Nonlinear Analysis of Cold-Formed Channels Bent about the Minor Axis

Olivia Oey¹, John Papangelis²

Abstract

The Direct Strength Method in AISI S100 and AS/NZS 4600 is an alternative design method to the Effective Width Method in calculating the design moment capacity of cold-formed channels. The Direct Strength Method uses elastic buckling stresses with an appropriate strength curve to calculate the design moment capacity. However, the current direct strength design equations have been calibrated for channels bent about the major axis and are known to be conservative for channels bent about the minor axis with the web in compression. In this paper, a nonlinear finite element analysis is described for cold-formed channels bent about the minor axis with the web in compression. The results are compared with the current direct strength design equations and a new design equation is proposed for channels bent about the minor axis with the web in compression.

1. Introduction

The Direct Strength Method (DSM) originally proposed by Schafer and Pekoz [1] provides an efficient and practical approach for the design of cold-formed steel (CFS) flexural members. The method uses elastic buckling stresses for the entire section with an appropriate strength curve to calculate the moment capacity and eliminates the tedious plate-by-plate calculations which are required for the Effective Width Method (EWM).

As the current DSM equations were derived through the calibration of flexural members bent about the major axis, the DSM approach adopted in the American AISI S100 [2] and Australian and New Zealand AS/NZS 4600 [3] standards provides conservative strength predictions for channels bent about the minor axis with the web in compression. This trend was first observed by Schafer and Pekoz [1] for the hat sections tested by Winter [4], which have a bending stress distribution that resembles that for lipped channel sections bent about the minor axis. Although flexural members are generally oriented to bend about the major axis, minor axis bending can still arise in eccentrically loaded columns [5] and purlins on roof pitches over 10°.

Currently, there are no existing DSM equations specifically for minor axis bending of channel members. Therefore, this study aims to propose a new DSM equation for channels bent about the minor axis with the long, slender web in compression. A series of nonlinear finite element analyses and elastic finite-strip buckling analyses performed by the programs Strand7 [6] and THIN-WALL-2 [7,8] respectively provided the ultimate bending moments \( M_u \) and local buckling moments \( M_{ol} \) of various cold-formed channel sections. The ultimate bending moment normalized by yield moment \( \frac{M_u}{M_y} \) was plotted against section slenderness \( \sqrt{\frac{M_y}{M_{ol}}} \) to calibrate a new direct strength curve. Therefore, a new DSM equation was proposed and compared against the current AISI S100 and AS/NZS 4600 DSM equation.

2. Design Rules

2.1 Effective Width Method

The slender plate elements comprising CFS sections are susceptible to local buckling under compression, where a series of buckle sine wavelengths develop and propagate along the length of the plate. The theoretical elastic local buckling stress \( f_{cr} \) of a plate element was defined by Timoshenko and Gere [9] and currently listed in AS/NZS 4600 as follows

\[
f_{cr} = \frac{k\pi^2E}{12(1-\nu^2)(b/t)^2}
\]

where \( k \) is the plate buckling coefficient, \( E \) is the elastic modulus, \( \nu \) is Poisson’s ratio and \( b/t \) is the plate slenderness, where \( b \) is the plate width and \( t \) is the plate thickness. In AS/NZS 4600, the uniformly compressed web of a channel

¹ Undergraduate Student, School of Civil Engineering, University of Sydney, Australia, ooeey6477@uni.sydney.edu.au
² Research and Consulting, School of Civil Engineering, University of Sydney, Australia, john.papangelis@sydney.edu.au
member bent about the minor axis has a value for $k$ equal to 4.0.

The post-buckling behavior of plates is characterized by the redistribution of stresses within the plate to the stiffer supported edges and occurs once $f_{cr}$ is attained [10]. In AS/NZS 4600, this phenomenon is represented by adopting an equivalent uniform stress acting over an effective width $b_e$ and is expressed as

\[ b_e = b \]  \hspace{1cm} (2)
\[ b_e = \rho b \]  \hspace{1cm} (3)

where $k$ is the plate slenderness given by

\[ k = \frac{f^*}{f_{cr}} \]  \hspace{1cm} (4)

in which $f^*$ is the design stress in the plate at the yield stress $f_y$, and $\rho$ is the effective width factor calculated as

\[ \rho = \left( 1 - \frac{0.22}{\lambda} \right) \leq 1.0 \]  \hspace{1cm} (5)

