ESSAYS ON GOVERNMENT DEBT AND SOVEREIGN DEFAULT

by

Tongli Zhang

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Abstract

The first chapter presents a sovereign default model to study how a monetary backstop affects the sustainable level of government debt and household welfare. The model features a government that adjusts fiscal policy infrequently. The monetary backstop can lift the government’s debt limit, giving the government more room to roll over its debt until it can adjust fiscal policy. The model can explain the long-run cycles of government debt-to-GDP ratios for a group of advanced economies. The quantitative analysis shows that the benefit of using a monetary backstop to avoid default outweighs the cost of inflation. A monetary backstop improves household welfare.

The second chapter studies how the government’s sovereign borrowing and default decisions and the financial sector’s health are affected by inequality among households. We extend a model of endogenous sovereign default and financial sector crises to include agents’ heterogeneity. Our results show that income inequality increases both the likelihood of sovereign default and financial crisis and reduces the level of policy response to financial instability risks. Tax progressivity is useful in restoring the efficiency of government guarantees.

The third chapter studies how the economic fundamentals of local government and regulatory policies affect the risk of local government finance vehicle (LGFV) debt. I find that LGFVs become riskier as the local government accumulates more debt, regardless of whether the local government borrows directly from investors or indirectly
through LGFVs. In addition, I examine how regulatory policy announcements impact the riskiness of LGFV debt. I find that regulatory policies significantly increase the yield spread of LGFV debt. Counterintuitively, the effect of regulatory policy is stronger for local governments with a lower debt-to-GDP ratio. I propose a model to show that the results can be explained by the seniority of official government debt over LGFV debt.

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Chapter 1

Monetary Backstop and Sovereign Default on Domestic Debt

1.1 Introduction

The Covid-19 pandemic has led to large increases in government debt across advanced economies, generating renewed discussion about the sustainability of government debt. Most of the government debt of advanced economies is denominated in domestic currencies, which allows the central bank to support the government by raising inflation to generate seigniorage revenue for the government. This set of unconventional policies is often referred to as the monetary backstop for government debt.

The importance of a monetary backstop in reducing the risk of government default was revealed in the European debt crisis. After the announcement of the Outright Monetary Transactions programme (OMT) by the European Central Bank (ECB), the yield of the government bond of indebted peripheral European countries dropped significantly Altavilla and Giannone (2014).  

\footnote{\textsuperscript{1}Removing this monetary backstop can expose the government to larger risks of default. During the Covid-19 pandemic, ECB president Christine Lagarde made a speech suggesting that it was not the ECB’s role to “close the spread” in sovereign debt markets. The market interpreted it as ECB retreating from being a lender of last resort to Italy. Italian sovereign bond prices fell by a record daily amount Arnold and Stubbington (2020).}
Despite being a useful tool to prevent government default, the prospect of providing a monetary backstop for government debt has raised concerns about “Fiscal Dominance”. This term describes the situation where the central bank uses monetary policy to accommodate the borrowing needs of the government. The term was first introduced by Sargent and Wallace (1981). In their paper “Some Unpleasant Monetarist Arithmetic”, they showed that, in the state of fiscal dominance, when the government deficit cannot be entirely financed by new borrowing, the central bank will be forced to tolerate excessive inflation in order to avoid government default.

The tension between the sustainability of government debt and the stability of prices indicates that there is a trade off for the central bank and for society at large. On the one hand, the central bank can prevent government default by providing a monetary backstop. Government default often causes disruptions of economic activity and significant income losses in the short run. On the other hand, commitment to the monetary backstop can pressure the central bank to increase the inflation to a costly high level.

In this paper, I study how a monetary backstop impacts the sustainable level of government debt and household welfare using a quantitative model of government default. The monetary backstop in the paper is defined as follows: in order to avoid the government default, the central bank raises the inflation rate to generate additional seigniorage for a sustained period of time. This is one of the channels through which the central bank can reduce government debt and avoid a default.
Figure 1-1. Debt-to-GDP ratios of selected advanced economies. Debt increase episodes are marked as red area and debt decrease episodes are marked as blue area.
One crucial feature of the model is that the government is slow in adjusting fiscal policy. This assumption is informed by the dynamics of government debt in advanced economies. As shown in Figure 1-1, the government debt-to-GDP ratio in advanced economies is marked by episodes of persistent debt increase and episodes of persistent debt decrease. The factors driving this very low frequency government debt cycle will be discussed in details in Section 1.4. To summarize, the debt cycle is linked to the fact that government revenues adjust much more quickly to a change in the trend growth rate than government expenditures as displayed in Figure 1-2. When the trend growth rate falls, government revenues fall immediately whereas expenditures tend to continue on their original path, leading to a persistent debt increase episode.

My model can reproduce debt increase and decrease episodes similar to those observed in the data by assuming that fiscal policy is adjusted infrequently. Debt increase episodes are triggered by falls in the trend growth rate and last until the government adjusts fiscal policy. The uncertainty about the fiscal adjustment reduces the government’s ability to roll over its debt and leads to government default. In my model, this type of rollover crises can happen even when the government debt-to-GDP ratio is relatively low if there is no monetary backstop.

The monetary backstop can remove the risk of rollover crisis and increases the debt limit of the government, giving the government more time to roll over its debt and adjust fiscal policy without default. Interestingly, the monetary backstop does not have to be actually enacted for a government to benefit from it. When the inflation target associated with the monetary backstop is sufficiently high, knowing that the central bank can provide that monetary backstop increases the debt limit of the government and allows the government to roll over its debt without actually receiving the monetary backstop. In equilibrium, the risk of high inflation is small.
In order to evaluate the impact of the monetary backstop quantitatively, I calibrate the model to match the data on the dynamics of government debt for advanced economies since 1980s. The calibrated model captures the long run cycle of the government debt-to-GDP ratio as well as the behavior of key economic variables such as the government primary balance, primary expenditure, and interest-growth differential. The simulation shows that a monetary backstop with mildly high level of inflation can significantly increase the debt limit of the government. Compared with an economy with no monetary backstop, allowing the central bank to raise inflation rate to 15 percent for 10 years on average can increase the debt limit by more than 200 percent of GDP and reduces the probability of government default to virtually zero. At the same time, the monetary backstop does not simply replace the risk of default with the risk of inflation. The monetary backstop is rarely enacted in equilibrium. The frequency of high inflation with the monetary backstop is much smaller than the frequency of default when there is no backstop.

Figure 1-2. The level of Output, Revenue, and Primary Expenditure around the beginning (set as time 0) of debt increase episode. The window covers from three years before the debt increase period to eight years into the debt increase period. All levels at year $-2$ are normalized to be 1.
Next, I used the calibrated model to study the optimal size of the monetary backstop which maximizes household welfare. My results indicate that, when the central bank provides a monetary backstop, it is better to choose a medium level of inflation target which is not too low or too high. If the inflation rate is too low, the seigniorage is not large enough to remove the risk of rollover crisis, the monetary backstop is triggered often in equilibrium, and the government can still default even after receiving the monetary backstop. As a result, both the risk of default and the risk of inflation are high. In this case, the welfare gain from introducing the monetary backstop is small.

By contrast, when the inflation rate is sufficiently large, knowing that the central bank can provide the monetary backstop increases the default threshold by a large amount. This allows the government to roll over its debt and adjust fiscal policy later rather than defaulting or actually receiving the monetary backstop. The monetary backstop is rarely enacted in equilibrium. As a result, both the risk of default and the risk of inflation are low. However, if the inflation rate is too high, enacting the monetary backstop will be too costly. In such cases, the cost of high inflation reduces the welfare gain of the monetary backstop.

The quantitative analysis suggests that, when the inflation target of the monetary backstop is between 2 percent and 30 percent, the benefit of avoiding the default outweighs the cost of inflation. In such cases, the monetary backstop improves household welfare. Under the calibration of the model, the optimal inflation target is 15 percent and the welfare gain is equivalent with 4 percent increase of the trend income.

1.1.1 Related Literature

The paper is related to several strands of literature.
First, this paper contributes to the large literature on fiscal dominance. The original analysis of Sargent and Wallace (1981) was derived from a deterministic model. Several papers have since extended the analysis to stochastic environments. Leeper (1991) introduced the definition of “active” fiscal policies and “passive” monetary policies. He shows that even when government debt is on average backed by direct taxes, innovations to debt may be completely monetized. Davig et al. (2011) use a rational expectation framework to study the implication of rising debt when there is a “fiscal limit”, which is defined as a point where the government can not increase taxes to finance higher debt levels, so either a fiscal adjustment or inflation must occur in order to stabilize debt.

These papers, however, do not allow for government default in their model.

Several other papers in this literature study the effect of monetary policy on the risk of government default. Uribe (2006) shows that when both monetary policy and fiscal policy are “active”, the probability of government default depends on the specifications of monetary policy. Bi et al. (2018) show that the effect of monetary policy depends on its specification: if the central bank targets the yield of risky government debt instead of the risk-free rate, the monetary policy accommodates the rise of sovereign default risk by lifting the inflation target even when the monetary policy is “active”. My paper differs from those contributions in two ways. First, I assume that the government adjusts fiscal policy infrequently to capture the inertia of fiscal policy of advanced economies, and that the adjustment is optimal when it happens. Jeanne (2019) and Gourinchas and Jeanne (2012) present a mechanism similar to the one in this paper in the context of a two-period model. By contrast, in Uribe (2006), the fiscal policy is exogenous. The government is not able to adjust

\[\text{In Leeper (1991), an active authority pays no attention to the state of government debt and is free to set its control variable as it sees fit. A passive authority responds to government debt shocks.}\]
the fiscal policy. In Bi et al. (2018), the government adjusts the fiscal policy in every period based on reduced form relations between the tax rate and the government debt-to-GDP ratio. Second, in all these previous models, government debt is nominal and inflation reduces the real value of the debt. In this paper, I assume that the government debt is real. Inflation affects the path of government debt by generating seigniorage for the government.

