructure.

Note that the CT-characterized porosity and heterogeneity distributions impose
restrictions on the techniques that can be used for mechanical characterization of the
sample. The CT investigation showed that the sample porosity was about 3%. Hence,
if one wishes to obtain overall properties for the chondrite, one must make measure-
ments with sample volumes sufficiently large to provide representative behavior given
these distributions.

3.3.2 Nano-indentation and local mechanical prop-
eries of phases

A main objective of this work is to probe the material properties of the hetero-
genous GRO meteorite by measuring the local properties of the individual phases.
In contrast to the effective properties of the chondritic material, the local properties
Figure 3.2: X-ray CT images to study the spatial structural elements distribution in a L6 chondrite. The presented serial section imaging alongside with the mineralogical composition analysis shows that the studied material (GRO 85209) is mainly composed of Fe-Ni, Chondrule, plagioclase and Mafic (the last two are considered as matrix in this study). The compositions of these phases are presented in the next section. Images (a) to (d) demonstrate Fe-Ni distribution throughout the volume. A chondrule starts to appear in snapshot (e) and by going through thickness (images (f) to (k)) we can observe chondrule structure through the thickness of the sample. Images (i) to (m) are used to show chondrule distribution in a different region of the sample.
associated with individual phases require a measurement technique that has sufficient spatial resolution to examine the individual grain types. The nanoindentation technique was selected to examine the mechanical properties of 10\(\mu\)m-sized inclusions with accuracy. Nanoindentation is a technique that uses a geometrically well-defined probe called the indenter to apply and remove a load onto a small volume of material (indents can be tens of nanometers in scale) \[120\]. During the nanoindentation process, the probe displacement is continuously measured, and an indenter force versus indenter displacement curve is produced. Quantitative nanoscale material properties can then be determined from the force-displacement curve. The small size of the indents allows the interrogation of very small sample volumes, and thus to interrogate individual phases (if the phase particle size is greater than about 100 nm). In this work, the continuous stiffness measurement (CSM) variant of the nanoindentation technique was used to measure the elastic modulus and hardness of the major constituent phases in the meteorite (see Table 3.1). CSM is a technique that essentially superimposes small-amplitude oscillations on the primary loading curve and provides a direct measure of dynamic contact stiffness during the overall loading portion of an indentation test (i.e. at any point of the experiment, and not just at the unloading point), while simultaneously allowing for more accurate observations since this approach is somewhat insensitive to thermal drift \[121\]. Note that using the CSM technique makes it possible to measure the mechanical properties of the phases without the need for discrete unloading cycles, and with a time constant that is at least three
orders of magnitude smaller than the time constant of more conventional methods for
determining the stiffness from the slope of an unloading curve [122, 123]. Nanoindenta-
tion measurements can be recorded at exceedingly small penetration depths (tens of nanometers),
making this method virtually nondestructive to the original material.

Fig. 3.3 shows the load-displacement profiles for the three primary phases present of the GRO sample: Olivine, Plagioclase, and Pyroxene. The contact stiffness can be measured from the loading portion of the indentation test using the CSM technique, or from the unloading portion using the conventional measurement techniques.
3.3.2.1 Conventional nanoindentation stiffness measurement

The conventional stiffness measurement technique in indentation experiments uses the slope of the unloading curve to calculate the sample elastic modulus. As the indenter is unloading, the deformation is purely elastic (by definition), and so elastic contact theorems can be used. The contact stiffness can then be calculated following the equations of [124] as:

\[ S = \frac{2}{\sqrt{\pi}} \kappa \sqrt{A} \pi E_{eq} = \left( \frac{dP}{dh} \right)_{h=h_{max}} \]  (3.1)

where \( \kappa \) is a constant related to the indenter geometry, \( A \) is the projected contact area, \( P \) is the load, and \( h \) the penetration depth. \( E_{eq} \) is a combined equivalent Young’s modulus of the indenter and sample, given by:

\[ E_{eq} = \frac{1 - \nu_s^2}{E_s} + \frac{1 - \nu_i^2}{E_i} \]  (3.2)

with \( E_i \) and \( \nu_i \) the Young’s modulus and Poisson’s ratio of the indenter, and \( E_s \), \( \nu_s \) those of the sample.

The unloading curve (which is non-linear) is then fit to a power-law type equation as:

\[ P = B(h - h_{max})^m \]  (3.3)

where \( B \) and \( m \) are fitting parameters, \( P \) is the load, and \( h \) the penetration depth.
reaching a maximum of $h_{\text{max}}$. Substituting Eqs. (3.2) and (3.3) into Eq. (3.1) gives:

$$E_s = (1 - \nu_s^2) \left(2\kappa \frac{1}{S} \sqrt{\frac{\pi}{A}} - \frac{1 - \nu_i^2}{E_i} \right)^{-1}$$  \hspace{1cm} (3.4)

### 3.3.2.2 Stiffness measurement using CSM

In the CSM technique, a harmonic force is imposed onto the driving load of the indenter, and the displacement response is measured continuously as a function of depth. The contact stiffness can then be directly calculated from the displacement signal and the phase angle difference between the force ($P = P_0 \exp(i\omega t)$) and displacement signals $h = h_0 \exp(i\omega t + \phi)$, as:

$$\tan \phi = \frac{\omega C}{(S^{-1} + K_f^{-1})^{-1} + K_s - m\omega^2}$$  \hspace{1cm} (3.5)

The indenter mass ($m$), the stiffnesses of the indenter frame ($K_f$) and the indenter support springs ($K_s$), and overall damping coefficient $C$ are known from the experimental apparatus setup, while the frequency, $\omega$ and phase angle $\phi$ are obtained from the recorded signals. The set of equations can then be solved for the only unknown, $S$. The sample’s Young’s modulus is then obtained using Eq. (3.4), evaluated at any point on the loading curve. The average elastic moduli values are shown in Table 3.1.
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Table 3.1: Mechanical properties of the major grain types in the L6 chondrite (GRO 85209).

<table>
<thead>
<tr>
<th>Material</th>
<th>Bulk Modulus (GPa)</th>
<th>Shear Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Coefficient of Thermal Expansion (K⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olivine</td>
<td>127±5 [this work]</td>
<td>80±5 [this work]</td>
<td>0.24 [123]</td>
<td>6×10⁻⁶ [126]</td>
</tr>
<tr>
<td>Pyroxyne</td>
<td>123±5 [this work]</td>
<td>70±5 [this work]</td>
<td>0.26 [125]</td>
<td>8×10⁻⁶ [110]</td>
</tr>
<tr>
<td>Plagioclase</td>
<td>85±5 [this work]</td>
<td>34±5 [this work]</td>
<td>0.32 [125]</td>
<td>4×10⁻⁶ [110]</td>
</tr>
<tr>
<td>Chondrule</td>
<td>83±5 [this work]</td>
<td>50±5 [this work]</td>
<td>0.25 [127]</td>
<td>10.4×10⁻⁶ [14]</td>
</tr>
<tr>
<td>Matrix</td>
<td>125±5 [this work]</td>
<td>53±5 [this work]</td>
<td>0.31</td>
<td>4.3×10⁻⁶ [Calculated]</td>
</tr>
<tr>
<td>(Plagioclase20%-Mafic80%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fe-Ni</td>
<td>156±5 [this work]</td>
<td>72±5 [this work]</td>
<td>0.30 [128]</td>
<td>12.6×10⁻⁶ [Calculated]</td>
</tr>
</tbody>
</table>

3.3.3 X-ray mineralogy

Once the major distributions were determined using CT measurements, an energy dispersive X-ray spectrometer (EDS) was used to obtain quantitative mineralogical composition maps. EDS is an analytical technique used for compositional analysis of a sample within an electron microscope, and utilizes the X-rays that are emitted from the sample surface when it interacts with the electron beam. Selecting a specific X-ray line from the emitted X-ray spectrum allows one to effectively build maps of the distribution of specific elements in the region being analysed.[129] Fig. 3.4 presents elemental and mixed data for the constituent phases. An image analysis algorithm was used to calculate the area fraction of each phase. It is seen that the L6-chondrite is mainly composed of three phases: Iron-Nickel (Fe-Ni(95%-5%)), chondrules, and a bulk matrix that consists of Plagioclase(20%)+Olivine(40%)+Pyroxene(40%).
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Figure 3.4: Optical micrograph of the L6 chondrite (GRO 85209) supplemented with corresponding compositional mapping obtained by energy dispersive X-ray spectrometer. Qualitative maps presented here suggest that the main large heterogeneities are Fe-Ni, and chondrules. It can be seen that the matrix is composed of plagioclase, olivine, and pyroxene.
3.3.4 Thermal Strain Measurements

In order to understand the response of the ordinary chondrite microstructure to thermal cycling, the evolving strain fields developed during each cycle were measured. The simultaneous access to the strain field, images of the microstructure, and the temperature field is essential. As shown in Fig. 3.1b, the thermal cycling experiments were coupled with digital and infrared thermography cameras to capture the strain evolution and to simultaneously record the full-field temperature map. Infrared thermography was conducted using a FLIR A325sc thermal camera with a spectral range of 7.5 – 12 µm, 320 × 240 pixel resolution and a maximum acquisition rate of 60 Hz allowing real-time tracking of surface temperatures. A 5 megapixel single-camera system was used to acquire images of the specimen surfaces while it was thermally cycled. The Digital Image Correlation (DIC) technique [130] was utilized to measure the evolution of the strain fields as a consequence of both the global temperature change and the local coefficient of thermal expansion (CTE) mismatches between phases during thermal-cycling. The meteorite specimen was subjected to thermal cycling in air using a programmable hot plate at a rate of 2°C/min from room temperature, 28°C, to 200°C and then back to room temperature. The temperature changes uniformly across the sample throughout the thermal cycle. This temperature rate and range is considered to be representative of the thermal rates calculated for some NEA surfaces with relatively low albedo, a thermal inertia similar to that of Itokawa, and a rotation period of 6h and at a heliocentric distance of 0.7-0.8 au from the Sun. [14][131]. Note
that the governing thermomechanical equations in this problem are controlled by the changes in temperature rather than the absolute temperature. The basic material properties deviate from our assumptions primarily when the homologous temperatures (ratio of current temperature to melting temperature) are significant, that is for temperatures beyond 1000K for these materials. In the range of temperatures considered here that would be appropriate for these bodies, the properties do not change significantly with temperature.

The sample dimension was $30 \times 18 \, \text{mm}$, and $6 \, \text{mm}$ thick. The sample’s surface was polished down to $1 \, \mu\text{m}$ using an alumina suspension. A photograph of the specimen is shown in Fig. 3.5: this sample had a naturally developed pre-existing crack starting from near the middle of the right edge. The presence of this natural crack provided an opportunity to monitor the crack propagation path and the associated strain fields as a consequence of thermal fatigue. However, note that the experimental optical system did not have the resolution to measure the crack-tip strain fields themselves (which have strong gradients).

Measuring strain using the DIC method relies on measurements of surface deformation by comparing an original (reference) configuration with subsequent states through tracking the movement of algorithmically identified distinct features. These distinct features are typically achieved by applying an artificial speckle pattern on the sample (e.g. by spraying paint markers on the surface). In contrast to most DIC measurements, no artificial speckle pattern was required for the studied specimen due
to the natural high-contrast microstructural features in the meteorite sample. From the image correlation analysis, one is able to compute the in-plane displacement vector \( \mathbf{u}(\mathbf{x}, t) \) that describes the motion of each material point (here subsets of pixels) at a time, \( t \), and with an initial position vector \( \mathbf{x} \). With the displacement components \( u_i \) known at every pixel in the image, the components \( E_{ij} \) of the infinitesimal strain tensor can be computed directly from the symmetric part of the displacement gradient:

\[
E_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) 
\]

Once the components of the strain tensor are known, the principal strains can be computed everywhere in the plane since they are the eigenvalues of the strain tensor (note that the principal directions are the corresponding eigenvectors). Figures 3.5a and 3.5b show the measured strain components (along the X and Y axes) in the meteorite during heating and cooling respectively. The image resolution and the correlation technique result in an error range of \( \pm 0.2\% \) in the reported strain values. Fig. 3.5c shows the variation of the magnitude of the largest principal strain (using the color map) across the surface during the heating phase. The arrows in this figure indicate the corresponding principal directions at some of the locations in the microstructure.

Fig. 3.5 shows the full-field evolution of two different components of the strain tensor, at different temperatures, during a single full cycle (heating/cooling) out of
the hundred cycles performed on this sample. It is evident, when the meteorite microstructure is overlaid on the strain fields, that the measured strain fields are directly correlated with the temperature-induced local displacements of the constituents. It is also observed that the local strain evolves heterogeneously with temperature, and this heterogeneity is strongly seen at the higher temperatures. This observation is consistent with the thermal strains arising from the mismatch of the coefficients of thermal expansion (CTE) values between the various phases.

Finally, the measured strain components were used to compute the magnitude and direction of the principal strains during the thermal cycling (this is also shown in Fig. 3.5). The spatial variation of the principal strains and the observed principal directions are consequences of the heterogeneous character of the meteorite for this kind of simple uniform thermal cycling condition. A series of images showing the major principal strain and the corresponding principal direction (arrows) are shown in Fig. 3.5 for a single heating cycle, demonstrating that the major principal strain distributions evolve with temperature (from 100°C to 200°C). By overlaying the principal direction on top of the principal strain maps, it is seen that the orientation of the major principal strain (near the preexisting crack) rotates with increasing temperature to line up normal to the crack growth direction. This is what we would expect if the crack grows, since the crack tip opening displacements would generate such an apparent principal strain at the resolution of these images. The thermal and DIC images thus suggest that the crack is growing as a consequence of the uniform
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Figure 3.5: L6 meteorite microstructure and its associated full-field strain map at different temperatures for one full thermal cycle (heating/cooling). Here we present the strain evolution along the X and Y axes as a function of the temperature during a) heating and b) cooling. c) the major principal strain distribution is also presented as a function of temperature. The local strain develops heterogeneous features as the sample is heated and cooled. The temperature always remains homogeneous.
thermal cycling of the meteorite sample. The specific conditions perceived by a similar sample on an asteroid surface and undergoing thermal cycling while surrounded by regolith will be somewhat different from that perceived by the sample in our laboratory environment. However, the impact of these differences is likely to be larger on the crack propagation rate than on the crack propagation path, since the latter is driven by the microstructure itself rather than the far-field boundary conditions.

### 3.4 Thermomechanical model

#### 3.4.1 Linear elastic model

The extensive characterization that was performed on the different phases in our sample allowed us to perform an experimentally informed numerical simulation of the response of the material to temperature variations. Note that the DIC recordings allow to calculate the strains, but these do not easily translate to stresses since thermal expansion could occur without resulting in any internal stresses. A numerical model that is informed by the experimental measurements is therefore required to compute internal stresses. The stresses themselves are needed to provide some indication of the reasons for the specific crack propagation path.

The object-oriented finite element code OOF2 and its predecessor OOF have been used extensively for many years in the engineering literature to study mechanical and thermal effects on real and simulated microstructures, e.g. [114] and references
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The OOF2 modeling framework has also been used to study the role of diurnal, thermally induced temperature and stress fields in model lunar mineral-grain microstructures in regolith production on the Moon [110]. Leveraging these earlier works, the high-resolution images of the real meteoritic microstructure were used to construct a microstructurally adaptive finite element mesh using OOF2 [132]. Since the CT scans (see Fig. 3.2) showed a nearly uniform volumetric variation indicating that the surface features are representative of the volumetric features, a 2D plane stress approximation is justified and provides sufficient information for the purpose of this study.

A “skeleton” consisting of a regular array of large elements is first imposed on the microscopic image. The coarse skeleton mesh is then refined through a series of adaptive methods by either adjusting the mesh nodal positions or by inserting new elements, until the mesh conforms (up to a specified accuracy) with the assigned image of the underlying microstructure. This adaptive process is efficiently performed by defining an effective potential, which is minimized as the mesh complies with the background image, while at the same time maintaining a set of well-shaped elements that do not introduce erroneous numerical artifacts due to irregular sharp or incompatible elements [133]. Hence, this potential is a combination of a homogeneity potential related to how many different materials, or distinct pixels, exist within the same element, and a shape potential related to the quality of the shapes of elements. The effective potential is therefore expressed as:

\[ \text{Effective Potential} = \text{Homogeneity Potential} + \text{Shape Potential} \]
where $U_{\text{hom}}$ and $U_{\text{shape}}$ represent the homogeneity and the shape potential of the mesh elements respectively, and $U_{\text{eff}}$ is the resulting effective potential. The parameter $\beta_{\text{hom}}$ is a weight whose value ranges between 0 and 1, depending on whether an improved shape quality or an improved homogeneity of the mesh elements is preferred.

Once the finite element mesh was generated and aligned to conform with the actual microstructure of the sample, the material properties (shown in Table 3.1) that were measured in our experiments were assigned to the microscopic features in the optically recorded microstructure. In our calculations we assumed that the physical and mechanical properties of the constituent phases remain constant at any given temperature. In this model, three distinct phases are identified: chondrules, an “Iron-Nickel” phase, and a “matrix” phase. The mechanical and thermal properties of the chondrule and Iron-Nickel phases are given in Table 3.1. We use the Mori-Tanaka micromechanical approach \[134\] to compute the effective bulk and shear moduli of the composite-like “matrix” phase by treating it as a composite of plagioclase, olivine, and pyroxene (the elastic properties of which are known). The effective thermal expansion coefficient (CTE) of the matrix is calculated following Levin’s \[135\] relationship between the effective expansion coefficient and the elastic moduli of the matrix con-
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...stituent phases [136]. Then, traction-free boundaries were applied conditions were applied and the longitudinal and transverse displacements at the bottom-left corner node were fixed. No external forces were imposed on the system, and an initial nodal temperature corresponding to the room temperature was assigned. Fig. 3.6 shows the resulting generated mesh with the applied boundary conditions. Note that the pin support in Figure 1, bottom-left, constrains both horizontal and vertical displace-ments while the roller support (bottom-right) constrains the vertical displacement only. The matrix and the different phases are assumed to be perfectly bonded, and a no-slip condition is applied.

It is worth noting that in the current configuration, the sample is allowed to expand so that no global stresses would develop from over-constraining the body’s expansion. The thermal stresses that develop within the sample are solely due to the mismatch in mechanical and thermophysical properties among the constitutive phases, which lead to natural internal constraints on the deformation of each phase.

In these calculations, the temperature of the entire microstructure is gradually ramped up to 200°C, and the nodal displacement solutions are obtained for different temperature increments. Strains are then calculated from the numerical displacement solutions. To evaluate the microscopic thermoelastic stresses that arise over the course of one thermal cycle within each phase of the microstructure, the thermomechanical model is used to solve the equilibrium equation for the sample with traction free boundary conditions. The effect of temperature in this thermoelastic problem is
Figure 3.6: Finite-element mesh generated from microstructure, together with the boundary conditions applied for the thermomechanical analysis. Note that the pin support (bottom-left) constrains both horizontal and vertical displacements while the roller support (bottom-right) constrains the vertical displacement only.
incorporated into the stress tensor as:

\[ \sigma_{ij} = C_{ijkl}(\epsilon_{kl} - \alpha \delta_{kl} \Delta T) \]  

(3.8)

where \( C \) is the stiffness tensor, \( \epsilon \) is the strain tensor, \( \alpha \) is the coefficient of thermal expansion in an element, \( \delta_{kl} \) are the components of the identity tensor and \( \Delta T \) is the increment in temperature from the reference room temperature. As evident from Eq. (3.8), an unconfined expansion of an isotropic material would produce zero stresses in response to homogeneous temperature increase. The solution converged with a relative residual error of \( 1 \times 10^{-14} \) using the conjugate gradient method.

### 3.4.1.1 Computational results and comparison with experiment

The computed numerical strain and stress fields at a temperature of \( T = 200^\circ C \) are shown in Fig. 3.7 together with the corresponding experimental results for comparison. This figure shows the original microstructure, the associated mesh generated for the simulations, the experimental principal strains measured using the DIC, the computed principal strain field from the numerical runs, and the computed major principal stress field.