Thus, for a channel bent about the minor y axis as shown in Figure 1, once the effective width of the horizontal web is determined, the effective section modulus about the minor axis $Z_{ey}$ can be used to calculate the section moment capacity defined as

\[ M_{sy} = Z_{ey}f_y \]  \hspace{1cm} (6)

This approach provides an efficient means of calculating the design capacity of thin-walled members by eliminating tedious effective width calculations. It also ensures inter-element compatibility, equilibrium at element junctures and takes into account the interactions between different buckling modes [12].

Minor axis bending of channel sections precludes the occurrence of distortional buckling and lateral-torsional buckling. With the edge-stiffeners or lips of the channels under tensile stresses, distortional buckling at the flange-web junctions is eliminated. Lateral-torsional buckling will also not occur as the beam is already bent about its weak axis. Therefore, this research only focuses on the DSM equation for local buckling as the slender web of the channel section is subjected to uniform compressive stress under minor axis bending.

Formally adopted in AISI S100 in 2004 and AS/NZS 4600 in 2005, the DSM equation for local buckling of flexural members was derived from the calibration of experimental test data by Schafer and Pekoz [1]. As stated in AISI S100 and AS/NZS 4600, the member moment capacity $M_{bl}$ for local buckling is presented as

For $\lambda_l \leq 0.776$: \[ M_{bl} = M_{be} \]  \hspace{1cm} (7)
\[ M_{bl} = \left[ 1 - 0.15 \left( \frac{M_{ol}}{M_{be}} \right)^{0.4} \right] \left( \frac{M_{ol}}{M_{be}} \right) M_{be} \]  \hspace{1cm} (8)

where $\lambda_l$ is the section slenderness given by

\[ \lambda_l = \sqrt{\frac{M_{be}}{M_{ol}}} \]  \hspace{1cm} (9)

in which $M_{be}$ is the member moment capacity for lateral-torsional buckling and $M_{ol}$ is the elastic local buckling moment

\[ M_{ol} = Z_{ef}f_{ol} \]  \hspace{1cm} (10)

where $Z_{ef}$ is the full section modulus and $f_{ol}$ is the elastic local buckling stress calculated from THIN-WALL-2.

The DSM equations (7) and (8) take into account the interaction between local buckling and lateral-torsional buckling. However, for channels bent about the minor axis, lateral-torsional buckling does not occur and so $M_{be}$ is replaced with the yield moment $M_y$ as follows

For $\lambda_l \leq 0.776$: \[ M_{bl} = M_y \]  \hspace{1cm} (11)
\[ M_{bl} = \left[ 1 - 0.15 \left( \frac{M_{ol}}{M_y} \right)^{0.4} \right] \left( \frac{M_{ol}}{M_y} \right) M_y \]  \hspace{1cm} (12)

2.2 Direct Strength Method

In comparison with the EWM, the DSM implements elastic buckling solutions for the whole section using computational tools such as THIN-WALL-2 [7,8] and CUFSM [11] to determine the three critical buckling stresses for local buckling, distortional buckling and lateral-torsional buckling.
and the section slenderness is given by

$$\lambda_t = \sqrt{\frac{M_y}{M_{ot}}}$$  \hspace{1cm} (13)

### 3. Section Dimensions

To investigate the behavior of cold-formed channel sections bent about the minor-axis, a total of 33 channel sections were analyzed. These consist of standard sections currently manufactured in Australia plus additional created sections with higher section slenderness $\sqrt{(M_y/M_{ot})}$.

The dimensions of a cold-formed channel section are shown in Figure 1. The channel sections in this study had depths $D$ which ranged from 80 mm to 500 mm and thicknesses $t$ which ranged from 0.45 mm to 3.2 mm. This range of dimensions allowed channels with section slenderness up to 4.0 to be investigated. Further, the channel sections all had a depth to width ratio $D/B$ less than 3.5 which is typical for cold-formed channel sections manufactured in Australia.