Second, the modeling approach of this paper relates to the sovereign default framework developed by Aguiar and Gopinath (2006) and Arellano (2008). Specifically, this paper is related to literature on the default of domestic government debt (D’Erasmo and Mendoza, 2012, 2016a; Casola and Sichlimiris, 2018; Mallucci, 2015; Niepelt, 2016; Pouzo and Presno, 2020). These papers, however, do not incorporate monetary policies in their model. More generally, this paper relates to recent research on the potential of using the monetary policy to avoid self-fulfilling debt crises (Aguiar et al., 2013; Bacchetta et al., 2015; Corsetti and Dedola, 2016; Camous and Cooper, 2019; Espino et al., 2020; Bianchi and Mondragon, 2018). In these papers, rollover crises happen when lenders believe that the government is more likely to default and bid down the prices of government bond. In my paper, rollover crises are caused by the inability of the government to adjust fiscal policy.

Third, this paper contributes to the literature studying the inertia in fiscal policy. Halleberg et al. (2001) provide empirical evidence for fiscal policy inertia in the event

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3D’Erasmo and Mendoza (2012, 2016a) explore the sovereign default on domestic debt in a heterogeneous agent economy with wealth inequality. They argue that the government may choose to default on its domestic debt in order to achieve the desired wealth distribution. Casola and Sichlimiris (2018) examine a case of default on domestic debt where the government maintains monopoly in issuing debt by separating the external debt market and domestic debt market. Mallucci (2015) introduces the banking sector into a sovereign default model, and study how domestic holdings of sovereign debt affect the risk and cost of sovereign default. Niepelt (2016) presents an model with overlapping generations of agents where the government can partially default on its debt. Pouzo and Presno (2020) analyzes the optimal taxation in an economy where the government can default on its domestic debt.

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of negative economic shocks for European countries. Fatas and Mihov (2002) show that forecast errors or inability for the fiscal policy to response to shocks can cause excessive deficit. Larch and Salto (2005) show that, for countries in the Euro area, when the growth rate increase/decrease suddenly, the inertia of budgetary process leads to automatic increase/decrease in the government budget balance to surplus/deficit. All three papers above study the impact of the fiscal inertia on the cyclical behavior of the government budget balance. My paper makes a contribution by showing that the inertia in the fiscal policy can persist for long time and drive the long run dynamics of the government debt. The way the fiscal inertia is modeled in my paper is close to Jeanne (2021) in the sense that the opportunity to adjust fiscal policy follows a Poisson process. However, my paper differs from Jeanne (2021) by introducing shocks to the trend growth rate and explicitly modeling the monetary backstop.

Finally, the empirical work of this paper contributes to the literature on the long-run dynamics of the government debt for advanced economies. Early papers such as Bohn (1998) and Greiner et al. (2007) show that the primary balance-to-GDP ratio is a positive function of government debt-to-GDP ratio in US and EU countries, which suggests that the sovereign debt of advanced countries is sustainable. Using more recent data for OECD countries, Beqiraj et al. (2018) provide evidences that the primary balance decreases in response to increases in the debt-to-GDP ratio for advanced economies. Turner (2012) estimates the contribution of interest-growth differential to the increase of the government debt of advanced economies. Mauro et al. (2013) show that the response of fiscal policy to changes in government debt varies across countries and over time, and this variation is driven in part by unexpected changes in potential economic growth. This paper makes a contribution by quantifying the impact of both primary balance and interest-growth differential on the dynamics of the government debt for advanced economies, and shows that the slow adjustment of
primary balance is the main cause for the large swing of the government debt-to-GDP ratio of advanced economies.

The remainder of this paper is structured as follows. Section 1.2 presents the model. Section 1.3 discusses the intuitions of the model using a special case where the income growth rate is constant. Section 1.4 calibrates the model to the long run cycles of the government debt of advanced economies. Section 1.5 and 1.6 contain the quantitative analysis of the effect of the monetary backstop on the sustainability of government debt and household welfare. Section 1.7 concludes.

1.2 Model

In this section, I present a model of government default on domestic debt. The model has two main assumptions. First, instead of having the government choose fiscal policy every period, the model assumes that fiscal policy is adjusted only when a window of opportunity arrives. Second, while the inflation rate in the model remains low in most periods, the central bank may raise inflation to increase seigniorage in order to avoid a government default. I use the model to explain the dynamics of government debt in advanced economies and examine the quantitative effect of the monetary backstop.

1.2.1 Environment

Time is discrete, with \( t \) denoting the current period. The economy is populated by a large number of identical households and has a government and a central bank. In each period, the representative household receives an endowment \( Y_t \) which grows with a stochastic growth factor \( G_t \):

\[
Y_{t+1} = Y_t G_{t+1} \tag{1.1}
\]
The growth factor $G_t$ can be either high or low and follows a two-state Markov process:

$$\text{Prob}(G_{t+1} = G^H | G_t = G^H) = p_h$$
$$\text{Prob}(G_{t+1} = G^L | G_t = G^L) = p_l$$

1.2.2 Household

The household derives utility from private consumption goods $C_t$, public expenditure $E_t$, and real money balance $M_t = \frac{M_t}{P_t}$. The felicity function $U(\cdot)$ is additive separable,

$$U(C_t, E_t, M_t) = C_t + \alpha Y_t^{\psi} E_t^{1-\psi} + \mu Y_t^{\rho} \left( \frac{M_t}{P_t} \right)^{1-\rho}$$

(1.2)

The scaling factors $Y_t^{\psi}$ and $Y_t^{\rho}$ ensure that the utility function is homothetic in $Y$.

The household discount future utility with factor $\beta$ and maximizes the infinite-horizon utility:

$$V_t = E_0 \left[ \sum_{0}^{\infty} \beta^n U(C_{t+n}, E_{t+n}, M_{t+n}) \right]$$

(1.3)

subject to the budget constraint:

$$C_t + q_t B_t (1 - \delta_t) + M_t = (1 - \tau_t - \eta \pi_t^2)(1 - \gamma \delta_t) Y_t + B_{t-1}(1 - \delta_t) + \frac{M_{t-1}}{1 + \pi_t} + F_t$$

(1.4)

where $B_t$ is the value of one-period real government bond purchased in period $t$ at price $q_t$. The inflation rate $\pi_t$ is determined by the central bank one period in advance. I will describe the central bank’s policy in detail in Section 2.4. Given that in the model, government debt is short-term and the inflation rate is determined in advance, whether the government issues nominal or real debt does not change the ultimate outcome. I assume government debt is real to simplify the notation. These assumptions prevent the real value of the debt to be inflated away, and allow the analysis to focus on the impact of the seigniorage on the risk of default.
Binary variable \( \delta_t \) denotes whether the government defaults or not, with \( \delta_t = 1 \) if the government defaults in the current period, 0 otherwise. If the government defaults on its debt, the household does not receive any payment on its government bond holdings. The government is unable to issue new bonds in that period. The government may resume debt issuance next period with zero debt. The default is associated with a temporary loss of income which equals \( \gamma \) times the trend income.

The household pays tax at rate \( \tau_t \). The tax is distortionary with a convex cost \( \eta \tau_t^2 \). The household receives a lump-sum transfer \( F_t \) from the government.

The problem can be normalized by dividing the utility function and the budget constraint by \( Y_t \). Hereafter I use lower case letters to denote normalized variables. (e.g. \( c_t = C_t / Y_t \)).

The normalized form of the household problem is

\[
v_t(b_{t-1}, m_{t-1}) = \max_{c_t, m_t} \{ u(c_t, e_t, m_t) + \beta \mathbb{E}_t [G_{t+1} v_{t+1}(b_t, m_t)] \} \tag{1.5}
\]

subject to

\[
c_t + q_t b_t (1 - \delta_t) + m_t = (1 - \tau_t - \eta \tau_t^2)(1 - \gamma \delta_t) + \frac{b_{t-1}(1 - \delta_t)}{G_t} + \frac{m_{t-1}}{(1 + \pi_t)G_t} f_t \tag{1.6}
\]

The normalized felicity function is

\[
u(c_t, e_t, m_t) = c_t + \alpha \frac{e_t^{1-\psi}}{1-\psi} + \mu \frac{m_t^{1-\rho}}{1-\rho} \tag{1.7}
\]

where \( e_t = \frac{E_t}{Y_t} \) is the primary expenditure-to-GDP ratio, and \( m_t = \frac{M_t}{Y_t} \) is the money to GDP ratio.

### 1.2.3 Government

The government chooses fiscal policy and whether to default. Fiscal policy includes the tax rate \( \tau_t \) and the primary expenditure \( E_t \).
The government can only make infrequent decisions on fiscal policy. In each period $t$, whether the government adjusts fiscal policy is determined by a random variable $\xi_t$ which takes the value 1 if the government adjusts its policy, and 0 otherwise.

\[
\xi_t = \begin{cases} 
1 & \text{with probability } 1 - \phi \\
0 & \text{with probability } \phi 
\end{cases} \quad (1.8)
\]

The intuition here is that the government budget decision is constrained by the political process and opportunities to reset the budget policy arrive infrequently. This assumption is consistent with the findings of literature on the political economy of fiscal reforms, which shows that fiscal reforms are often delayed due to election cycles or political gridlock (Alesina and Drazen (1991), Alt and Lassen (2006), Ma (2014), Yared (2018)).

When the output growth rate changes, the inability for the government to adjust fiscal policy produces inertia in government primary expenditure. Because the tax rate does not change, government revenue falls proportionately to output.

In addition, I assume that the government adjusts fiscal policy when it re-enters the debt market after a default.