A comparison of the experimental and computed strain fields shows that although the relative strain distributions on the surface are captured reasonably well by the
model, the magnitudes of the strains are underestimated by the model (i.e., the measured peak strains are larger - by a factor of almost 2 - than the computed strains). This is in some measure to be expected, since (a) the computations are inherently stiffer because the model assumes that all of the interfaces are perfect and remain bonded at all times, and (b) the computations ignore the porosity that we know to exist in the microstructure. In addition, the DIC measurements slightly overestimate the strains owing to the out-of-plane expansion due to heating, which leads to an overall magnification in the sample’s images 137. However, the reasonable qualitative comparison between the measured strain field and the computed strain field gives some confidence in the heterogeneity of the computed stress fields, with some relative magnitude uncertainty. Fig. 3.7 shows that thermal loading can produce stresses as large as 250 MPa at the interface of the constituent phases. It should be noted that the maximum stresses do not necessarily develop in the areas of maximum strain (e.g. within the Fe-Ni and Chondrule). Instead, the maximum principal stresses are located at the interfaces, especially where positive curvature exists for the softer material. For context, the bulk strength measures of several L type ordinary chondrites are presented in Table 3.2 noting that these strength measures are homogenized bulk stresses, the stress state is likely very different at the grain level and near stress concentration sites, and so these are not easily related to the driving forces for fracture (the stress intensity factor at the crack tip).

In this context, we note that fatigue crack growth occurs at sub-critical stress
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levels that are much smaller than the fracture toughness of the material. Therefore, linear thermoelastic numerical models such as that used here can serve only as an indicator of the stress concentration sites that would eventually act as promoters for sub-critical fatigue crack growth and of the crack path. Building a predictive model that can estimate the crack growth rate requires a careful examination of the stress intensity factor at the crack tip, which is not included in this numerical model. With respect to crack path, cracks will tend to grow along the planes of maximum tensile stress \[138\]. The maximum principal (tensile) stress plotted in Fig. 3.7 shows that the expected direction of the crack growth would be along the matrix/inclusion interfaces. Hence, the numerical results provide an explanation for the crack extension path observed in the experiment and shown in the SEM image in Fig. 3.7. Interestingly, these stresses are within the same range as those reported by \[110\] for a heterogeneous microstructure on the Moon.

Table 3.2: Typical peak stresses for L-type ordinary chondrites at different strain rates and stress states.

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Peak stress before failure (MPa)</th>
<th>Stress state</th>
<th>Strain rate (s(^{-1}))</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRO 85209</td>
<td>L6</td>
<td>90</td>
<td>Compression</td>
<td>150</td>
<td>[139]</td>
</tr>
<tr>
<td>GRO 85209</td>
<td>L6</td>
<td>10</td>
<td>Tension</td>
<td>35</td>
<td>[139]</td>
</tr>
<tr>
<td>GRO 85209</td>
<td>L6</td>
<td>200</td>
<td>Compression</td>
<td>500</td>
<td>[140]</td>
</tr>
<tr>
<td>GRO 85209</td>
<td>L6</td>
<td>350</td>
<td>Compression</td>
<td>500</td>
<td>[140]</td>
</tr>
<tr>
<td>MAC 88118</td>
<td>L5</td>
<td>50</td>
<td>Compression</td>
<td>0.001</td>
<td>[141]</td>
</tr>
<tr>
<td>MAC 88118</td>
<td>L5</td>
<td>150</td>
<td>Compression</td>
<td>450</td>
<td>[141]</td>
</tr>
<tr>
<td>Kyushu</td>
<td>L6</td>
<td>98</td>
<td>Compression</td>
<td>Quasi-static</td>
<td>[142]</td>
</tr>
<tr>
<td>La Criolla</td>
<td>L6</td>
<td>98</td>
<td>Compression</td>
<td>0.001</td>
<td>[143]</td>
</tr>
<tr>
<td>Kyushu</td>
<td>L6</td>
<td>11</td>
<td>Tension</td>
<td>Quasi-static</td>
<td>[144]</td>
</tr>
<tr>
<td>Ness County</td>
<td>L6</td>
<td>83</td>
<td>Compression</td>
<td>(\approx 0.002)</td>
<td>[145]</td>
</tr>
<tr>
<td>Arapahoe</td>
<td>L5</td>
<td>350</td>
<td>Compression</td>
<td>(\approx 0.002)</td>
<td>[145]</td>
</tr>
</tbody>
</table>

Note: the peak stress values presented in this table are homogenized bulk measures throughout the entire sample. The local stress state at the grain level could be largely different, especially near stress concentration sites.
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Figure 3.7: Comparison of experiment and numerical simulations. The figure on the top left is an optical micrograph of a region within the sample. The next figure shows the experimentally measured strain distribution in that microstructure, using DIC. The bottom left figure presents the computed strains and stress obtained using our microstructurally informed numerical model. Mechanical and thermal material properties assigned to all constituents are given in Table 1. Results show reasonable agreement in the relative strain distributions to those that were measured using the DIC technique. The strains within the chondrule appear more clearly in the numerical results, compared to the experimental measurements, due to the limited resolution in the DIC setup. The next figure shows the computed stresses. While the strains reach a maximum within the inclusions, the stresses concentrate at the inclusion/matrix interface. These stress concentrations constitute favorable pathways for crack growth. Finally, an SEM micrograph of the observed crack growth through the interfaces provides further evidence in support of this conclusion.
3.4.2 Numerical simulations of crack growth

In the previous section, a thermoelastic analysis of the stress field was performed. The peak stresses were found to occur at the interfaces around the individual grains, indicating that they may act as promoters for crack growth. Here, we use the XFEM model developed in Chapter 2 and track the growth of a single edge crack when subjected to a cyclic thermal excursion of $\Delta T = 200^\circ C$. Note that in this analysis we are only interested here in where the crack propagates, and not how long it takes to grow. For that, we set up our simulations such that the crack always propagates by a fixed length increment $da = 0.15mm$ at each timestep, and calculate its propagation angle $\theta$ using the maximum tangential stress criterion [79]:

$$\theta = 2 \arctan \left( \frac{1}{4} \sqrt{\left( \frac{K_I}{K_{II}} \right)^2 + 8} \right),$$

where $K_I$ and $K_{II}$ are the mode-I and mode-II stress intensity factors respectively. The stress intensity factors are calculated using the interaction integral approach Eqs. (2.11) and (2.10), as it was demonstrated [146] that the equivalent domain integral formulation is still valid even when an interface lies within the interaction integral radius.

Here, we do not consider any advanced contact-interaction laws between the individual grains and the surrounding matrix (the consequence of this assumption will be discussed in Chapter 4). Dunant et al. (2013) [147] demonstrated that a con-
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Figure 3.8: Starting with a closeup image of the microstructure (a), which is then converted to grayscale (b) for easier identification of the individual phases (c), a mesh is generated using OOF2 (d). A crack is then added starting at the left edge of the mesh (e). The temperature field is ramped up linearly to $\Delta T = 200K$ and back to the initial temperature, during which the crack tip stress intensity factors are calculated. The crack is then allowed to grow following the maximum tangential stress criterion (f-h).
forming FEM mesh can capture the essential material behavior assuming the mesh nodes closely trace the inclusion interface, which in this case is a direct outcome of the OOF2 mesher. Consequently, a perfect bonding (single displacement, no-slip) condition is assumed at the interfaces. The crack is allowed to propagate until final fragmentation, which we define as the instant when the crack reaches one of the mesh’s free boundaries. The recorded crack path is shown in Fig. 3.8. The traced path confirms the conclusions from Section 3.4.1.1 as the crack appears to favor the directions running along the different mineral interfaces, with the crack growing around (Fig 3.8f) or in-between (Fig 3.8g) the FeNi and chondrule mesh elements, but not through them.

### 3.5 Discussion

The high stress levels calculated at the interfaces are important from a fracture mechanics viewpoint as they are indicators for the preferential crack propagation path. Indeed, we find in the experiments that the crack grows along the interfaces between the inclusions and the matrix, precisely at the regions of the highest thermal stresses arising from the heterogeneous microstructure (as seen in the Scanning Electron Microscopy image in Fig. 3.7). An XFEM simulation of a crack growth showed that the crack had a tendency to grow towards the inclusions, and propagate along the interfaces. There are two main consequences for such a crack path. First, the result-
ing fragments will tend to be rounded, because the associated phases and interfaces in such microstructures have a rounded character. Second, a substantial fraction of the fragments are likely to be monomineralic since the interfaces are determining the crack path. Further, it is important to recognize that the overall fracture resistance and the crack growth rate is also affected by such compositional and morphological heterogeneities, spatial distributions, and the corresponding fracture process zones. Here we have focused on the crack path rather than the crack growth rate, which requires a more sophisticated simulation of the growing crack problem. Another important observation is that the stresses are themselves very heterogeneously distributed across the microstructure (see Fig. 3.7), which could lead to variability in the expected thermal fatigue time-to-fracture as a result of the variability in the microstructure.

One of the significant consequences of the observed crack path in this thermally cycled chondrite is that it is likely that the importance and efficiency of the thermal fatigue mechanism is a strong function of the microstructure of the specific material considered (at least at these cm length scales - at larger length scales, as Delbo et al. (2014) pointed out, it is the thermal gradient that counts rather than the local microstructural heterogeneity). These results suggest that boulders with few chondrules and iron-nickel phases may be less prone to disaggregation due to thermal fatigue. Further, the presented experimental evidence (Fig. 3.5) together with the stress analyses obtained through the data-driven thermomechanical model (Fig. 3.7) suggest
that thermal fatigue processes promote intergranular fracture. This is supported by the observation of the large number of mono-minerals \cite{8} in the Itokawa fragments as reported from samples returned by the Hayabusa mission. The results can thus perhaps be applied to differentiate (see Fig. 3.9) fragment generation resulting from thermal fatigue as opposed to fragmentation as a consequence of high velocity impact onto the surface of asteroids. Considering the typical micrometeorite impact velocity, it is expected that the fracture surface would show distinct multi-faceted cleavage surfaces \cite{148,149} that are different from the smooth surface features of the fragments collected by the Hayabusa mission. Tsuchiyama et al. (2011) \cite{9} suggested abrasion due to seismic-induced grain motion as a possible mechanism for these observed features. We note that there is nothing in our work that would suggest that this does not remain a feasible mechanism. However, the thermal fatigue process demonstrated in this investigation would not only generate fracture surfaces similar to the fragments collected from the surface of Itokawa, but would also result in the mono-mineral fragments that are observed.

Finally, it is noted that crack propagation is also a possible means for modifying existing regolith size and shape distribution on such bodies, leading to surface rejuvenation, both in terms of overall particle distributions on asteroidal surfaces and in terms of the development of relatively young surfaces on larger rocks.
CHAPTER 3. THERMOMECHANICAL CHARACTERIZATION OF METEORITE SAMPLES: EXPERIMENTS AND NUMERICAL SIMULATIONS

Figure 3.9: Comparison between a) lab generated fragments and b) fragment collected by the JAXA Hayabusa mission. Impact generated fragments show distinct multifaceted cleavage surfaces that are different from the smooth surface features of the fragments collected by Hayabusa. The thermal fatigue process we describe would generate fracture surfaces similar to the mono-mineral fragments collected from the surface of Itokawa.

3.6 Summary

In summary, a detailed mechanical characterization of an L6 ordinary chondrite specimen was performed to obtain information concerning its constituents and their corresponding thermomechanical properties. The sample contained a pre-existing natural crack, which provided an opportunity to observe the crack growth path during several hundred thermal cycles. The sample was subjected to heating/cooling cycles, while images were simultaneously recorded to calculate the full-field strain map with a DIC setup. High-resolution images of the microstructure were used to generate a representative mesh of the specimen and assign the experimentally measured material properties to the corresponding mesh elements. The thermally induced stresses were then calculated, and the results indicated high stress concentrations at the inclusion.
interfaces. The observed stress distribution is indicative of the role that inclusions could play in dictating the fatigue crack growth path and the nucleation sites for new fatigue cracks.

The preceding experimental investigation shows that cracks can grow in chondrites through thermal fatigue, and that the crack path is primarily along interfaces between phases. The numerical calculation provides an explanation for the observed crack growth by demonstrating that the peak stresses are concentrated at the inclusion interfaces. This crack growth mechanism is potentially a principal source of fragments and particles on small airless bodies. This is also a possible mechanism for modifying existing regolith size and shape distribution on such bodies, leading to surface rejuvenation. Because of the strong effect of microstructure, it is likely that this mechanism’s importance and efficiency is different for different materials, particularly at small (cm) scales.

Finally, it is concluded that crack growth due to thermal fatigue could, in principle, generate monomineralic fragments – such as those recovered from Itokawa – as a result of crack growth around the mineral interfaces.
Chapter 4

Thermal fatigue in meteorites: the role of inclusions and interfaces

In Chapter 3, thermal fatigue experiments were conducted on an ordinary chondrite (GRO 85209), and cracks were determined to grow primarily along the interfaces of the heterogeneities embedded within the meteorite matrix. A simple linear numerical model indicated that the grain boundaries act as stress concentrators due to the thermomechanical properties mismatch with the surrounding meteorite bulk material, and a larger mismatch in thermal expansion would lead to increased stresses along the boundaries. XFEM simulations of the crack path captured this tendency for cracks to grow along the interfaces, as observed in the experiments. It was then implied that iron-rich meteorites could fragment by thermal fatigue faster than those with less iron. However, these conclusions were derived from numerical models that
assumed perfect bonding among the individual grains and their surrounding matrix. Considering the formation histories of these meteorites (through flash heating, impact melting, ...), it is more likely that the perfect bond assumption is not a realistic representation of the actual interfaces. In this chapter, we explore the effects of interface properties on the overall thermal stresses in a meteorite. We start by deriving the analytical solution for the thermoelastic stress field caused by a single inclusion surrounded by an interphase material and embedded in an infinite matrix subjected to a uniform temperature change. The three-phase model assumes a homogeneous interphase surrounding the circular inclusion. Different interphase thermomechanical properties corresponding to varying degrees of bond strength are considered. We then apply an established semi-analytical model that computes the crack tip driving force for different meteorite and interface strength properties. Finally, the model predictions are compared to experimental observations, and we show that weak interfaces typical in iron-rich meteorites greatly reduce the crack tip driving forces for thermal fatigue.

4.1 Introduction and Background

Chondrites are some of the most common meteorites to have been collected on Earth, and are abundantly found in Antarctica, Australia, the Sahara, and New Mexico [150]. These meteorites are characterized by the presence of round particles
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called chondrules. Chondrites also contain particles rich in Ca-Al and metallic Fe-Ni (see Fig. 3.4), which have undergone extensive secondary processing on their parent bodies through shock metamorphism [151], thermal metamorphism [152], and aqueous alteration [153]. Given this rich formation and evolution history of chondrules, it is reasonable to expect that the individual grains are not always perfectly bonded to their surroundings.

An analogous problem occurs in the manufacturing of fiber-reinforced composite materials, where the fiber/matrix bond strength is often tailored to affect the overall mechanical response of the material. The mechanical properties of such composite materials are determined not only by the properties of the reinforcement and the matrix, but also by the characteristics of the bonding between them [154]. The bulk material properties of the composite can then be tailored by controlling the interfacial region (which is often referred to as the “interphase” material), for example through different fiber surface finishing or coating methods. In addition, there typically exists a large mismatch in thermal expansion coefficients between the reinforcements (such as Kevlar or carbon fiber) and the matrix (such as polymers). During processing of the composite, and upon cooling from the processing temperature, the expansion mismatch causes residual thermal stresses which are often high enough to nucleate microcracks. The extent of residual thermal stresses can sometimes be minimized by a careful selection of the fiber coating layer [155], though microcracking is often inevitable [156].
A number of analytical and numerical approaches have been developed to investigate the effects of the interphase region on matrix microcracks in composites. The interphase models are classified into two general categories: a) spring layer models, and b) interphase layer models. In the spring layer models [157, 158], the interfacial zone is modeled as a very thin layer of unspecified thickness surrounding the reinforcement (fiber or particle). The radial and tangential tractions are continuous across the interphase, but the displacements may be discontinuous from fiber to matrix due to the presence of the interphase in between. The interphase is thus characterized by radial and circumferential springs (or, alternatively by tension and shear springs) that can be varied to allow representation of various degrees of contact (from no contact to perfect contact between fiber and matrix). In the interphase layer approach, the interphase is considered as a distinct continuum of specified thickness. The mechanical properties of the interphase can be varied to represent different degrees of bonding, and can be represented as a homogeneous continuum with spatially uniform mechanical properties [159], or nonhomogeneous with radially varying properties [160].

In this chapter, the interphase layer approach is used to derive the stress field induced by the expansion mismatch of a circular inclusion that is imperfectly bonded to an infinite matrix. The solution is formulated for the single inclusion case, and the changes in stress field as a result of different interphase properties are explored. The infinite matrix assumption allows us to eliminate the boundary effects and focus only on the stresses arising from the expansion mismatch. Then, a periodic array
of inclusions in an infinite matrix containing a crack is used to represent different meteorite types. An iterative procedure [161] is employed to calculate the stress intensity factor at the crack tip for different interphase bonding configurations. The effects of interface bonding on the overall thermal fatigue response is addressed, and a possible implication for the fragmentation of iron-rich meteorites is discussed.

4.2 Thermal mismatch stresses for a circular inclusion in an infinite matrix

The analytical solution for the thermally induced stress field caused by a single inclusion in an infinite matrix will be detailed in this section. We first start with the simple case of an inclusion that is perfectly bonded to an infinite matrix, and solve for the stress field using the basic governing equations of thermoelasticity. A continuum interphase region is then added around the interphase, and the solutions are presented for different interphase characteristics.

4.2.1 Perfect bond between inclusion and matrix

Consider a circular inclusion embedded in an infinite matrix and subjected to a uniform temperature change of $\Delta T$. The problem is illustrated in Fig. 4.1a. The constitutive equations for a homogeneous, isotropic body in terms of the Lamé pa-
parameters ($\lambda$, $\mu$) are given in index notation as:

$$\sigma_{ij} = 2\mu\epsilon_{ij} + \lambda\epsilon_{kk}\delta_{ij} - \beta\Delta T\delta_{ij}$$

(4.1)

where $\beta$ is the thermoelastic constant related to the thermal expansion coefficient $\alpha$ by

$$\beta = \alpha(3\lambda + 2\mu)$$

(4.2)

Figure 4.1: A circular inclusion embedded in a matrix having (a) perfect bonding with the matrix, and (b) imperfect bonding with the interface modeled as a continuum interphase. The case of an infinite matrix is obtained by having $c \to \infty$. Typical radial stress profiles for the perfect bonding (c) and concentric interphase (d) cases are shown for an arbitrary temperature change $\Delta T$. 
CHAPTER 4. THERMAL FATIGUE IN METEORITES: THE ROLE OF INCLUSIONS AND INTERFACES

Since the geometry of the inclusion is circular, it is preferable to use cylindrical co-
ordinates (axes defined in Fig. 4.1). In addition, since both the geometry and loading
profile (uniform $\Delta T$) are axisymmetric, the displacement field has no $\theta$-dependence
and the only non-zero displacement component is in the radial direction. The funda-
mental solution of an axisymmetric circular region can then be used (see [162]), and
the radial displacement is given by:

$$u_r(r)(f,m) = A^{(f,m)}r + B^{(f,m)}r,$$  \hspace{1cm} (4.3)

where $A^{f,m}$ and $B^{f,m}$ are constants to be determined for the inclusion (superscript $f$)
and matrix (superscript $m$). The non-zero strains are then:

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r} = A - \frac{B}{r^2}$$ \hspace{1cm} (4.4)

$$\epsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} = A + \frac{B}{r^2}$$ \hspace{1cm} (4.5)

Note that a temperature change in a general three-dimensional solid would also
induce a thermal strain along the out-of-plane $z$-direction. In this work, we focus
only on the in-plane mismatch strain contributions and set $\epsilon_{zz} = 0$, which leads to a
constant and non-zero stress component in $\sigma_{zz}$. This out-of-plane stress has no effect
on the in-plane crack tip driving forces ($K_I$ and $K_{II}$).