### 4. Material Properties

The material properties of the CFS channel sections analysed in this study include the elastic modulus $E = 200,000$ MPa and Poisson’s ratio $v = 0.3$ while the shear modulus $G$ is given by [9]

$$G = \frac{E}{2(1 + v)} = 76,923$$  \hspace{1cm} (14)

In AS/NZS 4600, there are three common steel grades available for CFS of different material thicknesses. Table 1 shows the steel grades and corresponding thickness $t$, yield stress $f_y$ and ultimate tensile strength $f_u$.

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>$t$ (mm)</th>
<th>$f_y$ (MPa)</th>
<th>$f_u$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G450</td>
<td>$\geq 1.5$</td>
<td>450</td>
<td>480</td>
</tr>
<tr>
<td>G500</td>
<td>$1.0 &lt; t &lt; 1.5$</td>
<td>500</td>
<td>520</td>
</tr>
<tr>
<td>G550</td>
<td>$\leq 1.0$</td>
<td>550</td>
<td>550</td>
</tr>
</tbody>
</table>

### 5. Comparison between EWM and DSM

The section moment capacity $M_{ot}$ calculated using the EWM (Equation 6) normalized with the yield moment $M_y$ is plotted against the current DSM curve (Equations 11 and 12) for the CFS channel sections and is shown in Figure 2. It can be seen that the moment capacity predicted by the current DSM is quite conservative compared to the section moment capacity calculated by the EWM, especially at higher section slenderness. The reason the EWM maintains a higher strength is because it treats each individual plate comprising the section independently, which leads to only the web being ineffective. Therefore, the loss of strength in the compressive web corresponds to zero strength only for the web itself and not for the entire section.

### 6. Finite Strip Buckling Analysis

The elastic local buckling stress $f_{ol}$ used to calculate the elastic local buckling moment $M_{ol}$ in Equation 10 can be calculated by a finite strip buckling analysis [7,8,11], where the channel section is subjected to a uniform bending moment about the minor axis that puts the web in uniform compression.

The program THIN-WALL-2 [7,8] was able to show that local buckling of the web was the governing buckling mode for channels bent about the minor axis, as shown in Figure 3. The program also produces a signature curve in which the minimum indicates the lowest buckling stress $f_{ol}$ corresponding to the local buckling mode at a particular buckle half-wavelength, as shown in Figure 4.
7. Nonlinear Finite Element Analysis

7.1 General

When the response of a structure is not a linear function of the applied load, a nonlinear finite element analysis (NLFEA) is required to accurately predict the load-displacement response and the ultimate load of the structure. NLFEA involves iterative calculations of the element stiffness at every load or displacement increment until the solution converges. Such an analysis was performed in this study using the program Strand7 [6].

Nonlinear behaviour of a structure can be due to material nonlinearity and geometric imperfections. The nonlinear behavior exhibited by CFS can be accurately modelled using NLFEA provided the relevant input parameters are justified. The inclusion of material nonlinearity and geometric imperfections in the NLFEA allows for post-buckling behaviour to be represented in the analysis.

The arc-length numerical method as described by Riks [13] was adopted in Strand7 to ensure reliable convergence of the analysis and effectively obtain the post-ultimate load-displacement path. As the arc-length method simultaneously varies both the displacement and load vector coefficient while iteratively solving the nonlinear system of equations, solutions beyond the critical/failure points when local buckling instability occurs in the channel sections can be thoroughly attained. For the nonlinear finite element analysis in this study, the rotation about the minor y axis at each end of the cold-formed channel was incremented.

7.2 Finite Element Model

The channel sections were modelled as simply supported beams subjected to a prescribed rotation RY about the minor axis at each end, as shown in Figure 5. This produces a uniform bending moment along the complete length of the beam, causing the beam to sag with the web in compression. The restraints at each end were applied to a master node connected via rigid links located at the centroid of the cross-section. The length of the beam was at least 5 times the section depth to avoid any influence by the ends on the buckled shape of the web.

The beam was sub-divided into 4-node thin shell elements which have been developed based on the thin shell theory described by Jetteur and Frey [14]. The cross-section of any given channel was refined to have sufficient number of elements with a finer mesh at the corners, allowing for an accurate model to be achieved.