**1.2.3.1 Fiscal Policy**

At the beginning of each period $t$, the government observes $\xi_t$, and decides whether to default or not. Conditional on not defaulting, if $\xi_t = 1$, the government chooses tax rate $\tau_t$ and the level of primary expenditure $E_t$ to maximize its value. The primary expenditure will grow at constant growth rate $g_t$ until the next adjustment of fiscal policy. When an adjustment happens, $g_t$ is adjusted to the income growth rate of that
period.\footnote{Ideally, the government would choose the path of primary expenditures in all future periods \( \{E_{t+n}\}_{n=0}^\infty \). However, this would imply an infinite numbers of choice variables, which makes the model difficult to compute numerically.}

\[
g_t = G_t \tag{1.9}
\]

If \( \xi_t = 0 \), the government keeps the existing fiscal policy. The tax rate \( \tau_t \) and the growth rate of primary expenditure \( g_t \) remain the same as the last time when the government adjusts the policy. Let \( t_0 \) denote the time for the most recent policy adjustment.

\[
\tau_t = \tau_{t-1} = \tau_{t_0} \tag{1.10}
\]

\[
g_t = g_{t-1} = G_{t_0} \tag{1.11}
\]

The level of primary expenditure grows with the planned growth rate

\[
E_t = E_{t-1}g_t \tag{1.12}
\]

Consequently, the primary expenditure-to-GDP ratio evolves as follows

\[
e_t = e_{t-1}\frac{g_t}{G_t} = e_{t-1}\frac{G_{t_0}}{G_t} \tag{1.13}
\]

The process above does not guarantee that the expenditure-to-GDP ratio is bounded and remains below 100 percent. To ensure that the expenditure-to-GDP ratio fluctuates within a plausible interval, I assume that it is bounded

\[
e_t \leq e \leq e_h \tag{1.14}
\]

For example, if the expenditure-to-GDP ratio reaches its upper bound \( e_h \), \( e \) stays constant at \( e_h \) instead of following equation (1.13). When the government adjusts the fiscal policy, government choice of \( e \) is also constrained by condition (1.14).
1.2.3.2 Continuation

If the government adjusts fiscal policy to maximize household welfare (I refer to this case as state ‘a’), let $V^a$ be the value of continuation. The value of the government normalized by income $v^a = V^a / Y$ is as follows

$$v^a_t(b_{t-1}, m_{t-1}) = \max_{\tau_t, e_t} u(c_t, e_t, m_t) + \beta \mathbb{E}_t \left[ G_{t+1} v_{t+1}(b_t, m_t, e_t, g_t, \tau_t) \right] \quad (1.15)$$

Here $v$ (with no superscript) denotes the value of the government that has the option to default. The government’s optimization problem is subject to

$$e_t + \frac{b_{t-1}}{G_t} + f_t = \tau_t + q_t b_t + m_t - \frac{m_{t-1}}{(1 + \pi_t)G_t} \quad (1.16)$$

Seigniorage

If there is no policy adjustment (I refer to this case as state ‘n’), let $V^n$ be the value of continuation. The value normalized by the income $v^n = V^n / Y$ is

$$v^n_t(b_{t-1}, m_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1}) = u(c_t, e_t, m_t) + \beta \mathbb{E}_t \left[ G_{t+1} v_{t+1}(b_t, m_t, e_t, g_t, \tau_t) \right] \quad (1.17)$$

In this case, the government does not optimize. The current fiscal policy is determined by the last time the government adjusted its policy

$$\tau_t = \tau_{t-1} = \tau_{t_0}$$
$$e_t = e_{t-1} \frac{g_t}{G_t}$$
$$g_t = g_{t-1} = G_{t_0} \quad (1.18)$$

Here $t_0$ denotes the time of the most recent adjustment in fiscal policy. The government debt-to-GDP ratio $b_t$ evolves according to the government budget constraint.

In addition, I assume that the government debt-to-GDP ratio can not be negative. Once the debt-to-GDP ratio reaches 0, the government rebates all surpluses to the

---

5To differentiate the value of the government from the value of the household, I use symbol $V$ to denote the value of the government. The superscript “$a$” stands for “adjust”.

6The superscript “$n$” stands for “not adjust”.

---

15
household. This assumption allows the model to match the data where all advanced economies have positive debt throughout the history.

### 1.2.3.3 Default

A defaulting government is excluded from the bond market for one period and resumes bond issuance in the next period. When the government reenters into the bond market, it starts with zero debt and adjusts fiscal policy. The value of default is:

$$v_d^t(m_{t-1}) = \max_{\tau_t, e_t} u(c_t, e_t, m_t) + \beta E_t \left[ G_{t+1} v_a^{t+1}(0, m_t, \tau_t) \right]$$  \hspace{1cm} (1.19)

The tax rate $\tau_t$ and the expenditure-to-income ratio $e_t$ satisfy the government budget constraint:

$$e_t + f_t = \tau_t (1 - \gamma) + m_t - \frac{m_{t-1}}{(1 + \pi_t) G_t}$$  \hspace{1cm} (1.20)

Default can happen for two different reasons. First, the government defaults if the value of default is greater than the value of continuation. I refer to this type of default as a strategic default. In this case, the default indicator $\delta_t$ is determined as follows:

$$\delta_t(b_{t-1}, m_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1}, \pi_{t+1}) = \begin{cases} 
1 \text{ if } \xi = 1, v^a < v^d \text{ or if } \xi = 0, v^m < v^d \\
0 \text{ otherwise}
\end{cases}$$  \hspace{1cm} (1.21)

Second, in periods when the government cannot adjust fiscal policy, a default happens if the government cannot cover its financing needs with new borrowing. This is the case if:

$$\frac{b_{t-1}}{G_t} + e_t - \tau_t - s_t > \max_{b_t} \{q_t b_t\}$$  \hspace{1cm} (1.22)

Here $s_t$ is the seigniorage. In this case, the government will be forced to default due to lack of liquidity. I refer to this type of default as a rollover crisis.
The value of the government that has the option to default \( v \) satisfies:\(^7\)

\[
v = \begin{cases} 
\max\{v^a, v^d\} & \text{if } \xi = 1 \\
\max\{v^n, v^d\} & \text{if } \xi = 0
\end{cases}
\] 

(1.23)

### 1.2.4 Central Bank

In each period \( t \), the central bank determines the inflation rate one period ahead: \( \pi_{t+1} \). This assumption allows the inflation rate to be pre-determined which simplifies the state of money balance \( m_t \). I discuss the relationship between money balance and inflation target in Section 1.2.5.

I assume that the inflation rate is either low or high: \( \pi \in \{\pi^{\text{Low}}, \pi^{\text{High}}\} \). The central bank can provide a monetary backstop for the government by raising the inflation rate for a sustained period of time. Here instead of having the central bank optimize the inflation target, the inflation targets of the central bank are exogenous. The idea is to examine the impact of the monetary backstop by comparing models with different \( \pi^{\text{High}} \) and with a model where there is no monetary backstop. In the following paragraphs, I describe the central bank’s policy rule and explain the intuition behind the rule.

When the current inflation rate is low: \( \pi_t = \pi^{\text{Low}} \), the policy rule for the next period inflation target \( \pi_{t+1}(x_{t-1}, \tau_{t-1}, G_t, \pi_t) \) is as follows.

\[
\pi_{t+1}(x_{t-1}, G_t, \pi^{\text{Low}}) = \begin{cases} 
\pi^{\text{High}} & \text{if } v^d(\pi^{\text{Low}}) > v^c(\pi^{\text{Low}}) \\
\pi^{\text{Low}} & \text{otherwise}
\end{cases}
\] 

(1.24)

Here \( x_{t-1} = \{b_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1}\} \) denotes the fiscal state of the government. \( v^c \) is the value of continuation, which equals to \( v^a \) when the government adjusts the policy and to \( v^n \) otherwise.

\(^7\)For roll-over crises, the value of continuation is not well-defined. I assign a very small number for the value of continuation in the case of roll-over crises, so that equation (1.23) can be applied to both strategic defaults and roll-over crises.
The inequality $v^d(\pi^{Low}) > v^c(\pi^{Low})$ captures the intuition that the central bank raises the inflation rate if the government will default under the low inflation rate. If the government does not default under the low inflation, the central bank keeps the inflation rate low.

The policy rule implies that the central bank is committed to raise inflation rate to avoid a government default, even though letting the government default can yield higher values in some circumstances. Particularly, this assumption on the central bank’s commitment allows the model to examine the effect of the monetary backstop when $\pi^{High}$ is large. Alternatively, if the inflation policy is discretionary such that the central bank raises inflation only when it increases the welfare, the central bank will choose to never raise inflation when $\pi^{High}$ is above certain value. This is because when $\pi^{High}$ is large, the cost of raising the inflation is larger than the cost of default. In that case, there is effectively no monetary backstop in the model.

Once the economy is in the high inflation regime, $\pi_t = \pi^{High}$, the central bank exits this regime with a constant probability $p_\pi$, as long as the exit does not cause immediate government default. This assumption ensures that when a monetary backstop is triggered, it is expected to last for a sustained period of time.

1.2.5 Optimality Conditions

Bond Price

The bond price $q_t$ satisfies the Euler Equation. Since the household is risk-neutral in private consumption, the bond price equals to the expected payoff of the government bond multiplies the time discount factor.

$$q_t = \beta \mathbb{E}_t [1 - \delta_{t+1}]$$ (1.25)
Money

The no-arbitrage condition between money and bond ensures that \( m_t \) decreases with the expected next period inflation rate.

\[
m_t = \mu^{\rho \left[ 1 - \beta \left( \frac{1}{1 + \pi_{t+1}} \right) \right]^{\rho}} \tag{1.26}
\]

Seigniorage

The seigniorage is

\[
s_t = m_t - \frac{m_{t-1}}{(1 + \pi_t)} \tag{1.27}
\]

When the current inflation rate is low, the seigniorage revenue is rebated to households as a transfer. When the central bank raises the inflation to provide the monetary backstop, the government receives the seigniorage revenue.

\[
f_t = \begin{cases} 
  s_t & \text{if } \pi_t = \pi_{\text{Low}} \\
  0 & \text{if } \pi_t = \pi_{\text{High}} 
\end{cases}
\]

Resource Constraint

The government internalizes the impact of fiscal policy on the household’s consumption choices. As a result, the government recognizes the aggregate resource constraint

\[
c_t + e_t = (1 - \eta \tau_t^2)(1 - \gamma \delta_t) \tag{1.28}
\]

1.2.6 Timing

The timing of decisions in the model is the following:

1. The exogenous shocks \( G_t \) and \( \xi_t \) are realized.
2. Given the state variables \((b_{t-1}, m_{t-1}, e_{t-1}, g_{t-1}, \tau_{t-1})\) and the value of shocks, the central bank decides the inflation rate \(\pi\).