The boundary conditions in this problem require that: 1. the displacement in the
CHAPTER 4. THERMAL FATIGUE IN METEORITES: THE ROLE OF INCLUSIONS AND INTERFACES

inclusion is a non-singular function, 2. the matrix boundary at infinity is traction free; 3. the displacement field across the inclusion/matrix boundaries is continuous; and 4. the tractions across the inclusion/matrix boundaries are continuous as well (perfect bonding).

The non-singular condition in the inclusion gives \( B^f = 0 \), and the remaining boundary conditions are expressed as follows:

\[
\lim_{r \to \infty} \sigma_{rr}^m = 0 \quad (4.6a)
\]
\[
u|^{|}_{r=a} = u^{|}_{r=a} \quad (4.6b)
\]
\[
\sigma_{rr}^f|^{|}_{r=a} = \sigma_{rr}^m|^{|}_{r=a} \quad (4.6c)
\]

Solving for \( A^f, A^m, \) and \( B^m \), we get:

\[
A^m = \frac{\beta^m \Delta T}{\lambda^m + \mu^m} \quad (4.7a)
\]
\[
B^m = -a^2 \beta^m (\lambda^f + \mu^f) - \beta^f (\lambda^m + \mu^m) \Delta T \quad (4.7b)
\]
\[
A^f = A^m + \frac{B^m}{a^2} \quad (4.7c)
\]

The radial stress field resulting from an Iron-Nickel and a Chondrule inclusion in a matrix is illustrated in Fig. 4.2 In this solution, the heterogeneous meteorite grain
(iron-nickel, chondrule) is modeled as a circular inclusion that is perfectly embedded in the meteorite with material parameters given in Table 4.1. In the next section, this “perfectly bonded” assumption is relaxed through a three-phase model that allows for some relative motion (free expansion and slip) between the inclusion and matrix.

Figure 4.2: Radial stress as a function of radial distance for a 0.5 mm inclusion subjected to a temperature change $\Delta T = 200$ K. The heterogeneous meteorite grain (iron-nickel, chondrule) is modeled as a circular inclusion that is perfectly embedded in the meteorite matrix.

### 4.2.2 Thermoelastic stress field for an imperfectly bonded inclusion

Since chondrites are naturally occurring materials typically formed through accretion, it is expected that the mineralic grain inclusions would not be perfectly bonded
to their surrounding matrix. Here, we explore the effective thermoelastic stress field when the bonding is imperfect. We do so using the three-layer interphase model. The problem in Section 4.2.1 is modified to include an additional continuum circular region surrounding the inclusion (see Fig. 4.1). The problem remains axisymmetric, and so the general solution in Eq. (4.3) is valid in the inclusion, matrix, and interphase (indicated by superscript \(i\) hereafter). However, the inclusion is no longer perfectly attached to the matrix, but rather the inclusion/interphase and matrix/interphase boundaries are perfectly bonded. The change in interphase thermomechanical properties allows for modeling an effective bonding “strength” by acting as a buffer region enabling some relative deformation between the matrix and the inclusion.

The new boundary conditions are therefore expressed as:

\[
\lim_{r \to \infty} \sigma_{rr}^m = 0 \quad (4.8a)
\]

\[
u^i \bigg|_{r=b} = u^m \bigg|_{r=b} \quad (4.8b)
\]

\[
\sigma_{rr}^i \bigg|_{r=b} = \sigma_{rr}^m \bigg|_{r=b} \quad (4.8c)
\]

\[
u^f \bigg|_{r=a} = u^i \bigg|_{r=a} \quad (4.8d)
\]

\[
\sigma_{rr}^f \bigg|_{r=a} = \sigma_{rr}^i \bigg|_{r=a} \quad (4.8e)
\]

Solving for \(A^f\), \(A^i\), \(B^i\), \(A^m\), and \(B^m\), we get:
CHAPTER 4. THERMAL FATIGUE IN METEORITES: THE ROLE OF INCLUSIONS AND INTERFACES

\[ A^m = \frac{\beta^m}{\gamma^m} \Delta T \]  \hspace{1cm} (4.9a)

\[ B^i = \Lambda \left( A^m + \frac{1 + \gamma^i/2\mu^m}{\gamma^f - \gamma^i} (\beta^i - \beta^f) \Delta T - \frac{\beta^i}{2\mu^m} \Delta T \right) b^2 \]  \hspace{1cm} (4.9b)

\[ A^i = \frac{1}{\gamma^f - \gamma^i} \left( (\gamma^f - 2\mu^i) \frac{B^i}{a^2} - (\beta^i - \beta^f) \Delta T \right) \]  \hspace{1cm} (4.9c)

\[ B^m = \frac{b^2}{2\mu^m} (\gamma^f A^i - 2\mu^i B^i b^2 - \beta^i \Delta T) \]  \hspace{1cm} (4.9d)

\[ A^f = A^i + \frac{B^i}{a^2} \]  \hspace{1cm} (4.9e)

where \( \gamma = 2(\lambda + \mu) \) and

\[ \Lambda = \left[ \left( 1 + \frac{\mu^i}{\mu^m} \right) + \left( 1 + \frac{\gamma^i}{2\mu^m} \right) \left( \frac{\gamma^f - 2\mu^i}{\gamma^f - \gamma^i} \right) \frac{b^2}{a^2} \right]^{-1} \]  \hspace{1cm} (4.10)

Substituting Eq. (4.9) into Eqs. (4.1) and (4.5) gives the thermoelastic stress field for an inclusion with the imperfect bonding to the matrix modeled as an interphase continuum. Note that when \( b = a \), the solution reduces to the perfect bonding case of Section 4.2.1.

Different possible thermoelastic properties for the interphase material can now be considered. First, the interphase is modeled as a competent bi-material composite that is made of a homogenized mixture of inclusion and matrix materials whose thermomechanical properties are obtained through a rule-of-mixture approach. For a volume fraction \( V_f \) of inclusion material in the interphase, the effective Young’s
modulus is given by:

\[
E^i = (1 - V_f)E^m + V_f E^f
\]  \hspace{1cm} (4.11)

Then, the effective thermal expansion coefficient \( \alpha^i \) of the homogenized interphase is calculated using a concentric cylinder model as \[163\]:

\[
\alpha^i = \frac{V_f E^f \alpha^f + (1 - V_f)E^m \alpha^m}{V_f E^f + (1 - V_f)E^m}
\]  \hspace{1cm} (4.12)

Next, we recognize that the inclusion interphase (see Section 4.3.1) would typically contain some number density of microcracks and microporosity due to its formation and alteration history. The additional case of an interphase that is weakened by some effective microporosity is considered using a simple porosity model \[164\] with the Young’s modulus of the porous interphase, \( \bar{E}^i \) given by:

\[
\bar{E}^i = E_0^i \left(1 - \frac{\phi}{\phi_c}\right)^\xi,
\]  \hspace{1cm} (4.13)

where \( \bar{E}^i \) is the Young’s modulus of the interphase with porosity \( \phi \), \( E_0^i \) is the Young’s modulus of the solid interphase, \( \phi_c \) is the porosity at which the effective Young’s modulus becomes zero, and \( \xi \) is a material parameter dependent on the grain morphology and pore geometry. Fits to experimental data for different materials shows that \( \phi_c \) is near unity \[165\]. With a lack of accurate experimental measurements for the
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thermoemchanical properties of the interphase, we only consider the case of \( \alpha = 1 \), reducing Eq. (4.13) to a linear porosity model.

In the next section, the effects of changing the interphase properties (for the same inclusion and matrix material) are explored. The change in the thermoelastic stress field relative to that from a perfectly bonded inclusion is examined as a function of interphase thickness, composition, and porosity.

4.2.3 Results

We explore the effects of interphase properties on the thermoelastic stress field caused by the expansion of a single inclusion embedded in a matrix. We consider Fe-Ni and olivine chondrule inclusions in a matrix composed of 20% Plagioclase and 80% Mafic. The thermoelastic properties for the different materials are given in Table 4.1.

Table 4.1: Thermoelastic properties for the matrix and inclusions.

<table>
<thead>
<tr>
<th>Units</th>
<th>Matrix</th>
<th>Fe-Ni</th>
<th>Chondrule</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus, E</td>
<td>GPa</td>
<td>139</td>
<td>187</td>
<td>125</td>
</tr>
<tr>
<td>Poisson’s Ratio, ( \nu )</td>
<td>0.31</td>
<td>0.30</td>
<td>0.25</td>
<td>[104,127]</td>
</tr>
<tr>
<td>Expansion coefficient, ( \alpha )</td>
<td>K(^{-1})</td>
<td>(4.3 \times 10^{-6})</td>
<td>(12.6 \times 10^{-6})</td>
<td>(10.4 \times 10^{-6})</td>
</tr>
</tbody>
</table>

Typical solutions for the radial stress fields for a perfect bonding between the fiber and matrix (Eq. (4.7)) and for an imperfect bonding modeled as a continuum interphase (Eq. (4.9)) are shown in Fig. 4.1. In the following parametric analysis, we explore the change in the thermoelastic stress fields induced by a temperature variation of \( \Delta T = 200K \) as a function of different inclusions and interphase thickness,
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composition, and porosity.

Figure 4.3: Normalized radial stress as a function of normalized radial distance for varying interphase thickness. Dashed lines represent the chondrule-rich case, and continuous lines represent the metal-rich meteorites. The stresses are normalized by the stress solution for a perfectly bonded inclusion, and the interphase is assumed to be formed of equal amounts of inclusion and matrix materials. An increase in interphase thickness leads to a decrease in matrix stresses. This stress relaxation is more prominent in the chondrule-rich meteorites. In addition, thicker interphases lead to larger stresses inside the inclusion in the iron-rich meteorites.

First, the interphase is assumed to be homogeneous, non-porous, and composed of equal amounts of inclusion and matrix material (mixture with $V_f = 0.5$). Fig. 4.3 shows the effects of increasing the interphase thickness for the same inclusion radius on the thermal mismatch stresses relative to the perfectly bonded case. An increase in interphase thickness leads to a decrease in matrix stresses (30–45% relaxation), with the chondrule-rich meteorites showing more signs of relaxation than iron-rich meteorites. In addition, thicker interphases lead to slightly larger stresses within
the inclusion in the iron-rich meteorites. Inclusion stresses in the chondrule-rich meteorites were relatively insensitive to the interphase thickness because the elastic moduli of the chondrule and matrix materials (Table 4.1) are very similar.

![Figure 4.4: Normalized radial stress as a function of normalized radial distance for varying interphase composition. An increase of “matrix” material to the interphase composition leads to a decrease in thermal stresses in the matrix itself. A smaller volume fraction of inclusion material in the interphase leads to a smaller effective inclusion radius. That is, $V_f = 0$ is the case of a perfect interface for a small inclusion, and $V_f = 1$ is the case of a perfect interface for a larger inclusion. Dashed lines represent the chondrule-rich case, and continuous lines the metal-rich meteorites. The inclusion is arbitrarily chosen to be 0.5 cm surrounded by a 0.3 cm interphase.](image)

Next, the effect of interphase composition is considered in Fig. 4.4 and the thermal stresses are normalized by the case of $V_f = 1$, which corresponds to an interphase made of purely the inclusion material (or, a perfect bonding between the inclusion and matrix). The composition of the interphase has a significant effect on the ther-
moelastic stresses in meteorites. For the same inclusion and interphase thickness, a decrease in the volume fraction of the inclusion material inside the interphase leads to a decrease in the thermal stresses of matrix (with a small increase in stresses within the inclusion itself for the iron-rich meteorite cases). A stress relaxation of 23% is observed when the interphase is composed of equal amounts of inclusion and matrix materials.

Figure 4.5: Normalized radial stress as a function of normalized radial distance for varying interphase porosity. An increased interphase porosity leads to a decrease in the thermal stresses of the entire meteorite. Dashed lines represent the chondrule-rich case, and continuous lines the metal-rich meteorites.

Finally, the effects of porosity (resulting from micropores or microcracks in the interphase) are shown in Fig. 4.5. It can be seen that the introduction of porosity in the interphase relaxes the stresses within the inclusions as their expansion is less constrained by the surrounding matrix. Unlike the changes in interphase width and
composition (Figs. 4.3 and 4.4), an increase in porosity leads to an overall decrease in thermal stresses within both the inclusion and the matrix. In the perfect bonding cases (no interphase, Fig. 4.2), the matrix thermal stresses near the inclusion can exceed the material’s bond strength (see Table 3.2). This indicates that these meteorites would have been easily fractured in the early days of their formation, with cracks forming around the inclusions. The initiation of these cracks results in an effective porosity at the interphase and shields the meteorites from further elevated stresses.

The thermomechanical interaction between inclusions such as chondrules or Iron-Nickel and the surrounding meteorite matrix depends largely on their bonding characteristics. Using an analytical solution to the thermomechanical problem of a coated inclusion embedded in an infinite matrix, the effects of bond strength on the thermally induced stresses are explored. The parametric study demonstrates that a perfect bonding between inclusions and matrix leads to large thermal stresses near the interfaces, which are highly dependent on the way the inclusions are bonded to the bulk matrix. These stresses could be large enough to exceed the matrix’s fracture strength and propagate microcracks along interfaces. This demonstrates that the formation of microcracks as a result of the material’s exposure to elevated temperatures would have easily occurred early on after the formation of these meteorites. The cracks may later grow sub-critically in thermal fatigue. In the next section, we calculate the stress intensity factor for cracks in an idealized representation of a me-
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teorite microstructure. Then, SEM images of the inclusion/matrix interfaces are used to constrain the possible composition of the interphase material, and the implications on thermal fatigue are discussed.

4.3 Thermal fatigue driving force in an idealized meteorite microstructure

The previous section demonstrated the influence of interface bonding on the overall thermally induced stress field caused by a single inclusion. In Chapter 2, it was indicated that thermal fatigue is typically characterized in terms of the stress intensity factor, which is a measure of the singularity at the crack tip. The analysis presented in the previous section shows that interfacial bonding properties should largely influence the meteorite’s response to thermal cracking, but does not quantify how the interphase would affect the crack tip driving force. For that, we here use an established semi-analytical iterative solution procedure [161] to calculate the stress intensity factor. The meteorite microstructure is idealized as a infinite periodic array of circular inclusions. The thermal stresses in the matrix decay as $1/R^2$, following Eqs. (4.1), (4.5) and (4.9). Thus, when the individual mineralic grains are not sparsely separated, the interaction among neighboring inclusions becomes important. Pijaudier-Cabot and Bažant [161] presented a numerical method to capture this interaction based on the superposition of subproblems consisting of single inclusions whose
solution is already known. They derived an iterative procedure for calculating the
stress field, accounting for the stress concentration caused by a crack. The method
was later generalized [167] to the case of multiple cracks and randomly distributed
microstructures. In this section, we consider the idealized case of a periodic regular
array of inclusions in an infinite meteorite matrix. Since the microstructure is an
infinitely repeating array, macroscopic boundary effects are eliminated and stresses
are only driven by the mismatch in coefficient of expansion. Due to the periodic
distribution, each inclusion will be exposed to the same amount of interaction. Using
the principle of superposition of solutions, the main problem is decomposed into
two subproblems: I) an uncracked material with multiple inclusions, subjected to a
uniform temperature change; and II) a cracked material with traction loads on the

The iterative process is illustrated in Figs. 4.6 and 4.7 and the major steps outlined hereafter. In subproblem I, the idealized uncracked microstructure is composed
of a regular array of inclusions. The temperature change causes thermal stresses due
to the expansion mismatch between the matrix and the inclusions. Owing to the pe-
riodicity, the thermal stress in all inclusions will be the same. The problem can then
be further decomposed into a series of individual subproblems, where each inclusion
is alone in the matrix, causing some perturbation stress on its neighboring inclusions
(Fig. 4.7). The stress field caused by a single inclusion in an infinite matrix is known
(see Eq. (4.9)). What remains is to account for the contributions of the interactions
CHAPTER 4. THERMAL FATIGUE IN METEORITES: THE ROLE OF INCLUSIONS AND INTERFACES

Figure 4.6: A regular array of inclusions embedded in an infinite matrix subjected to a temperature change of $\Delta T$. The problem is divided into two subproblems, using superposition. Subproblem I captures the stress field in the uncracked body due to the temperature change $\Delta T$, and the opposite equivalent tractions are applied on the crack faces in Subproblem II.

of neighboring inclusions. Pijaudier-Cabot and Bažant \[161\] demonstrated that this interaction stress can be easily captured through an iterative solution based on the equivalent inclusion method. In this equivalent inclusion method, the theory of eigenstrains and eigenstresses is applied such that the heterogeneous solid is transformed
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to a homogeneous material, but with the areas occupied by an inclusion containing eigenstresses $\Delta \sigma$. This eigenstress, also referred to as the unbalanced stress field, exists inside the boundary $\Gamma_i$ (Fig. 4.7) and vanishes in the matrix outside $\Gamma_i$. The eigenstresses create a disturbance in the stress field such that tractions $p_i = -\Delta \sigma n_i$ act at the circular boundary $\Gamma_i$ with unit outward normal $n_i$.

The stress field may then be written as:

$$\sigma^* = \sigma^* + \Delta \sigma \quad \text{inside} \enspace \Gamma_i \quad (4.14)$$

$$\sigma = \sigma^* \quad \text{outside} \enspace \Gamma_i \quad (4.15)$$

where $\sigma^*$ is the stress field when the thermoelastic properties of the inclusion are changed to those of the matrix while the tractions $p_i$ are acting at the interface $\Gamma_i$. This unbalanced stress has to satisfy

$$\Delta \sigma = (C_i - C_m) : \epsilon, \quad (4.16)$$

where $C_i$ and $C_m$ are the inclusion and matrix elastic tensors respectively, and $\epsilon$ is the mechanical strain in the matrix. In this equivalent problem, the unknown parameter is $\Delta \sigma$ and it is solved for using an iterative procedure.

Starting from the known solution of a single inclusion in an infinite matrix, the stress field inside the inclusion is calculated, and the equivalent tractions $p_i$ are obtained. From these tractions, a new unbalanced stress field is recalculated using
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Figure 4.7: Subproblem I is composed of the superposition of the individual stress fields from each inclusion, while accounting for their interaction. The perturbation caused by a single inclusion is represented by the pressure field $p_i$ on the boundary $\Gamma_i$ using the Duhamel-Neuman analogy.

Eq. (4.16), and the procedure is repeated until the change in stress field becomes small (i.e., the numerical solver has converged). The reader is referred to [167] for a detailed derivation.

In subproblem II, a crack is introduced and loaded with tractions that are equal and opposite to those calculated along the fictitious crack location in subproblem I. The superposition of subproblems I and II gives the solution for the traction-free crack in the original problem. Following the Pijaudier-Cabot and Bažant procedure, the stress field on the crack is calculated directly by averaging the interacting pressure distributions along its faces so that it is loaded by a pressure $p_c$ on its contour $\Gamma_c$. The mode-I stress intensity factor can then be calculated at the crack tips ($\pm c$) as:
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\[ K_I(\pm c) = \frac{1}{\sqrt{\pi c}} \int_{-c}^{+c} \sqrt{\frac{c+x}{c-x}} p_c(x) \vec{n} dx, \quad (4.17) \]

where \( \vec{n} \) is the outward normal to the crack surface \( \Gamma_c \) shown in Fig. 4.6.

4.3.1 Stress intensity factor in different meteorite microstructures

The semi-analytical solution procedure is used to calculate the stress intensity factor in different assumed meteorite types. We consider the idealized properties for Allende and ordinary chondritic (OC, GRO 85209) meteorites shown in Table 4.2, which are compiled from [14,168,169].