7.3 Stress-Strain Curves

A nonlinear material property was assigned to the shell elements using the two-stage Ramberg-Osgood stress-strain curves for cold-formed steels proposed by Gardner and Yun [15]. The stress-strain relationship is given by

For \( f \leq f_y \):

\[
\varepsilon = \frac{f}{E_0} + 0.002 \left( \frac{f}{f_y} \right)^n \quad (15)
\]

For \( f_y < f \leq f_u \):

\[
\varepsilon = \frac{f - f_y}{E_{0.2}} + \left( \varepsilon_{0.2} - \frac{f_u - f_y}{E_{0.2}} \right) \left( \frac{f - f_y}{f_u - f_y} \right)^m + \varepsilon_{0.2} \quad (16)
\]

where \( n \) and \( m \) are the strain hardening exponents taken as 7.6 and

\[
m = 1 + 3.3 \frac{f_y}{f_u} \quad (17)
\]

and \( E_{0.2} \) is the tangent modulus of the stress-strain curve at the yield stress \( f_y \) (0.2% proof stress) determined from
\[ E_{0.2} = \frac{E}{1 + 0.002n \frac{E}{f_y}} \]  

(18)

The strain \( \varepsilon_{0.2} \) is the strain at the yield stress \( f_y \) (0.2% proof stress) expressed as

\[ \varepsilon_{0.2} = 0.002 + \frac{f_y}{E} \]  

(19)

and \( \varepsilon_u \) is the strain corresponding to the ultimate tensile strength \( f_u \) obtained from

\[ \varepsilon_u = 0.6 \left(1 - \frac{f_y}{f_u}\right) \]  

(20)

The stress-strain curves for the steel grades in Table 1 are shown in Figure 6.

7.4 Geometric Imperfections

Geometric imperfections are required to ensure that the true ultimate strength and post-buckling behavior of CFS members in numerical analysis is correctly considered [16]. This study utilized the local buckling mode to model the distribution of geometric imperfections along the beam length [17]. The local buckling mode obtained using linear buckling analysis in Strand7 was factored by the maximum geometric imperfection previously provided in a probabilistic study performed by Schafer and Pekoz [18], which was then superimposed onto the perfect geometry to model the geometric imperfections for the nonlinear analysis.

To account for the large variations in geometric imperfections, a sensitivity analysis for cumulative distribution function (CDF) values of 25% and 75% was performed on several CFS specimens to investigate its implications on the NLFEA results. A CDF value is defined as the probability that a randomly selected imperfection value is less than a discrete deterministic imperfection. The first analysis used a larger geometric imperfection with a 75% CDF value, which corresponds to 0.66t. The second analysis used a smaller geometric imperfection with a 25% CDF value, which corresponds to 0.14t. Based on the sensitivity analysis conducted on 12 channel sections, there was no significant variation in the NLFEM results for the CDF values of 25% and 75% with an average difference of 0.41% in ultimate moment \( M_u \). Therefore, the remaining specimens were analyzed with a geometric imperfection of 0.66t.

8. Moment-Rotation Behaviour

The moment-rotation curves from the NLFEA for three channel sections C20324, C20312 and C35012 with section slenderness of 1.101, 2.221 and 3.735 respectively are plotted in Figure 7.

The maximum point in the moment-rotation curves denotes the normalized ultimate bending moment \( M_u/M_f \) of the channel sections. In general, the normalized ultimate bending moment for a given channel section is lower with increasing section slenderness, accompanied by a reduction in the corresponding end rotation. It can be seen that channel sections with a low section slenderness have a more rounded moment-rotation curve and a gradual post-ultimate response whereas those with a high section slenderness experience a sudden failure after the ultimate moment is reached.

9. Deformed Shapes and Stress Distributions

The NLFEA performed in Strand7 allows for the deformation and stress behaviour along with the corresponding failure of the CFS channel sections to be examined in detail. Figure 8 illustrates the deformed shapes and longitudinal membrane
stresses at ultimate moment for the three channels with moment-rotation curves shown in Figure 7.

Due to the low section slenderness of the C20324 section, the beam has an ultimate moment $M_u$ that is greater than the yield moment $M_y$. From the stress diagram, it can be seen that the lips of the channel section have yielded before the ultimate moment is reached. At the ultimate moment, the onset of strain hardening has allowed for the stresses at the extreme fibers of the lips to exceed beyond the yield stress. Hence, plastic collapse governs the failure of channel sections with low section slenderness.