3. The government decides whether to default or not.

4. If the government defaults, it sets the fiscal policy \(\tau^d\) and \(e^d\) for the period in default. The government resets the fiscal policy when it re-enters the debt market.

5. Conditional on no default, if \(\xi = 1\), the government adjusts the fiscal policy \((e_t, \tau_t)\). If \(\xi = 0\), the government keeps the existing fiscal policy.

6. The household chooses his private consumption \(c_t\) and real money demand \(m_t\).

1.2.7 Recursive Equilibrium

**DEFINITION.** A Recursive Equilibrium is defined by: (i) a set of value functions: \(v, v^a,\) and \(v^n\); (ii) the government default rule \(\delta\); (iii) policy functions for the primary expenditure to income ratio and the tax rate when adjusting: \((e^{\xi=1}, \tau^{\xi=1})\); (iv) a policy function for inflation \(\pi\); (v) private consumption \(c\); (vi) real money balance \(m\); and (vii) a real bond price \(q\) such that

1. Given \(v^a, v^n\) and \(v^d\), \(v\) satisfies equation (1.23), \(\delta\) satisfies equation (1.21)

2. Given the value function \(v\), and the bond pricing function \(q\), \(v^a\) and \((e^{\xi=1}, \tau^{\xi=1})\) solve the government optimizing problem (1.15).

3. \(v^n\) satisfies equation (1.17).

4. \(v^d\) satisfies equation (1.19)

5. The inflation rate \(\pi\) satisfies the central bank policy rule (1.24).
6. Private consumption c and money demand m solve the household problem.

7. The real bond price q satisfies optimality condition (1.25)

1.3 Constant Growth Rate

To illustrate how the slow adjustment of fiscal policy and the monetary backstop affect the sustainability of government debt in the model, in this section, I discuss the model in a special case where the income growth rate is constant i.e. G^H = G^L. I show that when the government has a fiscal deficit, the slow adjustment of fiscal policy can significantly reduce the government default threshold (defined in next paragraph). The monetary backstop can lift the default threshold by turning the government deficit into surplus. The propositions in this section formalize the intuition.

1.3.1 Default Threshold

To study the impact of a monetary backstop on the sustainable level of government debt, I define the default threshold as the maximum debt-to-GDP ratio that the government can carry without default. In state ‘a’, the default threshold is a function of the GDP growth rate and the current inflation rate. In state ‘n’, the default threshold also depends on the existing fiscal policy.

\[ \bar{b}_t = \begin{cases} \bar{b}^a(G_t, \pi_t) & \text{if } \xi = 1 \\ \bar{b}^n(G_t, e_{t-1}, g_{t-1}, \tau_{t-1}, \pi_t) & \text{if } \xi = 0 \end{cases} \]  

(1.29)

As discussed in the Model section, default occurs either as a rollover crisis or a strategic default. In state ‘a’, because the government can adjust fiscal policy, there is no rollover crisis. The default threshold \( \bar{b}^a \) is the strategic default threshold. In state ‘n’, both rollover crisis and strategic default can happen. The default threshold \( \bar{b}^n \) is the minimum between the threshold of strategic default and the threshold of rollover crisis.
Lemma 1 shows that the default threshold is smaller in state 'n' than it is in state 'a'.

**Lemma 1.** For any given \((e_t, g_t, \tau_t, \pi_t)\), the default threshold in state 'n' is smaller than the default threshold in state 'a'.

\[
\bar{b}_n(e_t, \tau_t, g_t, \pi_t) \leq \bar{b}_a(\pi_t) \tag{1.30}
\]

Proof: See Appendix A.1.1.

The intuition is the following. When the government can adjust fiscal policy, default can only happen strategically. When the government cannot adjust fiscal policy, default can happen due either to a rollover crisis or a strategic default, depending on which threshold is lower.

In either case - strategic default or rollover crisis - the inequality holds. In the case of strategic default, because the government optimizes fiscal policy in state 'a' but cannot do so in state 'n', the value of repaying the debt is smaller in state 'n' than in state 'a'. As a result, the threshold for strategic default is lower in state 'n' than state 'a'. In the case of a rollover crisis, which only happens in state 'n', the government defaults before the debt-to-GDP ratio reaches the threshold for a strategic default. The default threshold for state 'n' is even lower in this case.

In addition, because the government does not adjust fiscal policy in state 'n', the projected path for government debt-to-GDP ratio plays an important role in determining the risk of government default. In particular, a rollover occurs only if the government debt-to-GDP ratio is on an increasing path. When a rollover crisis occurs, the default threshold tends to be much smaller than the threshold for a strategic default, especially if the probability for adjusting fiscal policy \(\phi\) is small. Proposition 1 characterizes the default threshold in case of rollover crises.

\[\text{8Here because the growth rate } G_t \text{ is constant, I drop it from the list of state variables.}\]
Proposition 1. A rollover crisis happens only if the government is in state 'n' and the debt-to-GDP ratio is increasing over time. In this case

\[ \overline{b}^n \approx \phi \overline{b}^a \]  

(1.31)

Proof: See Appendix A.1.2.

Proposition 1 states that the threshold for rollover crises is proportional to the probability of adjustment \( \phi \). When \( \phi = 1 \), the government adjusts fiscal policy in every period, the model reduces to a standard default model with no stickiness in government policy, \( \overline{b}^n = \overline{b}^a \). When \( \phi \) is small, the threshold for a rollover crisis is much lower than the threshold for a strategic default. The government may default with a relatively small amount of debt.

The intuition behind equation (1.31) is backwards induction. When the government cannot adjust fiscal policy and is running a fiscal deficit, the debt-to-GDP ratio is increasing. Once the government debt-to-GDP ratio reaches the default threshold for state 'n', \( \overline{b}^n \), the government will default in the next period unless it adjusts fiscal policy in the next period. The probability of repaying the debt is thus equal to the probability of adjustment \( \phi \). A small \( \phi \) would significantly decrease the bond price and reduces the amount of funds that the government receives from new borrowings. Given that the maximum amount of debt that the government can sustain in the next period is \( \overline{b}^a \), the maximum of debt that the government can roll over in the current period is close to \( \phi \overline{b}^a \).

Finally, if its debt-to-GDP ratio is on a downward path, the government can always roll over debt without the need to adjust fiscal policy. The default can only happen strategically. In this case, the default threshold in state 'n' will be close to the default threshold in state 'a' and significantly larger than \( \phi \overline{b}^a \).
1.3.2 Monetary Backstop

How does the monetary backstop affect the default threshold of the government? It depends on whether the default is a rollover crisis or a strategic default.

**Rollover Crisis** If the government default is caused by a rollover crisis when there is no monetary backstop, the effect of the monetary backstop on the default threshold depends on whether the seigniorage is large enough to cover the financing gap of the government. Proposition 2 characterizes the result.

**Proposition 2.** If the government defaults due to a rollover crisis when there is no monetary backstop, the effect of introducing a monetary backstop on the default threshold depends on the size of the seigniorage. If the seigniorage satisfies condition

$$s_t \geq \frac{1 - \beta_G}{G} b_{t-1} + e_t - \tau_t$$

the default threshold increases from the threshold of rollover crisis which is close to $\phi_b^a$ to the threshold of strategic default which is close to $\bar{b}_a$.

Proof: See Appendix A.1.3.

The intuition is the following. When a rollover crisis happens, Proposition 2 implies that the government must be in state ‘n’, and the debt-to-GDP ratio is on an increasing path. The monetary backstop creates a seigniorage revenue for the government, which improves the government budget condition. The condition (1.32) ensures that the seigniorage is large enough to cover the financing gap of the government, which equals to the primary deficit $(e_t - \tau_t)$ plus the interest-growth differential $(\frac{1-\beta_G}{G} b_t)$. Once condition (1.32) is satisfied, the government debt-to-GDP ratio is on a downward path. As shown in Proposition 2, rollover crises do not happen when the government debt-to-GDP ratio is decreasing. In this case, the relevant default threshold is no longer the rollover crisis threshold, but the higher strategic default threshold.
Proposition 2 highlights that the monetary backstop can increase the default threshold of the government by removing the risk of rollover crises. This effect is particularly crucial for the sustainability of government debt when $\phi$ is small. First, when $\phi$ is small, the threshold for a rollover crisis $\phi b^a$ is small. Removing the risk of rollover crisis increases the default threshold by a large amount. Second, when $\phi$ is small, it is more likely that the government is in state ‘n’, and lifting the default threshold in state ‘n’ has a large impact on the sustainability of the government debt.

The benefit of monetary backstop exists only if the seigniorage is large enough to cover the financing gap of the government. If the seigniorage is too small to satisfy condition (1.32), rollover crises can happen even with the monetary backstop. In this case, the impact of the monetary backstop on the default threshold is quite small.\(^9\) The intuition is as follows. When rollover crisis happens, as I shown in Proposition 2, $\bar{b}^n \approx \phi b^a$. The monetary backstop can increase $\bar{b}^n$ by increasing $\bar{b}^a$. However, when $\phi$ is small, the increase in $\bar{b}^n$ is much smaller than the increase of $\bar{b}^a$. As a result, the impact of the monetary backstop is small.