Table 4.2: Thermoelastic properties for the idealized meteorite microstructures.

<table>
<thead>
<tr>
<th></th>
<th>Allende</th>
<th>GRO 85209</th>
<th>Metal-rich OC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix Young’s Modulus (GPa)</td>
<td>44</td>
<td>74</td>
<td>74</td>
</tr>
<tr>
<td>Matrix Poisson Ratio</td>
<td>0.24</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>Matrix Expansion Coeff. (K(^{-1}))</td>
<td>(4.30 \times 10^{-6})</td>
<td>(8.50 \times 10^{-6})</td>
<td>(8.50 \times 10^{-6})</td>
</tr>
<tr>
<td>Inclusion</td>
<td>Chondrule</td>
<td>Chondrule</td>
<td>Fe-Ni</td>
</tr>
<tr>
<td>Inclusion Young’s Modulus (GPa)</td>
<td>125</td>
<td>125</td>
<td>187</td>
</tr>
<tr>
<td>Inclusion Poisson Ratio</td>
<td>0.25</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>Inclusion Expansion Coeff.</td>
<td>(1.04 \times 10^{-5})</td>
<td>(1.04 \times 10^{-5})</td>
<td>(1.26 \times 10^{-5})</td>
</tr>
<tr>
<td>Inclusion Avg. Radius (mm)</td>
<td>0.425</td>
<td>1.16</td>
<td>0.78</td>
</tr>
<tr>
<td>Inclusion Avg. Spacing (mm)</td>
<td>1.5</td>
<td>1.40</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Note that we do not intend to simulate an exact replication of the meteorite microstructure, but rather to investigate the effect of different interface bondings on the thermal stress intensity factor in meteorites whose constituent phases have het-

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erogeneous thermomechanical properties. Following the analysis in Section 4.2.2, we consider the cases of an array of perfectly bonded inclusions, an “average” interphase made of equal amounts of inclusion and matrix material with no porosity, and a very “weak” interphase with a porosity of $\phi = 0.7$. We solve for the thermoelastic stresses induced by a temperature change of $\Delta T = 200$ K, and then use Eq. (4.17) to calculate the driving force at the crack tip for varying crack lengths.

The normalized stress intensity factor as a function of normalized crack length is shown in Fig. 4.8.

Figure 4.8: The effect of interface bond strength (interphase properties) on the crack tip stress intensity factor. The horizontal axis is normalized by the radius of the inclusion, and the vertical axis is normalized by the fracture toughness of $1.6 \text{ MPa } \sqrt{\text{m}}$. It can be seen that weak interphases lead to a considerable drop in the stress intensity factor, whereas the perfect bonding case leads to increased stress intensity factors that are of the same order as the material’s fracture toughness.
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It can be seen that for the metal-rich meteorite case, the normalized stress intensity factors are generally larger than for the chondrule-rich meteorites. In the perfectly bonded iron-rich case, the stress intensity factor for relatively large cracks \((a/R > 6)\) is very close to the fracture toughness of the material. This is also in-line with the finite element results in Section 3.4.1.1 and the analytical solution in Section 4.2.1, where the thermal stresses approached the mechanical strength of the meteorite when grains were assumed to be perfectly bonded to the matrix. When the interphase is weakened (by microcracks, for example), the stress intensity factor drops considerably. This reduction in stress intensity factor corresponds to the relaxation of thermal mismatch stresses (see also Fig. 4.5). For very weak interphases, the stress intensity factor for small cracks \((a/R < 2.5)\) is generally small and increases weakly as a function of crack size, meaning that thermal fatigue would be highly inefficient in those cases.

So far, the results shown here demonstrate that relaxing the interface bonding would weaken the meteorite’s overall response to thermal fatigue. The model does not state which configuration is more likely to be present in a given meteorite type. For that, experimental characterization of the inclusion/matrix interface is required.

A detailed inspection of the inclusion interfaces of a real sample was performed by [170]. They noted extensive alteration around metal grains, where multi-layered inner rims with frequent intrusions into the metal were prominent, which are likely a result of corrosion penetration (Fig. 4.9). In contrast, altered chondrules exhibited a very sharp contact between the core and the matrix (i.e.: small “interphase” region). As
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Figure 4.9: A metal grain in a CR chondrite, showing a thick, multi layered inner rim surrounding a metal grain. An outer rim of coarse-grained materials surrounds the inclusion. The interphase consists of alternating iron and sulfur-rich layers with considerable microcracks. In addition, the SEM images reveal frequent intrusions into the metal. Image reproduced from Morlok et al. (2012).

such, metal-rich meteorites have a diffuse chemical composition between the inclusions and the surrounding matrix, with considerable radial microcracks forming around the core grain. Consequently, the SEM images suggest (qualitatively, at least), that this configuration is better represented in our model by the “average” or “weak” interphase case of Fig. 4.5 rather than by a perfect bonding. On the other hand, chondrules could be approximated as perfectly bonded to the matrix. As such, our results would imply that thermal fatigue would be less efficient in fracturing a heavily altered iron-rich meteorite as opposed to a chondrule-rich meteorite such as Allende, which is almost completely lacking in Fe-Ni metal.
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4.4 Conclusions

In this chapter, we explored the role of inclusion interface bonding and its effect on the thermal-induced stresses and stress intensity factor on the crack tip. Using the analytical solution of a coated inclusion embedded in an infinite matrix, we showed that a weak interface could efficiently relax the thermally induced stresses in the matrix. Then, we used an established method to calculate the stress intensity factor in an idealized microstructure representation of different meteorites. It was shown that weak interphases could also reduce the thermal stress intensity factor at the crack tip, rendering thermal fatigue less efficient in these meteorites.

In the current analysis, we have ignored the effects of residual thermal stresses in the meteorites. When these meteorites are formed, the large thermal mismatch among the individual grains gives rise to some residual stresses. This is similar to the problem of manufacturing fiber-reinforced composites, where the expansion mismatch causes residual thermal stresses upon cooling from the processing temperature, and which are often high enough to nucleate microcracks. We assume here that the residual thermal stresses have been relaxed due to some long-term stress relaxation mechanism (such as creep) or through the nucleation of microcracks at the interphase. We also limit our study to the thermoelastic response of the meteorites and ignore any plasticity effects. The analysis is thus focused on meteorites linked to S-type asteroids and is likely not applicable to metallic M-type asteroids.

The parametric analysis in this chapter considered the effects of different inter-
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Phase composition. With the lack of characterization of the interface thermomechanical properties, it is not possible to know which configuration is the best representation of a real microstructure. However, characterization experiments on CR chondrites [170] showed that metallic inclusion interfaces were formed of multi-layered cracked inner rims with frequent intrusions into the metal, which constituted the diffuse interphase material. This interface was different from that of a chondrule, which had a smaller external rim and very sharp contact between the inclusion and the matrix, meaning that chondrites had a diminishingly small interphase region surrounding the chondrules. By linking these experimental observations to the numerical results from Section 4.3.1, we conclude that heavily altered metal-rich meteorites may be less prone to thermal fatigue cracking (for a uniform temperature change) than chondrule-rich meteorites. Ongoing thermal fatigue experiments on Allende and Murchison (G. Libourel, personal communication) appear to support the conclusions derived from our model. In these experiments, Allende samples, which were almost lacking in metallic inclusions, showed stronger evidence of fatigue crack propagation than the other iron-rich meteorites.

Therefore, thermal fatigue could be less efficient in breaking down the very small rocks (where thermal gradients are small and thermal expansion mismatch is the main contributor) in iron-rich meteorites with weak interfaces than in chondrule-rich meteorites where the interfaces are sharp. It may then be expected that their parent asteroid’s regolith would, on average, consist of coarser grains than a similar asteroid.
with more chondrule-rich rocks. It would therefore appear that metal content, which may be derived from remotely sensed information (overall light absorption efficiency, infrared absorption bands, etc...), could be an indirect indicator for rock sizes in asteroids with regolith evolution driven by thermal fatigue.
Chapter 5

A hybrid framework for simulating hypervelocity asteroid impacts and gravitational reaccumulation

5.1 Introduction

Asteroid impacts ranging from small-scale cratering events to catastrophic disruption have played a crucial role in the evolution of the asteroid belt. Studying the impact history of asteroids also provides a glimpse into the active processes from the early formation of the solar system. Detailed information about asteroid shapes,

†This chapter is based on a paper submitted to Icarus and entitled “A new hybrid framework for simulating hypervelocity asteroid impacts and gravitational reaccumulation”. Prof. Derek C. Richardson provided the pkdgrav source code and is a co-author on the paper.
composition, and orbital dynamics has been collected through remote sensing [19] and spacecraft missions [20]; however, the internal structure of those asteroids remains poorly constrained. Is an asteroid a fractured monolith, or a rubble pile of gravitationally bound fragments? This distinction is difficult to make even for asteroids with known bulk densities. As pointed out in [21], the same data for asteroid 433 Eros could be interpreted to support a monolithic structure with impact-induced grooves [22], or to support a rubble pile [23]. Since laboratory experiments alone cannot sufficiently reproduce the scales and conditions within asteroid impacts, numerical modeling has become a valuable approach for providing insight into the internal structure of asteroids, asteroid surface modification, and the formation of asteroid families and satellites.

Studies of asteroid hazard mitigation, together with the prospect of kinetic impactor-based space experiments such as the DART mission [27], also call for detailed computational modeling of hypervelocity impacts onto small (sub-km) asteroids. A primary result of interest in asteroid mitigation studies is a measure of the momentum transfer characterized by the momentum enhancement factor [28, 29]. The momentum enhancement factor is the ratio of the total momentum imparted to the post-impact target and the initial momentum of the impactor. If no material is ejected, then only the momentum of the impactor has been added to the target (momentum enhancement = 1). If material is ejected from the target, this can enhance the momentum transfer to the body (momentum enhancement > 1). As such, tracking the fate of
ejecta from fragmentation to ejection and reaccumulation is crucial in capturing the momentum enhancement of the target following a high speed impact.

Numerous studies have demonstrated the importance of multi-stage modeling of asteroid impact processes so as to follow the ejecta from the early stages of impact to the late stage of gravitational reaccumulation \cite{171,172,173}. \cite{171} simulated the catastrophic disruption of a large parent asteroid and the subsequent formation of asteroid satellites using a two-step process where material fragmentation during the first few seconds was captured using a Smoothed Particle Hydrodynamics (SPH) code. In their simulations, the target was fully disrupted at the end of the SPH calculation. The smoothed particles in SPH were then transformed into discrete rigid spheres, and their positions and velocities handed off to a gravitational $N$-body code ($\text{pkdgrav}$) to trace the evolution of the system over several days post-impact. Since particles in SPH do not have a physical radius, and owing to the smoothing kernel in SPH codes, an iterative approach is often employed in which the particles’ radii are sequentially reduced until there is no overlap among the spheres.

The feasibility of this two-step approach was demonstrated through a detailed parametric study \cite{174} and was later used to explore the origin of asteroid families and satellites \cite{172,173,175}. The approach also suggested a possible origin of asteroid (25143) Itokawa as a rubble-pile aggregate by-product of a catastrophic disruption event \cite{176,177}. It is notable that in all of these studies, the initial impact leads to the complete disruption of the target asteroid, down to the SPH numerical resolution, and
the number of fragments was therefore equal to the number of SPH particles. This total disruption outcome was insensitive to the chosen resolution, leaving no individual fragment larger than the particle discretization size at the end of the fragmentation phase [174].

Recently, [178] investigated the impact history of asteroid (433) Eros using a new material model for brittle materials and a very different numerical approach. The Tonge-Ramesh material model incorporates multiple physical processes that occur during an impact event, such as dynamic crack propagation and damage accumulation, porosity growth and pore compaction, and granular flow of highly damaged materials. [178] simulated the formation of the largest craters on Eros (Himeros, Psyche, and Shoemaker) through sequential impact events onto an assumed shape model of a “young” monolithic Eros. Interestingly, their results showed the shattering but not complete disruption of the target throughout all of the three impact events. In addition, a final porosity of 20% was predicted within Eros as a result of the granular flow of the fragmented material. This porosity level is not far from the current observed bulk porosity of about 25% and suggests that Eros could be heavily fractured rather than a rubble pile aggregate that reaccumulated following a catastrophic disruption [179]. However, the [178] results focused on the short-timescale fragmentation phase and did not account for long-term gravity effects such as reaccumulation of material or the target’s compaction due to self-gravity.

In this work, we follow the hybrid technique of [171] by separating the impact event
CHAPTER 5. A HYBRID FRAMEWORK FOR SIMULATING HYPERVELOCITY ASTEROID IMPACTS AND GRAVITATIONAL REACCUMULATION

into two main timescales: the short-timescale fragmentation phase, and the long-timescale gravitational phase. We use a modified version of the Tonge-Ramesh model that incorporates the Tillotson equation of state and is implemented in a Material Point Method (MPM) framework. We validate the material model by comparing the predicted dynamic tensile strengths with Brazilian disk experiments on basalt. We use the validated model to first simulate the fragmentation phase, which is dominated by the stress waves traveling through the target. Then, a suitable cut-off time is selected to hand off the simulation to the $N$-body code \texttt{pkdgrav} for the long-timescale gravitational phase. Unlike SPH-based hand-offs, the conversion from particles to discrete spheres is greatly facilitated by taking advantage of the Eulerian-Lagrangian nature of MPM. We implement this approach to study the collisional outcome of a 1.21 km impactor onto a 25 km target (as in [177]) at the early stages and then hand off to the $N$-body gravity code, \texttt{pkdgrav}, for the long-timescale gravity evolution.

5.2 Numerical Modeling

5.2.1 The Material Point Method

The Material Point Method (MPM) is an extension of the particle-in-cell (PIC) method in which a continuum body is discretized into a set of Lagrangian material points (or particles) moving within a background grid [180, 183]. State variables are stored at the material points, while the computational grid is used to calculate the
necessary gradients for solving the equations of motion. This background grid is
discarded and reset after each timestep, avoiding the undesirable mesh tangling and
advection errors of mesh-based methods when modeling large deformations (Fig. 5.1,
top row). This dual particle-grid nature defines two numerical resolutions related
to a) the cell size or spacing, and b) the number of particles per cell. In effect,
the cell spacing defines the smallest lengthscale for which a particular gradient can
be calculated, and the number of particles per cell determines the integration order
within each cell. One of the advantages of MPM is that the use of the grid renders
it trivial to apply boundary conditions and resolve material contacts. The numerical
difficulties in SPH related to surface particles (when the kernel support is not fully
included) or the tensile instability [184] (from kernel approximation) are therefore
eliminated in MPM. In addition, contacts are resolved trivially in MPM by means
of the background grid, eliminating the need for the expensive neighbor-search algo-

As such, MPM is well suited for simulating large deformations of solids and has
been used in a number of applications including modeling injury in soft biological
tissues [186, 187], response of granular materials [188, 190], and asteroid impacts [178].
Figure 5.1: The top row illustrates the particle/grid interpolations performed during an MPM timestep. A continuum is discretized into a set of material points where all state variables are stored. At the beginning of a timestep, mass and velocity at grid nodes are calculated by interpolating the values at the material points. The equations of motion are solved on the grid, and the displacements are interpolated back to the material points. The background grid is discarded and reset at the end of the timestep. The bottom row illustrates our MPM-to-pkdegrav hand-off method that is consistent with the MPM interpolations. This hand-off procedure is discussed in Section 5.4.2.
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In MPM, a body is first discretized into a set of material points at which all variables (mass, velocity, temperature, ...) are stored. An integration for a timestep $\Delta t$ starting at a time $t$ begins by interpolating the material point mass $m_p$ and momentum $p'_p$ onto the grid such that the total nodal mass $m_n$ and nodal momentum $p_{n}^{t_0}$ are conserved. The nodal mass is obtained by interpolating the contributions from surrounding material points using a shape function $S_{np} = S_n(x_p)$ as:

$$m_n = \sum_p S_{np} m_p \tag{5.1}$$

$$p_{n}^{t_0} = \sum_p S_{np} p'_p \tag{5.2}$$

The grid mass matrix is constant throughout a single timestep, but is not necessarily the same at a subsequent timestep. Since material points can move across cell boundaries, the individual components of the grid mass matrix could change and should be computed at the beginning of every timestep. Consequently, a lumped mass matrix is generally preferred in order to minimize the computational cost related to matrix inversion at each timestep. Following Eq. [5.2], momentum is conserved by construction.

The grid velocities can then be calculated by dividing the nodal momentum by the nodal mass:

$$\mathbf{v}_n^{t} = \mathbf{p}_n^{t} / m_n \tag{5.3}$$

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After solving on the grid the weak form of the potential energy minimization, the nodal velocities at the end of the timestep are incremented as:

\[ \mathbf{v}^{t+\Delta t}_n = \mathbf{v}_n + \mathbf{a}_n \Delta t \] (5.4)

where \( \mathbf{a}_n \) is the nodal acceleration.

The material point positions and velocities are then updated by interpolating the velocities and accelerations from the grid nodes:

\[ \mathbf{x}^{t+\Delta t}_p = \mathbf{x}_p + \sum_n S_{np} \mathbf{v}^{t+\Delta t}_n \Delta t \] (5.5)

\[ \mathbf{v}^{t+\Delta t}_p = \mathbf{v}_p + \sum_n S_{np} \mathbf{a}_n \Delta t \] (5.6)

Note that Eqs. (5.1) and (5.2) are particle-to-grid interpolations whereas Eqs. (5.5) and (5.6) are grid-to-particle interpolations.

Finally, the material point strains and Cauchy stresses are incremented using the strain rate tensor, which is the symmetric part of the velocity gradient.

\[ \Delta \mathbf{\epsilon}^{t+\Delta t}_p = \mathbf{E} \Delta t = \frac{1}{2} \sum_n (\nabla S_{np} \mathbf{v}^{t+\Delta t}_n + \mathbf{v}^{t+\Delta t}_n \nabla S_{np}) \Delta t \] (5.7)

The choice of functional expressions for evaluating the shape functions \( S_n(\mathbf{x}_p) \) has direct consequences on the stability and convergence of MPM algorithms. The
classical MPM approach assumed that the entire mass associated with a material point is concentrated at the location of that point, and as such the same shape functions from finite element (FEM) codes could be directly used; i.e., $S_{np} = N_{np} = N_n(x_p)$ \[182\]. Bardenhagen et al. (2004) \[191\] showed that this assumption leads to cell-crossing artifacts in which artificial internal forces are developed as a result of a particle crossing a cell boundary, and proposed instead a generalized form for the shape functions given by:

$$S_{np} = \frac{\int_{\Omega_p} \chi(x_p) N_{np}(x_p) \, d\Omega}{\int_{\Omega_p} \chi(x_p) \, d\Omega},$$  \hspace{1cm} (5.8)

where the material point $x_p$ is assumed to occupy a domain $\Omega_p$. If the material point occupies an infinitesimally small domain (a true point), then $\chi(x_p)$ is a Dirac delta function, and the classical MPM formulation is recovered. Otherwise, if the material point is assumed to occupy a finite domain (i.e., $\chi(x_p) = 1$ in some particle domain $\Omega_p$ and 0 elsewhere), then $\chi(x_p)$ is a Heaviside step function. A number of alternative MPM shape function formulations such as the Undeformed Generalized Interpolation Material Point (UGIMP), the Convected Particle GIMP (CPGIMP), the Convected Particle Domain Integration (CPDI), and the second order CPDI (CPDI2) are then derived, all of which resolve the cell crossing issues of classical MPM. The particle domain $\Omega_p$ is typically rectangular (in 2D) prior to deformation. In UGIMP, these domains are assumed to be unchanged throughout the deformation, whereas in CPGIMP the area can deform while keeping its rectangular shape (i.e., stretched). On the other
hand, CPDI and CPDI2 convect the initially rectangular particle domains into parallelograms or general quadrilaterals, respectively. The concept is similarly applied to general 3D shapes, and the particular details of the different functional expressions for $\chi(x_p)$ are well discussed in Sadeghirad et al. and are summarized in Fig. 5.2.