On the other hand, the C35012 section with very high section slenderness appears to have failed by local buckling at an ultimate moment $M_u$ less than the yield moment $M_y$. The occurrence of local buckling prevents the full moment capacity of a slender section to be utilized as the stresses at the extreme fibers of the section develop below the yield stress at the ultimate moment. Hence, elastic local buckling governs the failure of channel sections with high section slenderness.

For the C20312 section of intermediate section slenderness, the ultimate moment $M_u$ is approximately equal to the yield moment $M_y$. Hence, failure is governed by a combination of yielding and local buckling (inelastic buckling) for channels with intermediate section slenderness.

It is of interest to observe in the stress diagrams that all the sections exhibit non-uniform stress distributions in the web. This is a consequence of the redistribution of the longitudinal membrane stresses in the web to the stiffer supported edges after the occurrence of elastic local buckling. This stress redistribution is the basis for the EW M mentioned previously.

Figure 8: Deformed shapes (left) and stress distributions (right) at ultimate moment
10. Proposed DSM Design Equation

The variation of the normalized bending moment $M_u/M_y$ as calculated by the NLFEA for the 33 channel sections with the section slenderness $\sqrt{(M_u/M_{ol})}$ is shown in Figure 9. Despite the variation in geometry and material properties, a clear trend exists between the normalized ultimate bending moment and section slenderness. Also shown in Figure 9 is the current DSM design curve calculated from Equations 11 and 12. It can be seen that the current DSM is very conservative compared to the NLFEA results.

![Figure 9: NLFEA and proposed DSM](image)

Since the ultimate moments for the channels were obtained from NLFEA and not from tests, the proposed DSM curve was conservatively derived to be lower than all the NLFEA results, as shown in Figure 9. Therefore, the proposed DSM design equation is as follows:

For $\lambda_l \leq 2.170$:

$$M_{bl} = M_y$$  \hspace{1cm} (21)

For $\lambda_l > 2.170$:

$$M_{bl} = \left[ 1 + 1.6 \left( \frac{M_{ol}}{M_y} \right)^{0.4} \right] \left( \frac{M_{ol}}{M_y} \right)^{0.4} M_y$$  \hspace{1cm} (22)

11. Conclusions

A nonlinear finite element analysis (NLFEA) has been described for cold-formed channels bent about the minor axis with the web in compression. The analysis takes proper account of material nonlinearity and geometric imperfections, which allow the ultimate moment to be determined. The arc-length numerical method was used in the analysis to ensure reliable convergence and to obtain the post-ultimate moment-rotation path.

It was shown that the current DSM design equation for channels bent about the minor axis is conservative with both the EWM and the NLFEA producing much higher moment capacities. Based on the ultimate moments from the NLFEA, a new DSM equation was proposed which provides a more accurate design moment capacity for channels bent about the minor axis with the web in compression.

The discrepancy between the current DSM and the proposed DSM (and EWM) is because of the fundamentally different local buckling modes associated with channels bent about the major and minor axes. The current DSM curve is based on local buckling of channels bent about the major axis, which involves buckling of the top part of the web, and the flange and lip, as shown in Figure 10(a). For channels bent about the minor axis, the local buckling mode is completely different as it involves buckling of the web only, as shown in Figure 10(b).

![Figure 10: Local buckling for (a) major axis and (b) minor axis bending](image)

It should be noted that the proposed DSM design equation is based on NLFEA which takes account of material nonlinearity and geometric imperfections. However, experimental testing is planned in the near future to confirm the proposed DSM design equation and conduct a reliability analysis to calculate a capacity design factor.

12. References

[1] Schafer BW and Pekoz T (1998a). Direct strength prediction of cold-formed steel members using numerical elastic buckling solutions, 14th International Specialty Conference on Cold-Formed Steel Structures, St Louis, USA, 69-76.


[11] Li Z and Schafer BW (2010). Buckling analysis of cold-formed steel members with general boundary conditions using CUFSM: conventional and constrained finite strip methods, 20th International Specialty Conference on Cold-Formed Steel Structures, St Louis, USA, 17-31.