It is worth noting that the benefit of the monetary backstop does not require the monetary backstop to be actually enacted. As long as the seigniorage is large enough to cover the financing gap of the government, knowing that the central bank can provide the monetary backstop removes the risk of rollover crisis and increases the default threshold of the government. This gives government more room to rollover its debt and adjust fiscal policy later without actually implementing the monetary backstop.

**Strategic Default** For strategic default, the impact of a monetary backstop is ambiguous. The monetary backstop has two opposite effects on the default threshold.\(^9\)See Appendix A.1.3 for details.
On the one hand, the monetary backstop reduces the cost of taxation by replacing the tax with seigniorage. Through this channel, the monetary backstop increases both the value of continuation and the value of default. However, the average tax rate in continuation is higher than the average tax rate in the case of default (and reentry). For the same amount of reduction in tax rate, because the cost of taxation is convex, the value of continuation increases more than the value of default. As a result, the monetary backstop increases the default threshold.

On the other hand, the high inflation reduces money demand which reduces the utility derived from money balances. Through this channel, the monetary backstop reduces both the value of continuation and the value of default. However, the monetary backstop tends to last longer in the case of continuation than in the case of default. This is because, in the case of continuation, the government debt-to-GDP ratio tends to stay high, which prevents the central bank from reducing the inflation target. As a result, the monetary backstop reduces the value of continuation more than the value of default, and therefore reduces the default threshold of the government. The total impact of the monetary backstop depends on the relative strengthen of these two effects. In the next section, I calibrate the model to evaluate the effect of the monetary backstop quantitatively.

1.4 Calibration

In order to analyze the impact of the monetary backstop beyond the special case of constant growth rate, I calibrate the parameters of the full model and show that the calibrated model is capable of explaining the dynamics of government debt in advanced economies. Using the calibrated model, I show that the monetary backstop increases the government’s default threshold. First, in Section 1.4.1, I show that
the government debt-to-GDP ratio of advanced economies swings in long-run cycles and identify episodes of persistent debt increase and persistent debt decrease. In Section 1.4.2, I compute moments for these episodes and calibrate the model to match moments. Lastly, in Section 1.4.3, I show that the calibrated model captures the behavior of government debt and key economic variables in advanced economies, and I investigate the impact of a monetary backstop on the default threshold of the government.

1.4.1 Empirical Analysis

I obtain data on government debt for advanced economies from the IMF World Economic Outlook (WEO) database.\textsuperscript{10} Figure 1-1 shows the government debt-to-GDP ratio for a group of selected countries.\textsuperscript{11} We can see that the government debt-to-GDP ratio swings in long cycles.

To characterize the dynamics of the government debt-to-GDP ratio in the data, I introduce a set of criteria to identify persistent debt increase episodes and persistent debt decrease episodes.

A debt increase episode is identified if this period satisfies the following conditions:

(1) starts from a year with $\Delta \text{Debt-to-GDP} \geq 1\%$.

(2) ends with one of the two following conditions:

i $\Delta \text{Debt-to-GDP} < 1\%$ for three years in a row

ii $\Delta \text{Debt-to-GDP} < -4.5\%$ for one year

(3) lasts for more than 10 years. This separates the long-run government debt cycle

\textsuperscript{10}See Appendix A.2.1 for details of the data
\textsuperscript{11}For the other countries, see Figure A-3 in Appendix A.2.4
from fluctuations due to the business cycle. (Beginning in 2008, this length is shortened to 7 years).\textsuperscript{12}

Conversely, I define a period as a debt decrease episode if it:

(1) starts from a year with the $\Delta$Debt-to-GDP < 0%.

(2) ends with one of the two following conditions:

i $\Delta$Debt-to-GDP $\geq$ 0\% for next two years

ii $\Delta$Debt-to-GDP $>$ 6\% for next year

(3) lasts for more than 10 years (Beginning in 2008, this length is shortened to 7 years).

Figure 1-1 shows debt increase episodes (red) and debt decrease episodes (blue) for selected economies.

In order to reveal the factors that drive the government debt cycle, I decompose the change of Debt-to-GDP ratio of each country into the contribution from the primary fiscal balance, the interest rate-growth differential, and the residual.\textsuperscript{13} Table 1.1 reports the summary statistics for each component.

For each component of the decomposition, I compute the difference between the average level of the variable during debt increase episodes and decrease episodes. I refer to this difference as the inter-episode difference. I compute inter-episode differences for each country and then take the average of this difference across countries.\textsuperscript{14}

\textsuperscript{12}Because the data series end at 2017, I shortened the length requirement to be 7 years for debt increase episodes after 2008 in order to capture debt increase episodes after the Global Financial Crisis.

\textsuperscript{13}See Appendix A.2.2 for details of the decomposition.

\textsuperscript{14}For countries with only debt increase episodes or debt decrease episodes, I calculate the difference between the mean of debt increase/decrease episodes and the rest of years. Countries that have neither debt increase episodes nor debt decrease are excluded from the calculation.
### Table 1.1. Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Diff.</th>
<th>Median</th>
<th>Std.Dev</th>
<th>Max</th>
<th>Min</th>
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</thead>
<tbody>
<tr>
<td>ΔDebt-to-GDP</td>
<td>1.40</td>
<td>5.70</td>
<td>0.92</td>
<td>4.50</td>
<td>20.86</td>
<td>-17.54</td>
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<tr>
<td>Primary Balance</td>
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<td>-4.14</td>
<td>-0.47</td>
<td>3.79</td>
<td>15.88</td>
<td>-10.80</td>
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<tr>
<td>$r - g$ Component</td>
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<td>1.05</td>
<td>-0.38</td>
<td>2.47</td>
<td>12.60</td>
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<td>Stock Flow</td>
<td>1.55</td>
<td>0.52</td>
<td>0.76</td>
<td>3.90</td>
<td>32.64</td>
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<tr>
<td>Revenue</td>
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<td>-0.46</td>
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<td>Primary Expenditure</td>
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<td>64.00</td>
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<td>Real Growth ($g$)</td>
<td>2.18</td>
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<td>1.99</td>
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<tr>
<td>Interest Rate($r$)</td>
<td>0.54</td>
<td>-0.69</td>
<td>1.55</td>
<td>4.91</td>
<td>12.92</td>
<td>-21.13</td>
</tr>
</tbody>
</table>

**Note:** Table 1.1 reports the summary statistics for advanced economies during 1981-2017 using data from World Economic Outlook (October 2018). Countries include: Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, UK, and USA. The second column reports the difference between the mean for debt increase episodes and the mean for debt decrease episodes. ΔDebt-to-GDP, Primary Balance, $r - g$ Component (interest-growth differential), stock flow, revenue, and primary expenditure are measured as percent of nominal GDP. Real growth rate and Interest rate are reported in percentage.
The second column of Table 1.1 reports the inter-episode difference for each variable. We can see that the annual increase in the government debt-to-GDP ratio is 5.7 percent higher in debt increase episodes than in debt decrease episodes. The majority of the difference in the debt dynamics can be attributed to the primary balance. After decomposing the primary balance into revenue and primary expenditure, we can see that higher primary expenditure is the main factor that causes debt to go up in debt increase episodes.

It is worth noticing that the average real GDP growth rate is lower in debt increase episodes than in debt decrease episodes. I plot the level of output, fiscal revenue, and primary expenditure in the first eight years of debt increase episodes in Figure 1-2 (shown in the introduction). The graph shows that fiscal revenue falls in pace with GDP, whereas the primary expenditure continues growing in its original path. The inertia in primary expenditure causes the persistent increase of the government debt-to-GDP ratio.

### 1.4.2 Parameters

This section describes my calibration strategy. Each period in the model is one year. The model has seventeen parameters. I calibrate eight parameters \((G^H, G^L, p_h, p_l, \alpha, \Xi, \eta, \beta)\) to match the moments of the government debt cycle. The parameters related to money balance \((\mu, \psi)\) are calibrated separately to match the evidence on money demand. The value of the remaining seven parameters are either chosen exogenously or taken from the literature. Table 1.2 reports the parameter values.

Among the parameters targeting the moments of the government debt cycle, the income growth rate \(G^H\) and \(G^L\) are calibrated to match the average and the inter-episodes difference for real GDP growth rate in the data. The probability \(p_h\) and \(p_l\) are chosen to match the frequency of debt increase episodes and the frequency of debt
decrease episodes. The coefficients \((\alpha, \Xi, \eta, \)\) are calibrated to match the average and the inter-episodes difference of the primary expenditure-to-GDP ratio, and the average of the revenue-to-GDP ratio in the data. The time discount factor \(\beta\) is calibrated to match the inter-episodes differences for the \(r - g\) component. Table 1.3 shows the value of targeted moments from the empirical data and the model counterparts.