In this work, we use a UGIMP interpolator. We note that UGIMP suffers from the so-called “numerical fracture”, which occurs when material points become separated by more than 1 cell and can no longer “communicate” through the background grid. This numerical anomaly is similar to the case of two SPH particles being separated by a distance larger than their radii of influence. The effects of this numerical fracture can be severe when modeling a material that can sustain very large deformations without fracturing (such as polymers), but is minimal for brittle materials simulated at high-enough resolutions. For the material and resolution used in this work, a separation of more than one numerical cell would only occur in the material points representing heavily fragmented blocks that are ejected from the impact crater, as will be demonstrated in Section 5.4.1 and the use of UGIMP is therefore acceptable.

In this work, we use the Uintah framework implementation of the MPM algorithm. Uintah is a highly parallel and adaptive computational tool equipped to tackle large-scale multi-physics problems spanning a wide range of length and time scales.

http://www.sci.utah.edu/download/uintah/
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Figure 5.2: The different types of MPM shape functions derived as particular cases of the Generalized Interpolation Material Point method by Bardenhagen et al. (2004). The advantages and drawbacks of each shape function are also shown. In this work, the uGIMP formulation is used. This figure is based on a similar tree graph from the NairnMPM documentation.
5.2.2 The Tonge-Ramesh material model for geo-

materials

The internal strength of a rocky asteroid has a great effect on its collisional evo-
lution. Following an impact onto brittle materials (such as rocks), stress waves travel
through the target and probe the internal structure causing the nucleation of cracks
and propagation of pre-existing flaws. Several key mechanisms should be considered
in this regime, including: a) the thermodynamic response, described by both the
elastic response and an appropriate equation of state; b) the fragmentation response,
described by the damage kinetics and interaction and growth of cracks; and c) the
motion of fragmented material, described by granular flow, pore compaction, and
tensile fragmentation.

The Tonge-Ramesh material model [195], which is now available in the public
domain and has an implementation in the Uintah framework, is a mechanism-based
material model for the high-strain-rate response of brittle materials that incorporates
self-consistent dynamically interacting crack distributions, granular flow, and pore
compaction. The material model was first used at the larger asteroid scales to demon-
strate the impact-induced lineament formation and porosity growth on Eros [178].

In this model, a brittle material is considered as a continuum with a spatial
distribution of flaws that are smaller than the numerical resolution of the discretized
body. The number density of flaws in a given representative volume is described by a
scalar damage parameter, $D$, which characterizes the deterioration in elastic moduli. The material is considered granular once a critical damage level is reached, at which point it follows a granular flow yield surface.

The flaw distribution is discretized into $N$ “bins” of similar flaw families, where the $k$-th family with number density $\omega_k$ represents flaws with an initial size of $s_k$ from which cracks of length $l_k$ may be nucleated due to the imposed loading conditions. As such, the damage within a given material point is defined as:

$$D = \sum_{k=1}^{N_{\text{bins}}} \omega_k (s_k + l_k)^3$$  \hspace{1cm} (5.9)

In this work, we use a bounded Pareto distribution for the flaw density whose probability density function (PDF) is given by:

$$g(s) = \frac{\gamma s_\text{min}^{-\gamma+1} s^{-\gamma}}{1 - \left(\frac{s_\text{min}}{s_\text{max}}\right)}$$  \hspace{1cm} (5.10)

where $s$ is the flaw size, and $s_\text{min}$ and $s_\text{max}$ are the minimum and maximum resolved flaws within each computational cell, respectively. The microstructure is then simulated by generating the local flaw distribution at every particle, following a discretization algorithm that takes the computational discretization and the global flaw distribution as an input and computes a specific realization of the distribution of flaws within the sample volume. The discretization procedure is described in detail in Tonge et al. \[195].
A self-consistent approach is used at each timestep to calculate the stress intensity factor resulting from the load as well as the crack environment. The crack growth rate ($\dot{l}$) is expressed as a function of the mode-I stress intensity factor $K_I$ at the crack tip as:

$$\dot{l} = \frac{C_R}{\alpha_c} \left( \frac{K_I - K_{IC}}{K_I - 0.5K_{IC}} \right)^{\gamma_c},$$

where $K_{IC}$ is the critical stress intensity factor and $C_R$ is the Rayleigh wave speed in the undamaged material. $\alpha_c$ and $\gamma_c$ are non-dimensional parameters that determine the maximum crack speed, and how fast cracks approach that limiting speed with increasing $K_I$, respectively [196]. Note that $K_I$ must exceed $K_{IC}$ for any crack growth to occur. It follows that rate effects [141, 197] become an inherent result of the interaction and growth of microcracks that have a finite propagation speed. In the Tonge-Ramesh material model, the nature of the rate sensitivity comes from a competition between the stress required to drive the activated cracks faster and the activation of the next set of available flaws. This interplay between local sampling, strain rate sensitivity, and specimen size is resolved for each flaw family at each material point. When the modeled geometry is larger, more realizations of the bigger flaws will be present in the local flaw distribution (compared to a smaller geometry of the same material), and lower stresses would be required (at the same strain rate) to initiate damage in the material by the initial activation of the largest flaw. The size dependence of strength and the transition strain rate then becomes a consequence of
the different flaw distributions in the material points of different sizes.

As cracks propagate and damage evolves, the material reaches a critical damage level at which most microcracks have intersected and created many small fragments of material. Thereafter, the material is considered to be granular, and granular flow is activated. The current model considers a critical level of damage of $D = 0.125$ for the onset of granular flow, corresponding to when the flaws have grown so that their size is on average equal to half of the average spacing between flaws. Damage can keep evolving until the crack sizes become equal to the average spacing between flaws, at which point no further cracking of the grains is possible. Extensions of this crack coalescence model have been developed by Huq et al. [198], but we do not use them in this work.

Once granular flow begins, we use a Drucker-Prager yield surface for granular plasticity defined as:

$$f(\sigma) = \sqrt{s : s} + A_{DP} \left( \frac{\text{tr}(\sigma)}{\sqrt{3}} - B_{DP} \right)$$  \hspace{1cm} (5.12)

where $s$ is the deviatoric part of the Cauchy stress tensor $\sigma$, with $s = \sigma - \text{tr}(\sigma)/3 I$ and $I$ the identity tensor. $A_{DP}$ is a positive parameter that controls the influence of the pressure on the yield limit and is related to the angle of friction used in a Mohr-Coulomb yield surface, while $B_{DP}$ is the gravitational overburden pressure.

Porosity evolution is handled through a $P - \alpha$ porosity model, which relates the
pressure $P$ to the distention $\alpha$ and defines an additional yield surface as:

$$f_p(P, \alpha) = \begin{cases} 
\frac{P}{P_c - P_0} - \frac{P_0}{P_c - P_0} \exp \left( - \frac{P_c - P_0}{2P_0(\alpha - 1)} (\alpha - \alpha_0) \right) & P < P_0 \\
(\alpha - 1) - (\alpha_0 - 1)\alpha^2 \left( \frac{P_c - P_0}{P_c - P_0} \right)^2 & P \leq P < P_c \\
\alpha - 1 & P > P_c 
\end{cases} , \quad (5.13)$$

where $\alpha = \rho_s / \rho$ is the distention defined as the ratio of the solid’s density $\rho_s$ to the total density $\rho$, and $J = \alpha \rho_0 / \rho_s = \rho_0 / \rho$ is the volume change ratio. $P_0$ and $\alpha_0$ are reference pressures and distentions respectively, and $P_c$ is the consolidation pressure. For this work, these parameters are fit to experimental crush curves for pumice [199].

### 5.2.3 Equation of State

The pressure at a material point is obtained from the equation of state (EOS). Here, we implement a Tillotson EOS [200], which can be regarded as a generalized form of the Mie-Grüneisen EOS that was used in the original Tonge-Ramesh model. In the Tillotson EOS, the equivalent Grüneisen parameter ($\Gamma$) is a function of both density and specific internal energy, as opposed to being only a function of density in the classical Mie-Grüneisen EOS. We only consider the “solid” part of the Tillotson EOS and do not account for any phase transformations into liquid or vapor states, a limitation reasonable for the range of impact speeds that we consider. As such, the pressure of an undamaged solid is given by:
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\[ P(\mu, \eta, e) = \left[ a + \frac{b}{e/(e_0/\eta^2) + 1} \right] \rho_0 \eta + A \mu + B \mu^2, \quad (5.14) \]

where \( a, b, A, B, e_0 \) are material-dependent Tillotson parameters, \( \eta = J^{-1} \), and \( \mu = \eta - 1 \). The specific internal energy, \( e \), contains contributions from the solid at zero-Kelvin (or a “cold” reference state) in addition to a thermal contribution. We use the material parameters outlined in \cite{201}, which are based on lunar gabbroic anorthosite \cite{202} parameters after substituting the basalt reference density and bulk modulus as reported by Nakamura et al. \cite{203}. Damage evolution leads to a deterioration of the elastic modulii, following the relations derived in Tonge et al. \cite{195}. The effect of damage on the volumetric response is then accounted for by scaling the computed pressure for the undamaged material by the ratio of the damaged bulk modulus to the undamaged bulk modulus.

5.3 Experimental Validation of the Material Model

5.3.1 Background

A number of numerical codes used in similar planetary-scale impact applications \cite{174, 204, 205} incorporate the tensile brittle failure of Grady et al. \cite{206} as initially
introduced into SPH by Benz et al. \cite{207,208}. In such models, the active number, \( n \), of flaws is assumed \textit{ad hoc} to be a function of strain \( \epsilon \) through a power law: \( n(\epsilon) = k\epsilon^m \), where \( k \) and \( m \) are so-called material-dependent Weibull parameters.

\cite{209} estimated a value of \( m = 2.9 \) for basalt, based on pulsed electron beam experiments. \cite{210} derived a relationship between the rate-dependent tensile strength and the \( m \) parameter. Using the strain rate sensitivity reported in \cite{211} \( (\sigma_\epsilon \sim \dot{\epsilon}^{1/3} - \dot{\epsilon}^{1/4}) \), they calculated \( m = 6 - 9 \). Their hydrocode simulations of impact experiments \cite{203} using \( m = 9.5 \) produced fits to fragment mass distributions. Benz et al. (1994) \cite{207} performed numerical simulations of the same Nakamura et al. (1999) \cite{203} experiments using \( m = 8.5 \) and reproduced comparable masses and velocities for the largest fragment, but underestimated the masses of the smaller fragments. Note that such a process of varying \( m \) to match fragment statistics from experiments calibrates the simulations as opposed to specifically validating the material model, since different computational schemes (with the same material model) will predict different degrees of fracture and fragmentation (see the Sandia Fracture Challenge: \cite{212}). \cite{23} reports a value of \( m = 9.5 \), derived indirectly from experimental measurements of tensile strength as a function of strain rate \cite{213}.

However, Nakamura et al. (2007) \cite{214} performed a series of dynamic tensile experiments on the same basalt of the documented high-velocity impact experiments \cite{203}, following the method proposed by Weibull et al. \cite{215,216} and demonstrated that the Weibull modulus \( m \) for their basalt should instead be in the range of 15–17.
Further, \[214\] shows that \( m = 17 \) for a loading rate of 0.035 mm/min and \( m = 39 \) for a 14 mm/min loading. However, the approach has been used with fixed \( m \) to simulate dynamic events in planetary science for many years. This demonstrates the importance of distinguishing between a) the calibration and validation of a material model, and b) the “validation” of a simulation approach that incorporates an otherwise unvalidated material model. The fact than an inaccurate material model embedded in an impact simulation can capture some features of an impact experiment does not necessarily validate the combination of model plus computational approach. Of course, lacking primary laboratory data on material behavior, one does the best one can with the available data.

Our material model makes no assumption on the relationship between activated flaws and strain, nor on a direct dependence of strain rate and fracture strength. Instead, flaw activation and growth are based on a sub-scale fracture mechanics approach that relates the effective stress intensity factor (\( K_I \)) to crack growth using Eq. (5.11). The initial flaw distribution itself was calibrated using independent dynamic compression and edge-on ball impact experiments \[195\]. Here, we validate the model using a set of dynamic Brazilian disk experiments published by Ramesh et al. \[140\] and demonstrate the model’s ability to capture dynamic tensile failure over a wide range of strain rates. The material parameters used in our model are shown in Table 5.1
5.3.2 Brazilian Disk Experiments

The Brazilian disk test is an experimental technique to indirectly measure the tensile strength of brittle materials (e.g. [217, 218]) in which a thin circular disk is diametrically compressed until failure. The compression along one direction induces tensile stresses along the perpendicular direction. From the measured dynamic load-to-failure ($F$) for a disk with diameter $D$ and thickness $t$, the tensile strength ($\sigma_t$) is defined as:

$$\sigma_t = \frac{2F}{\pi Dt}.$$  

[140] present dynamic strength measurements for basalt with the dynamic load applied on the sample (10 mm in diameter and 2 mm thick) using a conventional Kolsky bar setup [219]. A copper cushion is placed at the tungsten carbide platen/basalt sample interface to prevent stress concentrations at those surfaces. The experiments were carried out at strain rates ranging between 20–200 s$^{-1}$.

To validate the tensile failure in our material model, we set up the simulations with the same specimen geometry and use the velocity measurements recorded at the input bar as initial conditions. We discretize the sample using a numerical resolution that gives a cell spacing of 0.1 mm with one particle per cell. We assume an inverse power-law distribution of flaws that is characterized by a bounded Pareto distribution with a slope of $-3.0$. This slope gives self-similar scaling in the crack sizes where the
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Figure 5.3: The top row of images were acquired through a high-speed camera during a dynamic Brazilian disk experiment on a basalt sample (diameter is 10 mm). The bottom row shows a similar setup in MPM. The blue color refers to undamaged material, and the red is damaged material. The sample is dynamically compressed (by the incident plate on the left), and a state of local tension develops near the diameter’s center. A crack initiates due to the tensile stresses (middle picture) and propagates towards the platens, leading to failure.
average distance between flaws longer than a specified size scales linearly with the crack length and is consistent with the experimental observations of [220] where flaws followed a power-law spanning many orders of magnitude. The top row in Fig. 5.3 shows images acquired through a high-speed camera during a dynamic Brazilian disk experiment (30 s\(^{-1}\)) on a basalt sample, and the bottom row presents the damage profile in the corresponding MPM simulation. As the incident bar (on the left) compresses the sample, a tensile stress state is developed perpendicular to the loading direction. For a quasi-static test on a homogeneous, isotropic, and elastic material the maximum tensile stress would be at the geometric center of the specimen. In a dynamic test, wave interactions lead to a time-dependent shift in the location of maximum stress. Material heterogeneity also affects the location of the initial macro-scale crack nucleation. Consequently, at \( t = 160 \mu \text{s} \), the first subscale flaws are activated off-center but along the horizontal diameter. These cracks propagate orthogonal to the direction of maximum tensile stresses until fragmenting the disk. The damage profile at 220 \( \mu \text{s} \) is a typical failure pattern in Brazilian disk experiments [221]. The measured strengths are shown in Fig. 5.4 along with similar datasets from [222] and [214] performed on different basalts. We estimate the strain rates of [214] using the reported loading velocity and sample dimensions.

The numerical calculations were repeated for different simulated strain rates, and the computed dynamic tensile strengths are also shown in Fig. 5.4. These results illustrate the capability of the material model to reproduce tensile strengths within...
Figure 5.4: Basalt tensile strength dependence on strain rate. Brazilian disk experiments from Ramesh et al. (2017) are plotted along with Housen et al. (2009) and A. Nakamura et al. (2007). The Nakamura et al. (2007) strain rates are first-order estimates based on the platen loading speed and sample dimensions. Our model shows close agreement with the experimental data, especially at the higher strain rates.

the same range as those seen in the experimental measurements, with a trend that replicates the strain-rate dependence of the strength, using only the flaw distribution as input.
### EOS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 )</td>
<td>2700 kg/m(^3) m/s</td>
</tr>
<tr>
<td>( A )</td>
<td>2.67 ( \times ) 10(^{10}) J/m(^3)</td>
</tr>
<tr>
<td>( B )</td>
<td>2.67 ( \times ) 10(^{10}) J/m(^3)</td>
</tr>
<tr>
<td>( a )</td>
<td>0.50</td>
</tr>
<tr>
<td>( b )</td>
<td>1.50</td>
</tr>
<tr>
<td>( \epsilon_0 )</td>
<td>4.87 ( \times ) 10(^{8}) J/kg</td>
</tr>
</tbody>
</table>

### Small-scale flaw distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Flaw size (( s_{\text{min}} ))</td>
<td>5 ( \mu )m</td>
</tr>
<tr>
<td>Maximum Flaw size (( s_{\text{max}} ))</td>
<td>1 mm</td>
</tr>
<tr>
<td>Distribution Exponent (( \gamma ))</td>
<td>3.0</td>
</tr>
<tr>
<td>Flaw Density (( \eta ))</td>
<td>2 ( \times ) 10(^{12}) m(^{-3})</td>
</tr>
<tr>
<td>Number of flaw families (( N_{\text{bins}} ))</td>
<td>25</td>
</tr>
</tbody>
</table>

### Large-scale flaw distribution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Flaw size (( s_{\text{min}} ))</td>
<td>1.0 cm</td>
</tr>
<tr>
<td>Maximum Flaw size (( s_{\text{max}} ))</td>
<td>80 m</td>
</tr>
<tr>
<td>Distribution Exponent (( \gamma ))</td>
<td>3.0</td>
</tr>
<tr>
<td>Flaw Density (( \eta ))</td>
<td>250 m(^{-3})</td>
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<tr>
<td>Number of flaw families (( N_{\text{bins}} ))</td>
<td>25</td>
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### Micromechanics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fracture Toughness (( K_{IC} ))</td>
<td>1.6 MPa( \sqrt{\text{m}} )</td>
</tr>
<tr>
<td>Maximum Crack Speed</td>
<td>0.2 ( C_r )</td>
</tr>
<tr>
<td>Crack Growth Exponent (( \gamma_c ))</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### Granular Flow

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope of granular flow surface (( A_{DP} ))</td>
<td>0.6</td>
</tr>
<tr>
<td>Damage Cohesive Strength (( B_{DP} ))</td>
<td>0.3 MPa</td>
</tr>
<tr>
<td>Damage for Granular flow (( D_c ))</td>
<td>0.125</td>
</tr>
<tr>
<td>Maximum Damage (( P_{\text{max}} ))</td>
<td>0.2</td>
</tr>
</tbody>
</table>

### Pore Compaction

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference crush pressure (( P_0 ))</td>
<td>75 MPa</td>
</tr>
<tr>
<td>Reference distension (( \alpha_0 ))</td>
<td>1.25</td>
</tr>
<tr>
<td>Consolidation pressure (( P_c ))</td>
<td>200 MPa</td>
</tr>
</tbody>
</table>

**Table 5.1: Material model parameters for Basalt**
5.4 Numerical Simulation of Asteroid Impact

Now that we have demonstrated the material model’s capability to capture strain-rate effects using a micromechanics definition of sub-scale crack growth, we seek to simulate hypervelocity impacts onto km-sized asteroids. We consider the case of a 1.21 km diameter impactor striking head-on a 25 km diameter target with an impact velocity of 5 km/s, with both the impactor and the target modeled as basalt monoliths. These initial conditions and impact configuration are, in essence, identical to that of [177]. The difference is in the material model used (Tonge-Ramesh), and the numerical scheme (MPM) and computational framework in which these are implemented (Uintah). Simulating the same impact allows us to assess the consequences of using a realistic material model that properly captures rate-dependent behavior.

From a process perspective, we separate the problem of impact fragmentation and ejecta evolution into two sub-problems based on their characteristic timescales. In the short-timescale fragmentation phase (up to a few tens of seconds), stress wave interactions dominate the problem, and so we use the MPM implementation of the Tonge-Ramesh material model. Beyond that time, gravitational forces become important and so we formulate a consistent hand-off scheme to transition the MPM results into the $N$-body gravity code, pkdgrav, which will capture the evolution of ejecta for the hours following impact.
5.4.1 Short-timescale fragmentation phase

We choose a background grid resolution that consists of $200 \times 200 \times 200$ m cells with 1 material point per cell. This resolution is equivalent to 3 particles per impactor radius and results in a little over $10^6$ total particles in the system. Given the dimensions of the computational cell, we extend the initial flaw distribution to incorporate flaws as small as 1 cm and as large as 80 m with a flaw number density of 250 flaws/m$^3$. The 1 cm cut-off is chosen to maintain a balance between resolving the sub-scale flaw size distribution and the available computational resources, whereas the 80 m upper limit is chosen to be consistent with the homogenization process at the material point level. The flaw number density for this larger crack size population is obtained using the same Pareto distribution as the simulations of the Brazil disk experiments (Table 5.1).

The direct computation of self-gravity is ignored during the failure phase occurring in the first few seconds following impact since the fragmentation timescale in this hypervelocity collision is orders of magnitudes smaller than the dynamical timescale for gravity (further, the lithostatic pressure is small compared to the shock wave amplitude). Instead, we approximate the gravity effect at these times by setting the overburden strength of the granular material to the gravitational overburden pressure at the center of the asteroid [178, 225].

The fragmentation-phase calculation was carried out for a total simulated time of $\sim 37 \text{s}$. One example of the typical outputs of the simulation is shown in Fig. 5.5 in
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Figure 5.5: Short timescale fragmentation phase simulations performed within an MPM framework (Uintah) of the 1.21 km diameter impactor striking the 25 km diameter target head-on at the upper hemisphere. While these are full 3D simulations, in these snapshots a quarter-slice of the spherical target is removed to visualize its interior. A highly damaged region (in red) develops under the impact site, along with some visible radial lineaments. The entire target develops some level of damage (colored yellow) and granular plasticity is activated at all material points. A damaged “core” of low-velocity particles forms near the target’s center, which remains as the largest remnant in the long-timescale gravity simulations. We select \( t = 30 \text{ s} \) as a cut-off time for the \( N \)-body gravity code hand-off.
terms of snapshots of the damage in the target at different times. Note that sub-scale material heterogeneity breaks the symmetry of the problem. Each material particle is colored according to the degree of damage (Eq. (5.9)) in the particle. Note that granular flow begins in our material model when the damage parameter $D$ reaches a value of 0.125, and the damage in a particle saturates at $D = 0.2$ (colored in red in Fig. 5.5).

Immediately after impact (2 s), a highly damaged region develops just and below the impact site while shock waves propagate deeper into the target. As these waves propagate further within the target (4–6 s), more sub-scale cracks are activated, and by $\sim 6$ s all material points in the target have sustained some level of damage high enough for the onset of granular flow. Notice that the damage profile is not axisymmetric due to the inherent variability in the flaw distributions that is incorporated in the model. The reflected waves from the free surface (6–12 s) do not substantially change the overall damage profile since it is easier to grow preexisting cracks than it is to form new crack networks. By 18 s, the wave interactions from the impact have dissipated. Beginning around 12 s after impact, we observe ejecta coming from the impact site. These particles have speeds greater than 15 m/s, which exceeds the escape velocity on the surface of the body. Note that the “numerical fracture” limitation in the UGIMP implementation of the MPM equations that was discussed in the earlier section applies to these material points. The consequence here would be in overestimating the velocities of these particles as a trade-off to increased computational cost.
if we use $CPDI2$. In this simulation, only a small fraction of material points have been separated by a distance large enough for numerical fracture. These material points correspond to the high-velocity ejecta seen between 24–37 s in Fig. 5.5. We accept this limitation as these particles are unlikely to re-accumulate on the target and would not have a considerable impact on the damage and velocity profiles in the remainder of the target. A different treatment (such as using $CPDI2$) could be warranted when studying the formation of asteroid families or the low velocity ejecta in the context of asteroid mitigation simulations.

After 30s, the entire target has been damaged to some degree, with the bulk of the damage on the side that was impacted. Note that the extent of the high damage (shown in red) is much larger within the body than is visible on the surface. A crater has been formed (not easily seen in the visualization), but note that our simulation was not designed with the resolution to investigate crater dynamics. Ejecta from the crater formation event includes high ejecta velocities, as previously discussed, and essentially all of the surface particles all around the target begin to move away from the surface at velocities that range from 5 to 20 m/s. However, there remains a large core of particles in the target that have been damaged but have very low velocities. That is, the entire target is not disrupted by the impact event. This is a key difference between our results and those of [177]. The damage profile and velocity fields have been fully established and the pressure at all material points is below the lithostatic pressure. This indicates that the dominant physics of the problem has transitioned
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from a wave interaction and stress-dominated problem to one dominated by gravity and rigid-body forces. We therefore choose this time as a cut-off for handing over the simulation data to \texttt{pkdgrav}.

5.4.2 Hand-off from MPM to \texttt{pkdgrav}

Once the pressures within the target become comparable to the gravitational overburden pressure, the system’s evolution and the extent of reaccumulation are entirely determined by the velocity field of the individual fragments. The essential physics occurring in this longer timescale for reaccumulation can be properly captured by \textit{N}-body gravity codes such as \texttt{pkdgrav} \cite{226,227}.

The results from our MPM runs need to be transitioned into a format suitable for \textit{N}-body codes. In \texttt{pkdgrav}, materials are discretized into spheres. This spherical discretization requires having a set of well-formed elements that are non-overlapping or with extremely small overlaps as initial conditions in order to avoid unrealistic repulsive forces to reverse the overlap \cite{228}. Therefore, a consistent hand-off scheme is desirable that will ideally conserve the total momentum and energy of the system and result in a set of non-overlapping spheres at the initial step of the \textit{N}-body simulation. We have developed such a consistent hand-off procedure, and this is described below and illustrated in the lower half of Fig. 5.1.

First, we determine the positions of the material points at the final timestep of the MPM simulation relative to the undeformed grid, consistent with the last step in
the MPM algorithm (Fig. 5.1). We then interpolate the particle mass and momentum onto the background computational grid using the shape functions from MPM (Eqs. (5.1) to (5.3)). Nodal masses and velocities are calculated and interpolated back onto a single equivalent material point located at the center of each computational cell. Finally, the centered material point is transformed into a discrete sphere with a radius equal to half the grid cell length (last sub-plot in Fig. 5.1). The process is repeated for all MPM cells and the final data is written out into a format that can be processed in pkdgrav. This process conserves the total mass and momentum by construction. Small kinetic energy losses may occur in cells where a large number of material points are present, such as in highly compacted regions near the impact site. The order of this kinetic energy loss is equivalent to the energy dissipation in the MPM algorithm, which in our case is less than 1% of the total energy. In this hand-off, the projection of data from the material points to the grid preserves the total linear and angular momentum, but the total kinetic energy on the grid is less than the total kinetic energy in the material-points. The material-point-to-grid interpolation step dissipates kinetic energy, as do all GIMP implementations (which are either energy conserving or momentum conserving). This kinetic energy dissipation should always be quantified prior to running a pkdgrav simulation to ensure that the initial conditions in the gravity phase do not differ considerably from the last step of the MPM computation. The hand-off algorithm could be re-formulated to use a different sampling scheme in which the merging process results in two spherical particles in-
stead of one, hence minimizing energy losses. Such advanced merging processes have been described in [229][230]. In our case, the total energy lost in the hand-off process from MPM to pkdgrav was only 0.39%, and so we do not explore other algorithms for the hand-off. Note that a subsequent hand-off from pkdgrav back to MPM can be constructed in a similar manner and would allow the simulation of re-impact events following a reaccumulation stage.

In addition to the particle position, mass, and velocity, we also store the damage information by averaging the damage in each computational cell. In the long-timescale regime, we only use this damage value to label the “damaged” pkdgrav particles with different colors for visualization purposes.

5.4.3 Long-timescale gravity phase

After the position, mass, velocity, and damage information from the final step of the MPM simulations are handed off to pkdgrav, we set up the second phase of the simulation that explicitly captures the gravitational interaction and evolution of the materials. We here use the so-called “soft-sphere” discrete element method (SSDEM) [231], where the geometry is represented by a set of spherical particles that are allowed to have finite contact interaction times. SSDEM allows for the modeling of contact forces between particles in granular material, including rolling and twisting friction. Colliding spheres are allowed to slightly overlap and a linear spring-dashpot model is used to calculate the normal and tangential forces that oppose the deformation.
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This approach is different from that of [171], in which the “hard-sphere” version of pkdgrav was used [232]. In a hard-sphere implementation, collisions are assumed to occur instantaneously at a single point of contact. The hard-sphere assumption allows for larger timesteps than SSDEM, at the expense of under-resolving the details of the mechanisms occurring during contact. We set up the material parameters to be typical gravel-like [233], which were extracted from avalanche experiments of streambed rocks [234]. At this point, there is no material “strength” in the N-body gravity calculations. That is, gravitational attraction and frictional cohesion, including normal, tangential, rolling, and twisting frictions, are the only forces that tend to aggregate the particles. This “strengthless” approximation is likely to overestimate the evolution of the near-surface particle velocities, and the results of the pkdgrav calculation will be interpreted while being mindful to this limitation. The pkdgrav integration was carried out for a simulated time of 4.5 hours using 120 computing cores running for 11 days. A second-order leapfrog integrator with a timestep of 5 milliseconds is selected, which is limited by the particle velocities and the maximum allowed overlap (1% of the particle radius) in each timestep.
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Figure 5.6: Snapshots of the pkdgrav computations at different simulated times. The colors represent different damage levels, consistent with Fig. 5.5. Note that here $t = 0s$ refers to the beginning of the long-timescale gravity phase. In view of this hybrid MPM/pkdgrav approach, this time is in fact at 30s post-impact, which is the hand-off time from MPM. A violent ejection of surface material is observed during the first hour, followed by a reaccumulation phase. Throughout the entire gravity-timescale simulations, the heavily fractured “core” remains bound by gravity. The target is seen to be moving vertically downwards due to the momentum imparted to it.

The results of the second-phase simulation are presented in Fig. 5.6, with the particles colored according to the degree of damage developed during the first phase of the simulations. In the first $\sim 15$ minutes following impact, an ejection of near-surface material is observed. The majority of this ejecta blanket comes from the impact crater, although some particles as far as the end opposite to the impact site have also been mobilized. In the next hour, the low-speed ejecta begin to reaccumulate on the largest remnant, which remains held together throughout the entire gravity phase. In these runs, we observe a substantial ejecta fallback at the pole opposite to
the impact site due to the particles that were launched at velocities just less than the escape velocity, which is consistent with predictions from scaling relations [236]. By $\sim 2.5$ hours, almost 80% of the initial mass is recovered in the largest remnant. The surface of this largest remnant is covered with the \texttt{pkdgrav} particles labeled in red, which were the most heavily fragmented particles where granular flow was most active in the MPM simulations. By $\sim 3$ hours after impact, most of the reaccumulation events have occurred and the largest remnant is observed to be displacing vertically downwards due to the transfer of linear momentum. Some far-field particles may eventually cluster together to form smaller secondary asteroids, but our calculations did not run for long enough to capture such events. The mass ratio of the largest remnant to the parent body after 4 hours was $M_{LR}/M_{PB} = 0.85$, a much higher value than the 0.5 that is taken as a reference for disruption limit calculations. Note also the distinctive shape of the largest remnant in this non-rotating case (rotating targets are addressed in Appendix B). It is also apparent that the spatial distribution of damaged particles evolves as a result of the reaccumulation phase. The evolution of the spatial distribution of damage in the target is shown in Fig. 5.7 for three specific times: 3s after impact, 30s after impact, and 7230s after impact (2 hours after hand-off).
Figure 5.7: Predicted damage in the target as a function of distance along the diameter, in the direction of the impact. The dashed lines represent data extracted from the short-timescale (MPM) phases, and the solid line is from the long-timescale (pkdgrav) reaccumulation stage.

The predicted damage in the target is presented as a function of distance along the diameter, looking along the direction of impact. Note that the maximum value of damage allowed by the model is 0.2. Three seconds after impact, the region immediately beneath the impactor is fully damaged, but a significant fraction of the target is completely undamaged. At 30s, all particles along the diameter have developed some level of damage, with a highly damaged region reaching a depth of almost 10km. At 2 hours after the hand-off to pkdgrav, the high-velocity particles from under the impact site have been ejected and new surfaces have been exposed. This excavation is manifested in a shift to the left of the curve (notice the damage “dip” has moved
CHAPTER 5. A HYBRID FRAMEWORK FOR SIMULATING HYPERVELOCITY ASTEROID IMPACTS AND GRAVITATIONAL REACCUMULATION

from a depth of 10km to around 5km). In addition, the reaccumulation of mostly damaged particles at the target end opposite to the impact location is manifested as an extension of the original 25km diameter to around 28km. This new damage profile remains largely unchanged thereafter. Such predicted damage profiles may be useful in space mission planning.

5.5 Discussion

Using this new hybrid MPM/pkdgrav approach, we simulated the impact of a 1.21 km diameter basalt asteroid onto a 25 km diameter basalt target. We tracked the material response from material deformation and fragmentation through to gravitational ejection and reaccumulation. In contrast to SPH calculations with similar initial conditions, the 5 km/s impact does not lead to the complete shattering of the target (the SPH calculations of [177] led to fragmentation down to the smallest numerical resolution). Instead, our impact outcome shows a heavily damaged but coherent “core” under the impact site. The key difference between this work and [177] in the fragmentation phase is in the implemented (and validated) constitutive model. Here, the micromechanics definition of damage leads to inherent strain-rate dependence of the strength and deterioration of elastic moduli. Further, a “damaged” material is not generally “strengthless”. Furthermore, an initially non-porous target would develop internal porosity as the material is fragmented. Both the granular flow through
shearing of damaged material, and the crushing of pores due to pressure, constitute mechanisms for energy dissipation. In effect, the damage profile in Fig. 5.5 does not change substantially from 6s to 30s since further cracking of grains is less energetically favorable than the flow of fragments. Our model suggests that asteroids, even initially monolithic and non-porous ones, are “stronger” than is traditionally assumed. Impacts would have to occur at larger specific energies to reach complete disruption.

In the long-timescale gravity calculation, the fragmented “core” acted akin to a gravity well over which ejecta reaccumulated. Substantial ejecta fallback occurred at the pole opposite to the impact site, whereas an excavated surface emerged from beneath the impact location. The internal damage distribution in the asteroid after the ejecta reaccumulation phase shows the signature of both the impact and the fallback of damaged ejecta. The results indicate that such asteroids could have an interior that is a heavily fragmented shard of an initially intact parent body following a first impact. Additionally, they could have substantial porosity while not necessarily being a rubble pile of gravitationally bound reaccumulated fragments. Note that subsequent impacts may further break down, and possibly disrupt, the largest remnant.

The impact simulation examined in this work and in [177] correspond to a specific impact energy of $1.13 \times 10^8$ erg/g. In the [177] simulation, the largest remnant after gravitational accumulation had a mass ratio relative to the parent body of $M_{LR}/M_{PB} = 0.5$, which means that this impact is at the specific energy for disruption
(\(Q_D^*\)). A similar disruption threshold measure was previously obtained by [201] for impacts onto basalt with the same initial conditions. In contrast, the mass ratio of the largest remnant in our simulations was much larger, with \(M_{LR}/M_{PB} = 0.85\), meaning that the specific impact energy is smaller than the disruption threshold that would be predicted by our model. This indicates that intact monolithic parent asteroids may have a higher disruption threshold, \(Q_D^*\), than previously expected and require higher impact energies to be completely shattered. The limits of this observation for smaller targets remains to be explored, and we plan to use our model in a later work to explore the disruption threshold over a range of target sizes. These conclusions are similar to those of [205] where a significant increase in the catastrophic disruption threshold was noted due to energy dissipation through friction and pore crushing in granular material.

In contrast to [177] and [205], this work uses the soft-sphere implementation of pkdgrav, which has a better handle on individual fragment geometries and impact forces as opposed to the hard-sphere method where fragment collisions are treated as bounces or mergers resulting in a new spherical particle with a combined mass and equivalent diameter. The gravity response is therefore expected to be different, especially in terms of the formation of gravitationally bound aggregates. For the impact conditions considered in this work, we do not observe the formation of any asteroid families. However, the far-field ejecta could accumulate to form small secondary aggregates at much later times beyond those reached in our simulations.
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In Fig. 5.6 the target can be seen moving vertically downwards as a result of the momentum imparted by the impactor. The momentum enhancement factor is calculated to be 1.69, which is much smaller than previous estimates of impacts onto competent rocks [236, 237]. The low momentum enhancement in this case is primarily a result of the large amount of ejecta reaccumulation occurring in the gravity phase.

The consistent hand-off employed in our hybrid approach allows us to identify individual pkdgrav particles and map them back to their equivalent MPM particle. For instance, in this work the value of the damage parameter (Eq. (5.9)) from MPM is stored as an additional label in pkdgrav that we use to color-code the SSDEM spheres. Additionally, internal MPM state variables from the short-timescale fragmentation phase can be preserved throughout the pkdgrav simulations. A hand-off from pkdgrav to MPM can then be similarly performed for simulating subsequent impacts. Such a capability would be beneficial for detailed collisional history studies.

One caveat in the current work is that the target in the long-timescale gravity stage is modeled as an aggregate of spheres with no tensile strength. This is an oversimplification of the material state at the end of the MPM simulation where tensile strength is still present. In previous work [171, 174, 175, 177], such an assumption was justified given that all particles, down to the numerical resolution, were fragmented at the end of the SPH calculations. However, we note here that not all of the MPM particles were fully damaged by the time for hand-off. This simplification in the pkdgrav stage would likely lead to an overestimated velocity for the surface particles, partially
causing the violent ejection seen during the first timesteps of Fig. 5.6. [238] presented an implementation of strength in pkdgrav by bonding neighboring particles’ centers of mass with “springs” that result in a restoring force opposing the distention of the bonds. This implementation comes with an additional computational expense since the timesteps would be limited by the oscillation half-period of the spring [231]. One could use such a spring-based bonding approximation, and use the damage parameter from the MPM simulation as an indicator for the deterioration in the initial spring stiffnesses in pkdgrav. In future runs, we will be looking at the effects of strength in the long-timescale gravity regime [239].

Finally, we note that we do not experience any difficulties in MPM with capturing the low-velocity ejecta. The standard SPH formulation is often reported to induce small-scale sub-sonic velocity noise as a consequence of the gradient estimate error in SPH [240]. This numerical noise was also mentioned as an encountered difficulty in the work of Schwartz et al. (2016) [228] exploring Didymoon-scale impacts in the context of the DART mission. Given the scale of the target, its low escape speed, and the high impact velocity, the computation of the ejecta velocities become close to the numerical noise of the SPH simulations. Schwartz et al. (2016) [228] also notes that the results were affected by the wave reflections at the boundary of their discretized domain. In MPM, however, the use of a background grid for gradient calculations eliminates these gradient estimate errors. The coupled approach presented here may therefore be used to model the fate of low-speed reaccumulation in situations such as
CHAPTER 5. A HYBRID FRAMEWORK FOR SIMULATING HYPERVELOCITY ASTEROID IMPACTS AND GRAVITATIONAL REACCUMULATION

on Didymo on in the context of the DART mission.

5.6 Summary

We have presented a new coupled MPM/pkdgrav hybrid approach to simulate the collisional evolution of rocky asteroids from the early fragmentation stages to the later times of gravitational reaccumulation. A modified version of the Tonge-Ramesh material model that includes the Tillotson equation of state was implemented in an MPM framework to capture the material response for the first tens of seconds following impact. A consistent hand-off scheme was formulated to transition to the N-body gravity code pkdgrav for integrating the long-timescale gravity effects.