**Table 1.2. Calibration**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Growth Rate</td>
<td>(G^H)</td>
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</tr>
<tr>
<td>Low Growth Rate</td>
<td>(G^L)</td>
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<tr>
<td>Probability of High Growth</td>
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</tr>
<tr>
<td>Probability of Low Growth</td>
<td>(p_l)</td>
<td>0.71</td>
</tr>
<tr>
<td>Coefficient of Money in Utility</td>
<td>(\mu)</td>
<td>0.05</td>
</tr>
<tr>
<td>Coefficient of Expenditure in Utility</td>
<td>(\alpha)</td>
<td>0.19</td>
</tr>
<tr>
<td>Relative Risk Aversion (money)</td>
<td>(\rho)</td>
<td>4</td>
</tr>
<tr>
<td>Relative Risk Aversion (expenditure)</td>
<td>(\psi)</td>
<td>2</td>
</tr>
<tr>
<td>Time Discount Factor</td>
<td>(\beta)</td>
<td>0.97</td>
</tr>
<tr>
<td>Probability of Adjustment</td>
<td>(\Xi)</td>
<td>0.15</td>
</tr>
<tr>
<td>Cost of Taxation</td>
<td>(\eta)</td>
<td>0.15</td>
</tr>
<tr>
<td>Loss of Income in Default</td>
<td>(\gamma)</td>
<td>0.3</td>
</tr>
<tr>
<td>Low Inflation Target</td>
<td>(\pi^{Low})</td>
<td>2%</td>
</tr>
<tr>
<td>High Inflation Target</td>
<td>(\pi^{High})</td>
<td>15%</td>
</tr>
<tr>
<td>Probability of Inflation Restoration</td>
<td>(p_\pi)</td>
<td>0.1</td>
</tr>
<tr>
<td>Lower Bound of Primary Expenditure</td>
<td>(e_l)</td>
<td>36%</td>
</tr>
<tr>
<td>Upper Bound of Primary Expenditure</td>
<td>(e_h)</td>
<td>48.6%</td>
</tr>
</tbody>
</table>
The parameters $\mu$ and $\rho$ are calibrated to match the level and the interest elasticity of money demand. For the purpose of calibration, aggregate money is measured by Money Zero Maturity (MZM), which encompasses all non-term deposits, and money that can be accessed without notice and at par. Specifically,

$$\text{MZM} = \text{M2} - \text{Small-denomination Time Deposits} + \text{Institutional Money Market Funds}$$

Motley (1988) first introduced MZM as a broad money aggregate which gives private agents immediate command over goods and services.

By calibrating the demand for money to a broad money aggregate rather than just the monetary base, the seigniorage received by the government is large. In reality, this can be achieved when the higher inflation is associated with financial repression as shown by Reinhart and Sbrancia (2015). For the calibration of $\rho$, I obtain the data from Teles and Zhou (2005), where the interest elasticity of MZM is estimated to be 0.24. This corresponds to $\rho$ being almost equal to 4. For $\mu$, I set $\mu = 0.05$, so the money demand in the model under low inflation is 100 percent of GDP, which matches the current MZM-to-GDP ratio in the US.

In addition, I set $\psi = 2$, which is a standard value for the CRRA utility function. I assume the fraction of income loss in default is set to $\gamma = 30\%$, which is in line with typical estimates in the sovereign default literature (D’Erasmo and Mendoza (2012) and Rogoff and Reinhart (2011)). I take the average minimum and maximum primary expenditure-to-GDP ratio across countries from the data, hence, $e_l = 0.36$ and $e_h = 0.48$. I set $\pi^{Low}$ to be 2 percent which is a conventional inflation target for central banks of advanced economies. I assume that when the central bank provides the monetary backstop, the inflation rate increases to 15 percent and, on average, lasts for 10 years: $\pi^{High} = 15\%$, and $p_\pi = 0.1$. Later in Section 1.5, I relax the calibration for $\pi^{High}$, and show how the impact of monetary backstop varies with $\pi^{High}$.
### Table 1.3. Empirical Vs Simulation

<table>
<thead>
<tr>
<th>Target</th>
<th>Baseline Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave. Revenue</td>
<td>41.08</td>
</tr>
<tr>
<td>Ave. Primary Expenditure</td>
<td>41.21</td>
</tr>
<tr>
<td>Ave. Growth Rate</td>
<td>2.18</td>
</tr>
<tr>
<td>Probability of Debt Increase</td>
<td>0.40</td>
</tr>
<tr>
<td>Probability of Debt Decrease</td>
<td>0.19</td>
</tr>
<tr>
<td>Diff. Primary Expenditure</td>
<td>3.68</td>
</tr>
<tr>
<td>Diff. $r - g$ Component</td>
<td>1.05</td>
</tr>
<tr>
<td>Diff. Real Growth ($g$)</td>
<td>-1.98</td>
</tr>
</tbody>
</table>

**Note:** Table 1.3 compares the moments of the empirical data with the simulation results from the calibrated model. The first column reports the moments of the empirical data. The second column shows the same moments calculated using the simulation result of the calibrated model.
1.4.3 Quantitative Results

In this section, I compare the model’s predictions with the data and examine the impact of a monetary backstop on the default threshold. Using the event study method, I show that the model can explain the behavior of the main variables of interest in the government debt cycle.

For the event study, I run 10000 simulations of the calibrated model. Each simulation lasts for 400 periods. For each simulation, I identify debt increase/decrease episodes using the same method as in the data and collect simulated data for variables including: change of debt-to-GDP ratio, primary balance, interest-growth differentials, revenue-to-GDP ratio, primary expenditure-to-GDP ratio, and income growth rate. The goal is to compare the behavior of each variable above in the simulation with its behavior in the data around the time period when debt increase episodes begin.\(^\text{15}\)

The event in the study is the beginning of debt increase episodes. For each debt increase episode, I first obtain the value of each variable during a time window starting three years before the event and ending eight years after the event. I then compute the average trend across all debt increase episodes. Figure 1-3 plots the path of \(\Delta\) Debt-to-GDP ratio and its components at the beginning of debt increase episodes. The simulated path is represented by the black dashed line and the path from the empirical data is represented by red solid line. The beginning of the debt increase episode is marked by a vertical black line at time 0.

The model simulation captures the behavior of all six variables. Specifically, at the beginning of the debt increase episode, the real growth rate drops sharply. While the revenue-to-GDP ratio stays roughly constant, the primary expenditure to GDP ratio increases significantly. The large increase in the primary expenditure to GDP ratio increases significantly.\(^\text{15}\) The event study for the beginning of debt decrease episodes produces similar results. A brief analysis for debt decrease episodes can be found in Appendix A.2.3
The ratio reduces the primary balance, and causes the increase of the debt-to-GDP ratio.

I then use the calibrated model to study the impact of the monetary backstop on the default threshold in state 'a' $\bar{b}^a$ and state 'n' $\bar{b}^n$. Figure 1-4 compares the default threshold in state 'a' of the calibrated model with an alternative model where there is no monetary backstop. We can see that the monetary backstop in the baseline model increases $\bar{b}^a$ by about 50 percent of GDP.

The monetary backstop has a larger impacts on the default threshold in state 'n'. Figure 1-5 shows how the default threshold in state 'n' varies with the fiscal policy in the calibrated model (blue line) and in a model where there is no monetary backstop (red line). Here $e$ is fixed at 43 percent (which is the average in the data) and the default threshold is plotted as a function of tax rate $\tau$.

First, we focus on the relationship between the default threshold and the tax rate $\tau$. We can see that regardless of whether there is a monetary backstop, there are three different regimes in how the default threshold depends on $\tau$. When the tax rate is low, the government has a fiscal deficit, and the projected path for government debt-to-GDP is increasing. As shown by Proposition 1, in this case, the default is caused by roll-over crisis. The default threshold is close to $\phi \bar{b}^a$. As $\tau$ increases, the projected path for government debt-to-GDP ratio turns from an increasing path to a decreasing path. In this stage, the default threshold jumps from the threshold of roll-over crisis to the threshold of strategic default which is close to $\bar{b}^a$. Finally, when $\tau$ is large, defaults only happen strategically. The default threshold stays close to $\bar{b}^a$.

Second, we focus on the difference between the model with monetary backstop and the model with no monetary backstop. Compared with the case where there is no monetary backstop, the monetary backstop provides seigniorage revenue to the government. On the graph, this is equivalent with shifting the curve to the left. As a
result, the largest impact of the monetary backstop happens in the middle part of the graph where $\tau$ is between 43 percent and 52 percent. The intuition is as follows. When $\tau$ is between 43 percent and 52 percent, the seigniorage from the monetary backstop is able to turn the path of government debt-to-GDP ratio from increasing into decreasing. Consistent with Proposition 2, once the central bank provides the monetary backstop, the government default threshold increases from the threshold of rollover crisis (which is close to $\phi \tilde{\theta}^a$ ) to the threshold of strategic default (which is close to $\tilde{b}^a$). The default threshold increases by more than 200 percent.

However, if $\tau$ is too small (below 43 percent) or too large (larger than 52 percent), the impact of the monetary backstop is small. This is because, when $\tau$ is too small, rollover crises happens regardless whether there is a monetary backstop or not. If $\tau$ is larger than 52 percent, the government has a large fiscal surplus. Defaults happen strategically even when there is no monetary backstop. In both of these cases, the monetary backstop does not change the nature of the default.

1.5 Debt Sustainability

In this section, I use the calibrated model to investigate how the size of the monetary backstop affects the sustainability of government debt by varying the calibration of $\pi^{High}$. I find that, as long as the seigniorage is large enough to cover the financing gap of the government, the monetary backstop significantly increases the default threshold of the government and reduces the frequency of default. In addition, I show that, when $\pi^{High}$ is sufficiently large, the risk of inflation associated with the monetary backstop is low.
Figure 1-3. Mean of selected macroeconomic variables for simulations (dashed line) and empirical data (solid line) at the beginning of a debt increase episodes. The debt increase episodes starts at time 0 which is marked with a black line. All variables are measured in percentage of GDP.
Figure 1-4. The default threshold of government debt-to-GDP ratio in state 'a'. From left to right, each blue bar indicates $\bar{b}$ of the calibrated model for different states of $(G_t, \pi_t)$. Each red bar shows the default threshold from an alternative model where all the calibrations are the same except there is no monetary backstop.