The multi-physics material model is centered around the growth mechanism of an initial distribution of subscale flaws. Rate effects in the model are a natural outcome of the limited crack growth speed, which is explicitly computed based on the local stress state. In addition, porosity growth, pore compaction, and granular flow of highly damaged materials are captured at the material-point level. We validated the model’s predictive capability by comparing the dynamic tensile strength with high-strain-rate Brazilian disk experiments performed on basalt samples.

As an application of the hybrid technique, we considered an asteroid impact with initial conditions similar to [177]. In contrast to previous results, the impact event did not lead to the complete disruption of the target. The collision imparted substantial
damage onto the target, with most of the damage localized under the impact site, resulting in a heavily fractured but not fully damaged “core”. The material points were then converted into soft spheres and handed over to pkdgrav in a self-consistent manner to calculate the gravitational interaction of the ejected material. We observed substantial ejecta fallback onto the largest remnant of the parent body, with a recovered mass of the largest remnant being 0.85 that of the parent body, indicating that the disruption thresholds for such targets may be higher than previously thought.

The framework presented in this study can be applied to a variety of asteroid impact and deflection scenarios. In the future, we plan to incorporate a tensile strength model in the \( N \)-body simulations and use this newly developed framework to explore the disruption thresholds for a range of target sizes. The model may also be used for studying impacts onto initially rotating targets and their effects on the collisional evolution of asteroids and asteroid families.
Chapter 6

Summary and future work

6.1  Concluding remarks

This thesis covers vastly different mechanisms related to the time-dependent failure of materials subjected to the extreme environments related to our solar system, namely:

1. In-place fragmentation through thermal fatigue (cm, $10^3$–$10^6$ years),

2. Damage and ejecta generation through large-scale impacts onto asteroids (km, 10–30 seconds)

3. Gravitational evolution of fragmented material (km, 1–20 hours).

A common theme throughout this thesis is in developing predictive mechanisms-based models to help us understand applications that are beyond what can be replicated
in a traditional lab environment. This bridging is in terms of both the timescales (thermal cycling, shock wave and gravity timescales) and lengthscales (from cracks in rocks through impacts onto km-sized targets).

In Chapter 2, an efficient multiscale thermomechanical model was formulated to study the fragmentation of regolith through the thermally driven fatigue growth of cracks. Using the insights obtained from the numerical model, a simple scaling analysis was performed and a predictive analytical model was developed. The results revealed a characteristic lengthscale related to thermal fatigue, which may be reflected in size-frequency distributions of regolith in the asteroids most susceptible to thermal fragmentation.

In Chapter 3, experimental assessments of an ordinary chondrite meteorite sample were performed to determine its thermomechanical properties. The local mechanical properties of individual grains were probed using a nanoindentation technique. A finite-element mesh that closely traces the grain contours and captures the geometric features of the microstructure was generated. A linear elastic thermal analysis was performed to calculate the strain and stress fields and the results were compared to experimental measurements. The results indicated that the inclusions, primarily those that are iron-rich, act as stress concentration sites with the maximum stresses occurring at the inclusion/matrix interfaces. This suggests that cracks would preferentially grow along those interfaces, which was indeed observed in thermal cycling experiments.
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In Chapter 4, the effect of the inclusion/matrix interfaces on the efficiency of thermal fragmentation was closely examined. Using a semi-analytical thermomechanical model to calculate the crack-tip driving forces in different idealized micro-structures revealed that interfaces containing an increased amount of porosity (such as pre-existing cracks) or a diffuse composition (such as iron melts) greatly reduce the stress concentrations at the crack tips. SEM observations of the inclusion interfaces in the studied samples supported the conclusions derived from the model.

Finally, attention was shifted from the quasi-static crack propagation in thermal fatigue to the dynamic processes that occur during a hypervelocity impact onto kilometer-sized asteroids. A hybrid numerical framework was formulated in Chapter 5 to consistently bridge across two different mechanisms of interest: from stress-waves-dominated material response to gravitational interaction of individual particles. In the stress-waves-dominated regime, a material point method was used along with a modified version of the multi-mechanism Tonge-Ramesh material model. Then, a consistent hand-off was carried out to an N-body gravity code (pkdgrav) to resolve the individual gravitational forces on each discretized material block. The framework was applied to simulate a hypervelocity asteroid impact onto a 25-km diameter target. The simulation resulted in a largest remnant that is not fully disrupted down to the smallest numerical resolution, in contrast to previous published work on this particular scenario, implying that asteroid impacts would have to occur at larger specific energies to reach complete disruption.
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6.2 Suggestions for future work

The solar system is rich with new puzzles that challenge our understanding of the applicable material failure processes. This work was aimed at developing models for studying the processes that govern the surface evolution of airless bodies in the solar system, spanning a large range of timescales and lengthscales. The different numerical models developed in this thesis may find relevant applications related to planned and ongoing space missions. A few such areas of interests are briefly outlined.

Thermal fatigue growth of crack networks:

The efficiency of thermal fatigue in the generation of regolith in the solar system was shown to be of significant importance. The development of more sophisticated 3D models that capture the interaction, growth, and coalescence of a large network of cracks is therefore warranted. The analyses performed to date relied on the fatigue growth of a single crack driven by a one-dimensional temperature profile. It would be important, in the context of space missions such as Hayabusa 2 and OSIRIS-REx, to develop an advanced thermal model that can account for local topography of asteroid surfaces, generating a more accurate higher-order representation of the temperature profile that captures the effects of elevations, shadowing, irregular illumination, and orbital eccentricity will be captured. Then, the generation of small fragments through the coalescence of cracks, or through scabbing of surface materials, would provide a more accurate representation of the shapes and sizes of fragments that are expected
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Figure 6.1: Artistic representation of an asteroid surface where thermal fatigue was an active mechanism driving the surface evolution. Small rocks are preferentially depleted, while larger boulders have long through cracks and are surrounded by an apron of scabbed rocks. Image courtesy of Seth Izen.

to be found on asteroid surfaces targeted by these missions. One would then be able to identify the likely statistical distribution of regolith sizes as a result of thermal fatigue, and the temporal orders of magnitude required to fragment rocks on the surface of actual airless bodies in the solar system.

Thermal fatigue evolution as a function of rock mineralogy and petrology:

It was shown in Chapter 2 that the temperature gradient effects diminish for rocks much smaller than the thermal skin depth, and thermal fragmentation is more likely to be driven by the local composition and mineralogy, such as the thermal expansion
CHAPTER 6. SUMMARY AND FUTURE WORK

mismatch of the individual grain. For these scales, the crack path would depend on the local distribution of the individual grains in the meteorites (Chapter 3), and the interface strength between the inclusions and matrix (Section 4.3.1). A close investigation of the effects of various microstructures on the thermomechanical evolution of regolith would benefit our understanding of thermal fatigue fragmentation at those scales. Developing an experimentally informed model to explore the possibility of generating monomineralic grains through thermal fatigue fragmentation could help in distinguishing between a rock primarily formed through thermal fragmentation as opposed to impact. These key signatures for thermal fatigue may then be manifested in terms of remotely sensed measurements or derived properties (infrared absorption bands, surface roughness, thermal inertia, and so on...).

Asteroid impacts, disruption thresholds, and rotating targets:

Using the hybrid numerical framework developed in Chapter 5, we demonstrated that impacts onto asteroids could result in remnants larger than previously thought, leaving behind a solid core covered with a debris of ejecta. There will be value in applying this framework to perform a detailed parametric analysis and explore the outcomes of these impacts. Impactors with different sizes, angles of incidence, or speeds would produce different fragmentation profiles and varying largest-remnant mass, shape, and orbital characteristics (after reaccumulation). Such an analysis will likely provide a deeper insight into the interiors of many asteroids and asteroid
Figure 6.2: Artistic representation of asteroid regolith of different diameters (40cm, 10cm, 5cm, and 2cm; from left to right), illustrating the possible characteristic signatures of thermal fatigue and impact-generated regolith. The large 40 cm boulder shows prominent evidence of impacts (zap pits), while the intermediate 10cm boulder has a combination of both impacts and through cracks propagated by thermal fatigue. The 5 cm rock contains a through crack that was driven by thermal fatigue, whereas the 2cm rock shows less evidence of any thermal cycling. Advanced thermomechanical models could help in identifying the possible signatures and expected rock sizes as a consequence of each mechanism. Image courtesy of Seth Izen.
families. The impact conditions that lead to the loss of half of the mass of the initial target (commonly referred to as the disruption threshold), would likely need to be reevaluated. Our numerical simulations suggest the final largest remnant may either be a fully shattered object that is held together by gravity after reaccumulation (a true rubble pile), or a solid and coherent object that was left behind as a core from a larger parent, as in the results of Chapter 5. Disruption threshold limits should therefore reflect more information on the state of the largest remnant (fractured, shattered, reaccumulated) than just the mass ratio. Scaling theorems could also be tested against such numerical simulations.

In addition, asteroids in the solar system have an initial period of rotation that is on the order of a few hours (depending on the size of the asteroid). Most hydrocode numerical simulations to date ignored the spin of the target as an initial condition. There will be value in exploring the possible effects of spin on target strength (for example, from the reduced loading due to centrifugal forces), as well as the transfer of angular momentum in an impacts onto a rotating target. Impactors coming in at different directions relative to the directions of spin would likely produce distinctly different reaccumulation profiles and angular momentum enhancements. The hybrid approach may also be used to estimate spin-rate limits for asteroids that are not “rubble piles”. Preliminary results on impact cases involving rotating targets and impact energies near the disruption threshold are presented in Appendix B.
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**Multimechanism framework for regolith size-frequency evolution:**

While regolith generation in the solar system may have a single dominant actor driving its evolution, it is more likely that multiple mechanisms provide different levels of contributions. The combination of the thermal fatigue fragmentation model with a micrometeorite fragmentation model would be an interesting area to explore. A micrometeoritic impact would nucleate and grow a network of cracks, which can then continue to propagate by thermal fatigue, which would then increase the likelihood of fragmentation by a subsequent micrometeorite impact. The coupling of these two mechanisms at different timescales would provide a better insight onto the conditions likely experienced by rocks on airless bodies in the solar system.
Appendix A

The role of inclusions in the dynamic fragmentation of brittle geologic materials

Introduction

In Chapter 4, we explored the role of inclusions and interfaces in relation to the thermal fatigue driving forces. Some possible links between microstructure and expected fragment sizes by thermal fatigue were suggested, based on the content and spacing of the distinct grains in meteorites. A similar link between microstructure and fragment shapes and sizes has also been observed in the dynamic fragmentation

†This appendix is largely based on a collaborative work published by Hogan et al. [241]
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of brittle materials.

In this study we investigate the dynamic compressive failure and fragmentation of basalt, paying particular attention to the role each constituent mineral phase has in these processes. Our results indicate the existence of two fragmentation mechanisms: I. a mechanism that creates small fragments that is associated with the spacing of critically activated defects. These fragments are primarily comprised of pyroxene (which has the lowest fracture toughness in this material). II. a mechanism related to larger fragments that is associated with the structural failure of the sample. These fragments are primarily polyphase and polygrain in composition.

Then, we investigate the strength of fragmented basalt material for different initial fragment size distributions: 1. between 10 and 100 microns, 2. between 200 and 800 microns, and 3. between 300 and 1,800 microns. The porosity of each of the three samples was maintained between 30 and 35%. Understanding the composition of the fragments beforehand allows us to better interpret our experimental results, which indicate that the strengths of the fragmented material increased with decreasing fragment sizes, from 4 to 25 MPa, and then to 175 MPa. An increase in strength with smaller fragment sizes is expected because of the associated increase in frictional dissipation, and decrease in the relative contribution of compaction and fracturing mechanisms. However, we do note that fragments less than 100 microns fail as a result of the activation of a different critical defect type than in the bulk material and for fragments larger than 100 microns, where olivine grains are the key contributors.
to fracture. Altogether, these results highlight the influence of the composition and defect population of planetary materials on the associated length scales that arise from dynamic failure and fragmentation.

Understanding the dynamic behavior of planetary materials is central in interpreting planetary impacts [107] and asteroid collisions [175]. The initial fragmentation byproducts of these hypervelocity events are dependent on the material microstructure (e.g., constituent phases and number of flaws) and the complex multi-axial stress-loading history that manifests during impact. Additional fragmentation may occur from, for example, subsequent high-energy impact events (e.g., meteoroid bombardment) and through other mechanisms, such as thermal fatigue. An improved understanding of the fragmentation of planetary materials, as well as the behavior and the evolution of these fragmented materials, will lead to more insight into the processes that govern large-scale impacts.

In this study, we investigate the dynamic fragmentation of intact and granular basalt. We choose basalt because it is the most common type of rock in the inner solar system, as well as in the main asteroid belt. We seek to understand the effect of the microstructure (e.g., constituent phases, defects) on the dynamic compressive failure and fragmentation of the intact material. Once key defects are identified, they are quantified using image analysis techniques and are then linked to fragment size distributions. Lastly, we investigate the strength of the granular basalt for different initial fragment size distributions, and discuss the implications in terms of the im-
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important length and time scales arising from failure and fragmentation of planetary materials.

A.1 Methods and Materials

Experiments on intact and granular basalt are performed at strain rates between 200 and 500 s\(^{-1}\) using a Kolsky bar apparatus. Impact simulations by Ernst et al. [242] showed that these are common strain rates in the vicinity of the impact site for a 5 km/s impact by a 1 km-sized object. Time-resolved failure of the intact sample is imaged using a Kirana high-speed camera filming at 2 million frames per seconds with a 100 ns exposure time. Digital image correlation techniques are applied to the video images to measure the components of strain in the image. A Kolsky bar setup is used as described in Kimberly et al. [141]. Cuboidal samples are used for the intact material geometry, approximately 3.5 mm × 4 mm × 5 mm in dimension. Fragments were collected after the experiments for examination, and subsequent dynamic experiments were performed on granular material samples consisting of three selected fragment size populations: 1. between 10 and 100 µm (mean of 26 ± 11 µm), 2. between 200 and 800 µm (mean of 408±145 µm), and 3. between 300 and 1,800 µm (mean of 988±422 µm). The granular fragments are encapsulated in heat shrink tubing with an outer diameter of 7.6 mm and an inner diameter of 7.4 mm. As the granular material undergoes deformation, the tubing applies a confinement pressure (\(\sigma_{\text{conf}}\): Pa) given
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by:

\[ \sigma_{conf} = \frac{\sigma_y}{2} (\lambda^2 - 1) \]  \hspace{1cm} (A.1)

where \( \sigma_y \) is the yield strength of the confinement material (Pa), and \( \lambda \) is the ratio of the outer and inner diameter of the confinement tube. Here, \( \sigma_y = 10.3 \text{ MPa} \), and \( \lambda = 1.03 \), and, thus, \( \sigma_{conf} = 314 \text{ kPa} \). Two hardened steel cylindrical plugs are inserted to contain the material. The material for the plugs matches that of the Kolsky bar, and the dimensions of the plugs are approximately 7 mm in diameter and 7 mm in length. The gage section of material being tested has diameter of 7 mm and its length is varied to maintain similar porosities depending on the initial average fragment size. Two copper pulse shapers that were 0.7 mm thick were used with a projectile length of 25.4 mm.

The basalt was purchased from Coverall Stone, WA. A polarized thin section image is shown in Fig. A1a and an optical microscope image is shown in Fig. A1b. Olivine (< 500 \( \mu \text{m} \) and angular), pyroxene (< 150 \( \mu \text{m} \) and darker in shade), and feldspar (< 100 \( \mu \text{m} \), light and needle-like structures) are highlighted in the images. Our basalt has a Young’s modulus of 70 GPa and a density of 2,870 kg/m\(^3\). Past Kolsky bar experiments involving this basalt have shown that it has a uniaxial compressive strength of 385 MPa at a strain rate of \( 10^{-3} \text{ s}^{-1} \) and \( \sim 570 \text{ MPa at } 950 \text{s}^{-1} \) [243]. It has a tensile strength, as measured using the Brazilian disk technique, of 60 MPa at 60s\(^{-1}\) and 175 MPa at 22s\(^{-1}\) [243].
A.2 Experimental Results

A.2.1 Dynamic Failure of Intact Basalt

Initially, we examine the dynamic compressive failure of the intact basalt sample (Fig. A2). Shown on the left of Fig. A2 is the stress-time history of a Kolsky bar experiment with a peak stress of 500 MPa. The nominal stress-rate is shown using the dashed-line and is computed by taking the slope of the stress-time curve between 10%
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Figure A1: (a) Polarized image of basalt thin section and (b) non-polarized surface image of basalt microstructure, both with constituent phases labelled. (c) Optical microscope image of basalt fragments and (d) converted monochrome image of basalt fragments with major axis size (L: m), area (A: m²), and perimeter (P: m) labelled.
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and 90% of the peak stress. The stress-rate for this experiment is 30 MPa/µs. The strain rate may be approximated by dividing the stress rate by the Young’s modulus (E: Pa), resulting in a strain rate of approximately 430 s⁻¹. Also plotted with the stress-time curve are measurements of the axial (horizontal in the image; \( \epsilon_{xx} \)) and lateral strains (vertical in the image; \( \epsilon_{yy} \)). These are obtained using digital image correlation applied to the high-speed video images on the right. Also note that the times \( t_1 \) to \( t_6 \) are matched with the stress-time history and are plotted as blue dots on the left.

The dark regions on the surface of the sample in the video images are olivine grains (highlighted in Fig. A2 image at time \( t_1 \)). We first observe fracture located at the bottom right surface of the sample just prior to peak load (time \( t_2 \)). The fracture appears to intersect one of the dark olivine grains on the surface. Correspondingly at \( t_2 \), the strains computed using digital image correlation are \( \epsilon_{xx} = -0.9\% \) and \( \epsilon_{yy} = 0.4\% \). After peak stress (\( t_3 \)), additional fractures on the surface are visible and, again, these intersect some of the olivine grains. As a result of the fractures, the stress in the sample collapses and the strains increase to \( \epsilon_{xx} = -1.1\% \) and \( \epsilon_{yy} = 0.9\% \) as a result of dilation. The coalescence of the fractures between the olivine grains is more clearly observed at \( t_4 \), where some additional fractures have also grown. The speed at which these cracks propagate is measured as 650 ± 100 m/s. Note that not all of the cracks grow horizontally across the sample (i.e., in the direction of maximum compression). Pore collapse, material heterogeneity, local buckling and
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Figure A2: Stress-time history of dynamic uniaxial compression of basalt (black curve) with time-resolved high-speed video images showing meso-scale failure mechanisms. The dashed line with the red underneath in the stress-time plot is the linear fit of 10 and 90% of the peak stress and this corresponds to the stress-rate of 30 MPa/µs. The green curve is the associated axial strain ($\epsilon_{xx}$), while the yellow curve is the lateral strain ($\epsilon_{yy}$).

Angular olivine defect shapes (see Fig. A1b) may contribute to the non-horizontal cracking. The strains in the sample at $t_4$ now increase at a faster rate, and are $\epsilon_{xx} = -1.3\%$ and $\epsilon_{yy} = 2.1\%$. Cracks continue to grow and coalesce at $t_5$ and $t_6$, and the stress in the sample continues to collapse. By the time the stress has completely collapsed in the sample (i.e., around 40 µs), the strains are approximately $\epsilon_{xx} = -3.7\%$ and $\epsilon_{yy} = 12\%$.

A.2.2 Quantification of Olivine Grains

Olivine grains have been identified as a potential defect contributing to the fracture and failure of the basalt. We provide some additional evidence in the image shown as an inset in Fig. A3a. To obtain this image, fragments collected from the intact...
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experiment were mounted in resin and systematically polished to investigate internal features. In the inset image, olivine grains are observed to intersect the fracture surface at multiple locations inside of the fragment, and no internal fractures are observed. This could be evidence that olivine grains are contributing sites for fracture initiation, perhaps as a result of the hardness and stiffness mismatches between the olivine phase and the pyroxene and feldspar phases.

Once olivine grains are identified as key defects, the next step is to quantify them so that we can relate their characteristics to fragmentation mechanisms. This also allows us to document their microstructure, which is not something commonly done. Image processing techniques are applied to a collection of microstructure images (an example shown in Fig. A1b and the half-flaw size (s: µm), areal flaw density (#/m²), and spacing between the defects (µm) are computed. Shown in Fig. A3a is the cumulative distribution of olivine grain sizes, given by the variable s. Note here that the major axis dimension of the olivine grain is denoted as 2s, and we are plotting half of this because inputs for numerical models are commonly taken as the half-flaw size. We use the following cumulative distribution function G(s) to fit the data:

\[ G(s) = \frac{1}{1 + \left( \frac{s - s_{min}}{s'} \right)^n} \]  \hspace{1cm} (A.2)

where \( s_{min} \) is the minimum size considered, and \( s' \) and \( n \) are scaling parameters that are fit to the data, with values of \( s_{min} = 5 \) µm, \( s' = 10 \) µm and \( n = -2.3 \).

Next, we plot the areal defect density (#/m²) of flaws larger than those on the
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Figure A3: (a) Cumulative distribution of olivine grain sizes with distribution fit, and inset of optical microscope image of internal features of a basalt fragment highlighting olivine grains intersecting the fracture surface. The blue line is the experimental measurements, while the red dots are the fitted distribution. (b) Areal flaw density (#/m²) of olivine grains for sizes larger than s (blue dots) with power-law curve-fit shown using the hashed-line.

corresponding x-axis (Fig. A3b). This is denoted as $\eta(s)$. A power-law fit is used to fit the data:

$$\eta(s) = Cs^D$$

where $C$ is a scaling parameter and $D$ is the fractural dimension. The data is fitted well with $C = 3.6 \times 10^{10}$ and $D = -1.80$.

A.2.3 Dynamic Fragmentation of Intact Basalt

We examine dynamic fragmentation results in Fig. A4a, where we plot the cumulative distribution of fragments (red dashed-line) for fragments larger than approximately 10 $\mu$m. The first thing we observe is an inflection point in the distribution at around 100 $\mu$m, suggesting there are two different fragmentation mechanisms. To
investigate this further, we also plot the cumulative distribution of spacing between adjacent olivine grains. It appears that the offset of the spacing distribution approximately corresponds to the inflection observed in the fragment size distribution. This suggests that there is a direct relationship between the olivine grain spacing and the resulting fragment sizes for sizes less than 100 μm. This appears reasonable since fractures initiated from olivine grains will likely coalesce with fractures initiated from adjacent olivine grains. Their coalescence will form a fragment size that is approximately equal to the spacing between grains.

Next, we examine the two different fragmentation mechanisms by considering a scatterplot of the mean greyscale color intensity against fragment size (Fig. A4b). The two different fragmentation mechanisms become more clearly defined and we describe the two regimes as follows: (I) fragments < 100 μm in size that are related to the defect spacing between olivine grains. These fragments are optically bright (as they have high greyscale intensities), and, thus, these fragments are believed to be primarily comprised of pyroxene. Pyroxene is the weakest mineral phase in this material. The composition of these smaller fragments has also been confirmed with scanning electron microscopy and Energy Dispersive Spectroscopy. (II) Fragments > 100 μm that are believed to form from the coalescence of the macro-fractures observed in the high-speed video image (Fig. A2). The fragments in this regime are controlled by the structural failure of the sample, which are, in turn, dependent on the stress-state and strain rate.
APPENDIX A. THE ROLE OF INCLUSIONS IN THE DYNAMIC FRAGMENTATION OF BRITTLE GEOLOGIC MATERIALS

Figure A4: (a) Cumulative distribution of dynamic uniaxial compressive fragmentation sizes and spacing between adjacent olivine grains. (b) Scatter plot of mean greyscale color intensity and size with two regimes labeled.

A.2.4 Dynamic Strength of Granular Basalt

In this final subsection we investigate the effect of the initial fragment size distribution on the strength of the fragmented material. We consider three distributions of fragments, each of which are shown in Fig. A5a. Fragments were sieved for:

1. $10 < L < 100 \, \mu m$ (average: $26 \pm 11 \, \mu m$),
2. $200 \, \mu m < L < 800 \, \mu m$ (average: $408 \pm 145 \, \mu m$),
3. $300 \, \mu m < L < 1,800 \, \mu m$ (average: $988 \pm 422 \, \mu m$).

The porosity, $\Phi$, of the samples before loading is estimated as:

$$\Phi = 1 - \frac{\rho_{\text{frag}}}{\rho_{\text{intact}}} = 1 - \frac{m_{\text{frag}}/\nabla_{\text{gage}}}{\rho_{\text{intact}}}$$

where $\rho$ is the density (kg/m$^3$), $m_{\text{frag}}$ is the mass of fragments (kg) being tested, and $\nabla_{\text{gage}}$ is the volume (m$^3$) of the gage section of the heat shrink tubing in which materials are being tested. The porosities of the three granular samples are estimated as 30%, 32%, and 35%, respectively. The corresponding stress-time response of the three different granular samples are shown in Fig. A5b. The peak strength of the
Figure A5: (a) Cumulative distribution of fragment sizes used as initial conditions for compressive granular tests. (b) Stress-time history of compressive granular tests for different initial conditions of fragment sizes.

Material increases for decreasing fragment size from 4 MPa for larger fragment sizes, to 25 MPa for the intermediate size conditions, to 175 MPa for the smallest set of fragments tested. We note here that these are preliminary results and additional tests are needed.

A.3 Summary and Implications

The role of the microstructure on the dynamic compressive failure and fragmentation of basalt has been examined. Real-time failure processes were tracked with high-speed photography, and these were linked with stress-time and strain measurements in order to understand the high-rate deformation of the basalt. Strains at peak stress are $\epsilon_{xx} = -0.9\%$ and $\epsilon_{yy} = 0.4\%$. By the time the stress has completely collapsed in the sample, the strains are approximately $\epsilon_{xx} = -3.7\%$ and $\epsilon_{yy} = 12\%$. Crack speeds were also measured (650 ± 100 m/s), and these are important because, when coupled with a length scale (e.g., defect spacing), they provide guidance into
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inherent time scales associated with brittle failure.

High-speed photography was also used to identify olivine grains as sites for fracture initiation in basalt, and this is believed to be a result of the hardness and stiffness mismatches between the olivine and the adjacent grains. In order to better quantify the defect population, image processing techniques were used to determine the size and areal density ($#/m^2$) of the olivine grains, as well as the spacing between adjacent grains. Distributions were fit to the size and areal defect density results, while the spacing between grains was compared with fragment sizes. The quantification of defects provides insight into the olivine grain defect population in the basalt, and this is important when strength measurements and failure processes for different basalts are contrasted, and in the computational modelling of large-scale impact events into basaltic material.

The consequences of the microstructure on dynamic compressive fragmentation were considered in detail, and two fragmentation mechanisms were noted: 1. A mechanism that generates smaller fragments comprised primarily of the weakest mineral phase (i.e. pyroxene) that is related to spacing between a critically activated type of defect (i.e., olivine grains); 2. A mechanism that generates larger fragments associated with the structural failure of the sample. The fragments in this regime are polyphase and polygrain in nature. These results highlight the importance of the material microstructure on the dynamic compressive failure of planetary materials. In particular, the weakest mineral phase (in terms of fracture toughness) is preferen-
Appendix A. The Role of Inclusions in the Dynamic Fragmentation of Brittle Geologic Materials

tially fragmented to form the smaller fragments, and the sizes of these fragments are controlled by the spacing between some critically activated defects. In contrast, the sizes in Regime II are governed by the structural failure mechanisms that are activated during loading, which are dependent on stress-state and geometry. The initial fragmentation of the bulk material (in terms of size and composition) may have an effect upon the subsequent strength of the fragmented material.

The strength of the fragmented material was examined for three different size ranges: 1. $10 < L < 100$ (average: $26 \pm 11 \, \mu m$), 2. $200 \, \mu m < L < 800 \, \mu m$ (average: $408 \pm 145 \, \mu m$), and 3. $300 \, \mu m < L < 1,800 \, \mu m$ (average: $988 \pm 422 \, \mu m$). Again, the smaller fragment class are primarily comprised of pyroxene grains, while the intermediate and larger classes are comprised of polyphase and polygrain fragments made up of a combination of olivine, pyroxene and feldspar. The strength of the material was observed to increase for decreasing fragment size from 4 to 25 to 175 MPa. These strengths are < 1%, 5%, and 35% of the strength of the intact material, indicating: 1. mm-sized sized fragments have little to no strength, 2. the smallest fragments maintain notable strength, and 3. any impact into fragmented material (e.g., regolith on the lunar surface) would experience a range of strengths and failure mechanisms, and these should be considered in any such simulation. We note here that on-going work is needed across a wider range of conditions to confirm these interpretations.

Interestingly, there is a greater increase in strength between the intermediate and
smaller fragment subsets than between the larger fragments and the intermediate fragments. It is understood that the contribution of frictional dissipation increases for smaller fragments, and the relative contribution of compaction and breaking decrease. However, we note that inherent defect population will also change as the fragment size is reduced, and this reduction in defect size will affect the strength of the fragmented material. As an example, the smaller pyroxene ($< 100 \mu m$ in size) in our basalt material will contain different and likely smaller defects than those fragments larger than 100 $\mu m$ (which contain olivine grains). Altogether, the composition and defect statistics associated with the fragmented material should be considered when developing constitutive relation describing the strength of fragmented material.

The role of the microstructure on the dynamic compressive failure and fragmentation of basalt has been examined. We have demonstrated the importance of the microstructure (composition and defect population) on the inherent length and time scales arising from the failure and fragmentation of planetary materials. The initial fragmentation, in terms of resulting composition and size, has been shown to have significant effects on the subsequent strength of the fragmented, and this should be considered in any development of constitutive relations for fragmented materials.
Appendix B

Disruption thresholds and impacts onto spinning asteroid targets

B.1 Introduction

In Chapter 5, we developed a hybrid numerical framework to simulate the outcomes of a hypervelocity impact onto km-sized asteroids. In the example considered in Chapter 5, the mass ratio of the largest remnant to the parent body after 4 hours was $M_{LR}/M_{PB} = 0.85$, a much higher value than the 0.5 that is taken as a reference for disruption limit calculations. This indicated that intact monolithic parent asteroids may have a higher disruption threshold, $Q_d^*$, than previously expected and require higher impact energies to be completely shattered. In this section, we explore the extent of this observation over a range of target sizes. In addition, asteroids in
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the Solar System have an initial period of rotation that for some asteroids (depending on location and size) can be on the order of a few hours. Most hydrocode numerical simulations to date ignored the spin of the target as an initial condition. We present some simulation results for oblique impacts onto rotating targets, and demonstrate the distinct ejecta reaccumulation profiles as a result of different impact directions relative to the spin direction.

B.2 Disruption Thresholds

Traditionally, disruption thresholds are defined as the specific impact energy required to generate a largest remnant whose mass is half that of the target. Following this definition, there may be two means of obtaining such a largest remnant. An impact event could thoroughly shatter a target, and the individual fragments may then be reaccumulated by gravity to form a sort of true “rubble pile”. On the other hand, an impact event could break down (but not completely shatter) the target, and sufficient volume of material is ejected from the target to reduce its total mass to half its original mass. Evidently, these would require different specific energies to achieve for the same target size and mass, and the distinction between these two different remnants would have direct implications on asteroid deflection techniques.

The hybrid numerical model developed in this thesis allows for the identification of these thresholds through the simulation of multiple impacts with different impactor
APPENDIX B. DISRUPTION THRESHOLDS AND IMPACTS ONTO SPINNING ASTEROID TARGETS

sizes and/or velocities. Such calculations are computationally expensive, and so we perform several low-resolution studies on disruption threshold for a different km-sized target. In these simulations, we use a coarse grid spacing in MPM so that the total number of material points does not exceed 300,000 particles, leveraging some balance between resolving the target geometry and reducing the computational cost of each run. In the gravitational reaccumulation phase, we employ the hard-sphere implementation of pkdgrav, instead of the soft-sphere implementation used in Chapter 5. Particles are transformed into discrete hard spheres, and low-speed collisions between these fragments are assumed to result in mergers giving a new spherical particle of combined mass and equivalent diameter. This assumption allows us to identify the impacts a priori during each time step using a k-D tree neighbor search algorithm. The use of hard sphere allows for much larger timesteps in the gravity simulations than would be possible with soft sphere. Since we treat all low-velocity impacts as resulting in mergers, the simulations will provide information on the mass of the largest remnant, but will not capture its shape. The results from these runs are presented in Fig. B1 along with previous estimates from literature.

In addition, we plot the mass evolution of fragments (individual spheres) for different specific energies Fig. B2. Notice in the case of the $2.77 \times 10^7$ erg/g impact that at $t = 2.18$ h, the mass of the largest remnant is exactly 0.5 that of the parent body, and this increases to roughly 0.7 at $t = 4h$ and to 0.8 at $t = 6$ h due to ejecta reaccumulation. This demonstrates the importance of using a hybrid framework that
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Figure B1: A compilation of disruption thresholds from literature, with the specific energies simulated in this work labeled. All impacts are head-on at 5 km/s. Red circles indicate a final largest remnant whose mass is bigger than half of the initial target. The green circle indicates a mass ratio of the largest remnant to the parent body that is nearly 0.5. Red crosses indicate dispersion of the target.
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Figure B2: Distribution of the mass of the largest remnant relative to the mass of the target at different times during the reaccumulation phase. The y-axis is limited to the range between $0 - 10$, and the light-blue shades are for the bins with $N > 10$. The mass of the largest remnant increases (moves to the right) as the low-velocity ejecta is gradually reaccumulated.

This captures both the shock-dominated and gravity-dominated responses to accurately capture the disruption thresholds in asteroid impacts.

B.3 Impacts onto rotating targets

Next, we consider impacts onto asteroids having an initial spin. A large number of asteroids are rather slow rotators, with spin rates of a few hours to complete a single rotation. Catastrophic impacts onto asteroids have been suggested to result in an “angular momentum splash” when reaccumulation following a disruptive impact yields a “rubble pile” target of which a significant amount of angular momentum...
APPENDIX B. DISRUPTION THRESHOLDS AND IMPACTS ONTO SPINNING ASTEROID TARGETS

had been carried away by the escaping material. The reaccumulated core would be left with a lower rotation rate (or net spin-down) compared to the parent body. In addition, impact ejecta on a spinning body were found to preferentially escape in the direction of rotation \[245\], systematically draining away angular momentum and leading to collisions that can reduce the spin of some midsized asteroids.

In this section, we simulate an oblique impact onto a 25 km target having an initial period of rotation of 6 hours. As a first-order approximation, we capture the angular rotation through an effective velocity distribution of the MPM particles as:

\[
v_p = \frac{dr}{dt} = \Omega \times r \tag{B.1}\]

where \(v_p\) is the linear velocity of a material point having a position vector \(r\) on an asteroid with angular velocity \(\Omega\). The initial conditions and velocity profile for the target considered in this case are shown in Fig. B3.

We simulate two impact conditions, with the impactor striking the target at 30° along the direction of spin (a) and opposite to the direction of spin (b) (Fig. B4). The MPM simulation is executed for 30s, and the results are then handed off to pkdgrav where the effects of gravitational interaction are captured for a simulated time of 2 hours. The shapes of the largest remnants in each case are shown in Fig. B6. The ejecta fallback onto different areas as the target core is spinning gives distinct shapes for these remnants. Interestingly, the impact along the direction of spin (Fig. B6a) appears to have generated a rim of reaccumulated fragments around the equator.
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Figure B3: Equivalent linear velocity profile for a 25 km target with a 6 hour period of rotation.
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Figure B4: Initial conditions for the impact onto a spinning 25 km diameter target with a period of rotation of 6 hours. We define the impact direction shown in (a) as being “along” the direction of spin, and that of (b) as being “opposite” to the direction of spin. In both cases, the impactor is 1.21 km in diameter and the impact speed is 5 km/s. The rotation axis for $\omega$ is shown pointing out of the center of the target.

Figure B5: Ejecta fallback onto the largest remnant following an impact (a) along the direction of spin and (b) opposite to the direction of spin. Snapshots are at 0, 0.2, 0.8, 2.2, 2.5 hours, and particles are color-coded according to damage.
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Figure B6: Triangulated shape models for the largest remnants following the impact (a) along the direction of spin and (b) opposite to the direction of spin.

B.4 Future Directions

To improve our understanding of asteroid evolution, performing hypervelocity impact simulations onto asteroids with a variety of initial conditions seem as inevitable research directions. The development of more sophisticated multi-physics mechanical models for impacts, and their coupling with advanced $N$-body gravity codes, will provide new insights onto the interior of asteroids. The addition of asteroid’s period of rotation as initial conditions in hypervelocity impact simulations appears to be an attractive endeavor towards the understanding of the evolution history of asteroids in our solar system. The effect of this spin on the target’s strength and resultant transfer in angular momentum remain to be explored. Finally, as more physical mechanisms are understood and implemented into numerical codes, a better picture can be drawn of the largest remnants resulting from a hypervelocity impact. Such codes may be
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used to explore the history of asteroids that may constitute viable targets for landing and sample retrieval missions, as well as in determining optimal impact parameters for planetary defense applications.
Appendix C

Codes and Datasets

Throughout the thesis, multiple codes written with different programming languages were used to generate, process, and analyze the datasets. All the datasets and results used in this thesis are stored in the Ramesh group drives[1] under the charleselmir storage space.

The thermal fatigue crack growth code, input files, and post-processing scripts used throughout Chapter 2 and in Chapter 3 are written in MATLAB and are found in /charleselmir/MATLAB_XFEM along with the raw data sets. The results of the characterization experiments performed on the GRO 85209 sample and presented in Chapter 3 are archived in /charleselmir/GRO along with the MATLAB codes used for post-processing the experimental measurements. The analytical study on the effect of interface properties on thermal fatigue that is presented in Chapter 4[1]

[1]At the time of writing, the drives are accessible from the Hopkins intranet at the IP address 10.161.161.20
was performed with a MATLAB code archived in /charleselmir/Interfaces along with the datasets and post-processing scripts.

The results presented in Chapter 5 constituted the largest data sets (in terms of storage space) of this thesis. The modified version of the Tonge-Ramesh material model is written in C++ and is hosted on the HEMI git server under the charleselmir/Uintah-PTR repository. The pkdgrav code was provided by Derek C. Richardson. In addition, the code used to perform the hand-off from MPM to pkdgrav is written in Python and is hosted on the HEMI git server under the /charleselmir/mpm_handoff_pkdgrav repository. The repository also contains a set of custom tools (written in Python and bash) that provide useful features for data visualization (including a plugin code for visualizing pkdgrav data in VisIt). Finally, the raw datasets of all the Uintah simulations are archived in the Ramesh drives in the /charleselmir/Impacts folder. The folder contains a compressed (tarball) archive of the Uintah timesteps for the simulations described in Chapter 5 and Appendix B. These data sets are in the Uintah UDA format and can be visualized with VisIt. Similarly, the timesteps of the pkdgrav simulations are also archived as a tarball in /charleselmir/Impacts. These files can be visualized using the pkdgrav built-in visualization tools (ssdraw, mkmov, etc...) or with the custom VisIt visualization plugin hosted in the /charleselmir/mpm_handoff_pkdgrav git repository.

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2 At the time of writing, the git server can be accessed at https://gitlab.hemi.johnshopkins.edu
3 VisIt is the open source visualization tool that was used to visualize the Uintah simulation archives. The latest VisIt code can be found on https://wci.llnl.gov/simulation/computer-codes/visit
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oid collisional evolution. I. Angular momentum splash: Despinning asteroids

Vita

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