Figure 1-5. The default threshold of government debt-to-GDP ratio in state 'n' as a function of $\tau$. Here $\pi_t = \pi^{Low}$, $G_t = G_h$, and $e_t = 0.43$
1.5.1 Debt Limit

Figure 1-6a shows how the default threshold in state ‘n’ changes with $\pi^{High}$, and compares it with the default threshold in the case when there is no monetary backstop. The state variables ($G, e, g, \tau, \pi$) are chosen to match the empirical data for debt increase episodes.\footnote{Based on the empirical data, in debt increase episodes: (i) the income growth is low $G = G^L$; (ii) the expenditure-to-GDP ratio increases faster than income $g = G^H$; (iii) the revenue-to-GDP ratio is about 42 percent of GDP $\tau = 42\%$; (iv) the primary expenditure-to-GDP ratio is about 44 percent of GDP $e = 44\%$.}

The effect of the monetary backstop crucially depends on the value of $\pi^{High}$. When $\pi^{High}$ is below 8 percent, the impact of the monetary backstop on $\bar{b}^n$ is very small. This is because the seigniorage is too small to cover the financing gap of the government. The government can default due to rollover crisis even after receiving the seigniorage. When $\pi^{High}$ is larger than 8 percent, the seigniorage becomes large enough to cover the financing gap of the government. The monetary backstop increases the default threshold from the threshold of rollover crises to the threshold of strategic default. As a result, $\bar{b}^n$ increases from 40 percent of GDP to more than 200 percent of GDP. This result shows that, in order for the monetary backstop to have a large impact on the default threshold, the central bank needs to choose a sufficiently high inflation rate when providing the monetary backstop.

Figure 1-6b shows the default threshold in state ‘a’ in response to the variation of $\pi^{High}$.\footnote{The blue line in Figure 1-6b displays $\bar{b}^a (G^H, \pi^{Low})$ for the model with the monetary backstop. Default thresholds $\bar{b}^n$ for other states of $G_t$ and $\pi_t$ have similar relationships with the value of $\pi^{High}$. See Appendix D for details.} We can see that, compared with the case where there is no monetary backstop, the monetary backstop increases $\bar{b}^a$ by about 40 percent of GDP. As $\pi^{High}$ increases, $\bar{b}^a$ stays roughly constant, even though the present value of the seigniorage from the monetary backstop increases significantly. This is because, in state ‘a’, all
government defaults are strategic defaults, and the cost of inflation offsets the positive effect of the seigniorage. As discussed in Section 1.3, the monetary backstop increases the default threshold in state 'a' by replacing the tax with seigniorage. However, the high inflation of the monetary backstop reduces the money demand and lowers the default threshold. As a result, even though the size of the seigniorage increases with $\pi^{High}$, the default threshold stays constant.

Lastly, given that the monetary backstop increases the debt limit of the government, I examine whether the higher debt limit leads to more government borrowing. Figure 1-7 plots the average government debt-to-GDP ratio as $\pi^{High}$ varies. We can see that when $\pi^{High}$ is below 15 percent, the government debt-to-GDP ratio increases with $\pi^{High}$. Having larger monetary backstop increases the debt limit of the government and gives the government more space to accumulate more debt and adjust later. When $\pi^{High}$ is larger than 15 percent, further increases of $\pi^{High}$ reduces the average government debt-to-GDP ratio. This is because when $\pi^{High}$ is large, the cost of inflation surpasses the cost of default. As concern for inflation replaces concern for default, further increases in $\pi^{High}$ discourage government borrowing.

1.5.2 Frequency of Default

As the monetary backstop increases the default threshold of the government, it also reduces the frequency of government default. Figure 1-8 shows the frequency of government default as $\pi^{High}$ varies. We can see that when there is no monetary backstop, the government defaults once every 120 years on average. For the model with the monetary backstop, the frequency of default decreases significantly as $\pi^{High}$ increases. Specifically, for $\pi^{High}$ above 8 percent, the frequency of default reaches zero. There is no risk of government default in this case.

While the monetary backstop reduces the risk of government default, it does
not simply replace defaults with episodes of high inflation. The red dashed line in Figure 1-8 plots the frequency of high inflation in response to the variation of $\pi^{High}$. We observe that, when $\pi^{High}$ is small, the monetary backstop is enacted often in equilibrium. The risk of high inflation is high. However, as $\pi^{High}$ increases, the frequency of high inflation drops significantly. Specifically, when $\pi^{High}$ is larger than 10 percent, as the risk of default reaches zero, the frequency of high inflation associated with the monetary backstop is much smaller than the frequency of default when there is no monetary backstop.

The result indicates that when the central bank provides a monetary backstop, it is better to choose a relatively high inflation target for the monetary backstop. If $\pi^{High}$ is too small, the seigniorage is not large enough to remove the risk of rollover crisis, the monetary backstop is triggered often in equilibrium, and the government can still default even after receiving the monetary backstop. In this case, both the risk of default and the risk of inflation are high. By contrast, when $\pi^{High}$ is sufficiently large, knowing that the central bank can provide the monetary backstop increases the default threshold by a large amount. This allows the government to rollover its debt and adjust later rather than defaulting or actually receiving the monetary backstop. The monetary backstop is rarely enacted in equilibrium. As a result, both the risk of default and the risk of inflation are low.

### 1.6 Welfare

The previous section showed that the monetary backstop increases the default threshold of the government and reduces the risk of default. At the same time, the monetary backstop may lead to costly high inflation and inefficient debt accumulation. In this section, I examine this trade-off by looking at how the monetary backstop affects
To quantify the impact of the monetary backstop on household welfare, I perform two numerical exercises. In the first exercise, I examine the welfare benefit of unexpectedly introducing a monetary backstop. To do so, I first simulate an economy with no monetary backstop for \( t \) periods. This initial period is chosen to be sufficiently long to ensure that, at time \( t \), the state of the economy is random. I then introduce a monetary backstop at time \( t \). The welfare benefit of the monetary backstop is calculated as follows

\[
\Delta V = \frac{(V^\pi_t - V^\emptyset_t)}{V^\emptyset_t}
\]  

(1.33)

Here \( V^\pi_t \) is household welfare after the introduction of monetary backstop at time \( t \) and \( V^\emptyset_t \) is household welfare if the monetary backstop is not introduced into the economy. Because the value function is homothetic in the trend income, by dividing the difference with \( V^\emptyset_t \), the welfare benefit is measured in equivalent changes of the trend income.\(^1\)

I run 10000 simulations. Figure 1-9 shows the average welfare gain of introducing the monetary backstop across simulations. We can see that for any \( \pi^{High} \) below 30 percent, introducing a monetary backstop increases household welfare. Under the calibration of the model, the welfare maximizing level of \( \pi^{High} \) is 15 percent. Introducing a monetary backstop of this size is equivalent with permanently increasing the trend income by about 4 percent. When \( \pi^{High} \) is too low or too high, the welfare gain is small. In the next paragraph, I discuss the intuition by decomposing the welfare impact of the monetary backstop.

The monetary backstop affects household welfare through three channels: (i) reducing the frequency of default; (ii) reducing the cost of taxation; (iii) reducing

\(^1\)See Appendix A.4 for the derivation.
money demand. To analyze the contribution of each channel, I decompose the welfare benefit of the monetary backstop into these three effects.

\[ \Delta V = \Delta V_\delta + \Delta V_\tau + \Delta V_m \] (1.34)

Here \( \Delta V_\delta \) is the welfare benefit due to fewer defaults, \( \Delta V_\tau \) is the welfare benefit due to lower cost of taxation, and \( \Delta V_m \) is the welfare cost of high inflation. Figure 1-10 shows how each component varies with \( \pi^{High} \).

First, the black line in Figure 1-10 shows that the monetary backstop improves household welfare by reducing the risk of government default. When \( \pi^{High} \) is small, default happens frequently. As a result, the welfare gain of the monetary backstop is small.

Second, the blue line in Figure 1-10 shows that the monetary backstop reduces household welfare by reducing money demand. The magnitude of this effect depends on the frequency of high inflation as well as the cost of each high inflation episode. When \( \pi^{High} \) is small, the cost of each high inflation episode is small but the frequency of inflation is high. When \( \pi^{High} \) is large, each high inflation episode becomes costly, which reduces the welfare gain of the monetary backstop.

Lastly, the red line in Figure 1-10 shows the welfare gain from reducing the cost of taxation. This is due to (i) seigniorage directly replacing taxes and reducing the cost of taxation and (ii) the monetary backstop increasing the default threshold, which allows the government to smooth the tax rate by accumulating debt. Under my calibration of the model, I found that the second effect dominates.

In the second exercise, I examine the welfare cost for an economy that suddenly loses a monetary backstop. An incident like this can occur either when the central bank retreats from its position of providing the monetary backstop for government debt or when the market suddenly realizes that its initial belief in the central bank’s
commitment on providing the monetary backstop is false. This scenario is relevant for Euro countries, given that support from the ECB for government debt has been frequently questioned over the last decade.

In this exercise, I first simulate an economy with a monetary backstop for $t$ periods. Then I remove the monetary backstop at time $t$. The welfare cost of removing the monetary backstop in the model is computed as

$$\Delta V_t = \frac{(V^\pi_t - V^\varphi_t)}{V^\varphi_t} \quad (1.35)$$

Here $V^\varphi_t$ is household welfare after the monetary backstop is removed in period $t$, and $V^\varphi_t$ is household welfare if the monetary backstop is not removed. The welfare cost of removing the monetary backstop is measured in equivalent changes of the trend income.

Figure 1-11 shows the average welfare cost of removing the monetary backstop over 10000 simulations. Compared with Figure 1-9, the cost of removing the monetary backstop is larger than the benefit of introducing it. The largest welfare cost happens when $\pi^{High}$ is 15 percent, and is equivalent with permanently reducing the trend income by more than 5 percent.

Figure 1-12 decomposes the welfare cost of removing the monetary backstop into three effects: (i) Higher cost of taxation; (ii) More frequent default; (iii) Higher money demand. Compared with Figure 1-10, the cost of default has a larger welfare impact in removing the monetary backstop than it does in the introduction of the monetary backstop. This is because, in an economy which previously had a monetary backstop, the default threshold is high, and the government tends to accumulate a large amount of debt. A sudden removal of the monetary backstop abruptly reduces the government default threshold, exposing the government to large risks of default.

This intuition is confirmed by Figure 1-13, which shows the probability of default in
the period of removing the monetary backstop. We can see that the government has a
40 percent chance of defaulting immediately after the removal of the monetary backstop.
The large probability of default implies that removing the monetary backstop can be
very costly to the economy. By contrast, in an economy where there is originally no
monetary backstop, the government adopts a prudent fiscal policy, and the frequency
of default is lower. Even though the introduction of a monetary backstop reduces the
frequency of default, the welfare gain through this channel is not as large.

This result highlights the asymmetric nature of the monetary backstop. When
agents in the economy do not expect the central bank to provide the monetary
backstop, the welfare benefit for introducing it may be small. However, for an economy
which previously had the monetary backstop, indicating an intention to remove it
can exposes the government to large risks of default. The asymmetric effect of the
monetary backstop provides an explanation for the dynamics of the European Debt
Crisis. Before the European Debt Crisis, investors implicitly expected the ECB to
provide monetary backstop for Euro countries in time of crises. As a result, peripheral
countries were able to accumulate large amounts of debt at relatively low cost. Later
during the European Debt Crisis, investors started worrying that their initial belief of
the monetary backstop might have not been correct, which leads to large increases in
the yield of government bond of indebted peripheral countries. The panic subsided
only after the ECB showed that it was committed to avoid government default by
announcing the Outright Monetary Transactions programme (OMT).
Figure 1-6. The blue line shows the default threshold for different calibrations of $\pi^{High}$. The red line indicates the default threshold when there is no monetary backstop.
**Figure 1-7.** The average government debt-to-GDP ratio for an economy with a monetary backstop (blue line) and an economy with no monetary backstop (red line).

**Figure 1-8.** The frequency of defaults (blue solid line) vs the frequency of high inflation episodes (red line). The blue dashed line indicates the frequency of defaults when there is no monetary backstop.
Figure 1-9. The welfare gain from introducing the monetary backstop. The welfare gain is measured as fraction of permanent income.

Figure 1-10. Decomposition of the welfare gain from introducing a monetary backstop. Here $\Delta V_m$ is the welfare decrease due to a lower demand for money, $\Delta V_T$ is the welfare gain due to lower cost of tax, and $\Delta V_\delta$ is the welfare gain due to lower cost of default.
Figure 1-11. The welfare loss from removing the monetary backstop. The welfare loss is measured as fraction of permanent income.

Figure 1-12. Decomposition of the welfare loss from removing a monetary backstop. Here $\Delta V_\gamma$ is the welfare cost due to higher cost of tax, $\Delta V_\delta$ is the welfare cost due to more frequent default, and $\Delta V_m$ is the welfare increase due to a higher demand for money.
Figure 1-13. Frequency of government default in the period immediately when the monetary backstop is removed
1.7 Conclusion

This paper studies how a monetary backstop affects the sustainability of the government debt and household welfare. I present a model where the government adjusts fiscal policy infrequently and the central bank can raise the inflation rate for a sustained period of time in order to avoid government default. The model is able to explain the long-run dynamics of the government debt-to-GDP ratio of advanced economies. In particular, the model predicts that when the trend growth rate falls, the government is slow in adjusting primary expenditure, which continues along its original path and leads to a persistent increase in government debt. I test model predictions using the event study method and find consistent evidence. Finally, I calibrate the model and conduct quantitative analysis.

The results indicate that a monetary backstop can significantly increase the government’s debt limit. When the government has a fiscal deficit, uncertainty about fiscal adjustment exposes the government to risks of a rollover crisis. The government can end up defaulting even with a relatively low debt-to-GDP ratio when there is no monetary backstop. The monetary backstop provides the government with seigniorage, which can remove the risk of a rollover crisis and increase the government’s debt limit.

The quantitative analysis highlights that the risk of inflation crucially depends on the size of the monetary backstop. When the monetary backstop is sufficiently large, knowing that the central bank can provide the monetary backstop lifts the debt limit of the government and allows the government to roll over its debt without actually enacting the monetary backstop. In equilibrium, the risk of inflation is low and the monetary backstop increases household welfare.

This paper suggests avenues for future research. Given that the inflation rate is determined one period in advance and the government issues short-term debt, the
model addresses the impact of the seigniorage but avoids the possibility of using surprise inflation to reduce the real value of government debt. The model could be extended to include long-term nominal government debt, and thereby used to study the impact of surprise inflation on government debt dynamics.

Finally, the political economy of slow fiscal adjustment deserves further study. In particular, income inequality coincides with political gridlock in advanced economies (Ma, 2014; Voorheis and McCarty, 2015; Yared, 2018). One could extend the model in this paper to include income heterogeneity among households and study how income inequality mutually interacts with fiscal inertia and a monetary backstop.
Chapter 2

Sovereign Debt Crises and Banks Bailouts: Why Does Inequality Matter for Fiscal Policy Efficiency?

Co-author: Sandra Valentina Lizarazo-Ruiz and Horacio Sapriza

2.1 Introduction

This paper aims to understand the role of the economy’s income distribution in determining the optimal level of government guarantees for banks’ liabilities. It considers the potential benefits (reducing financing cost in the economy) and the potential costs (increasing the potential for over-lending by the banks plus the possible transfer of risk from the financial sector to the government) of the guarantees and how they change in relative terms depending on the share of the population that benefits more directly from the guarantees versus the share of the population that bears the cost burden of those guarantees.

Government guarantees to banks’ liabilities have been used in numerous countries in recent history, mostly around periods of increased financial distress.1 The rationale

1For the period between 1983 and 2003 Laeven and Valencia (2008) list 18 countries that provided government’s guarantees to banks’ liabilities, either on the mist of a banking crisis or just when financial pressures started to show - Chile 1983, Finland 1992, Sweden 1992, Mexico 1993, Turkey
for the use of these guarantees lies in the fact that they can help reduce funding
costs for individual institutions by enhancing the market perception of their resilience
to negative shocks. However, the expectation of banking sector rescues by the
government can create moral hazard for banks to participate in excessive risk-taking
and unsustainable lending practices. In addition, government guarantees to banks
also lead to risk transfer between the financial sector and the government. Studies
that have focused on this trade-off include, for example, Bianchi et al. (2021) and
Hur et al. (2021). The conclusion on the desirability of these guarantees vary widely
across studies with many studies find that guarantees might have a potential negative
impact leading to “financial excess” where credit is larger than what would be the
optimal level while other studies find that these type of guarantees have the potential
to stabilize the economy under the right circumstances.

Beyond the analysis of the trade-offs between incentives and liquidity support,
another kind of consideration that takes a central role in the discussion is the social
fairness of government guarantees to the financial sector. Most of the time, rescues
across the world have elicited two opposing views in public opinion: On one side, those
who consider the banking sector bailouts a necessary evil to prevent the collapse of the
financial system and the dire repercussions to the economy as a whole. On the other
side, those who consider the transfer of large sums of money to support the solvency
of financial institutions after periods of excessive risk-taking on their side as socially
unacceptable, corresponding to the socialization of private losses, benefiting few while
generating a fiscal burden to all. Financial sector bailouts by the government are

Republic 2003. According to IMF (2009) during the Global Financial Crisis as many as 13 of the
G-20 countries extended government guarantees of wholesale borrowing - Australia, Canada, France,
Germany, Italy, Mexico, Netherlands, Russia, Saudi Arabia, Spain, South Korea, United Kingdom
and United States-.
therefore likely to face some degree of public opposition.

In addition to different political views in the population on the role of the government, an element that might affect the level of public support for government guarantees to the financial sector is the income distribution among the population. It is reasonable to expect that, independently, if the expected cost per-capita of these guarantees is large in comparison to the income of the less well-off segments of the population, the program might face opposition, consequently constraining the policy response of the government.

Evidence of the large cost of government support to banks’ liabilities between 2008-2015 in Europe is shown in Table 2.1, with the average maximum approved cost per-capita for the guarantees of €17,131 for the group of countries in the table (those that decided to offered guarantees to banks), and an average cost of actual per-capita guarantees granted (those that had to be honored) of €7,552. These two values correspond respectively to 156.99% and 69.21% of the average highest income level for the first quartile of the disposable income distribution or 56.75% and 25.02% of the average per-capita income for the countries in the table (see Table 2.2 for individual countries values).

At the same time, the direct benefits of the guarantees accrued to the relatively small share of population in the economy working in the financial sector (according to Table 2.1 employment in domestic credit institutions in the countries in the table was about 1.73% of total employment), though indirect effects that are much harder to measure reached a much larger population. Regarding the intrinsic unequal impact of the guarantees to banks’ liabilities, Denk et al. (2014)’s empirical findings suggest that in OECD countries where bank creditors didn’t incur losses in bank failure resolution cases, implicit bank debt guarantees tended to benefit bank creditors, shareholders,
and financial sector employees.

In this paper, we extend the model of endogenous sovereign default and government guarantees to the financial sector developed in Bianchi et al. (2021) to consider heterogeneity in the population. In the model, different agents derive their income from different factors of production (labor vs capital) while being subject to a common tax regimen that pays for the government provision of guarantees and other transfers from the government to the population. The government decides on the level of guarantees and debt default to maximize the social welfare which is the average utility of workers and capital owners weighted by the fraction of each group in the population.

In order to isolate the role of the income distribution, we focus on economies with identical levels of production but different income distributions. While this has the cost of ignoring additional possible interactions between distribution and macroeconomic performance, it allows us to derive a clear vision of whether income inequality matters in terms of sovereign and financial risk and in terms of the limitations for fiscal policies in responding to those risks.

Using the calibrated model, we find that higher income inequality reduces the optimal level of government guarantees to the financial sector and makes financial crises deeper. While the government might find it optimal to provide guarantees to banks’ liabilities in order to reduce the likelihood of banks’ failures in a representative agent setting such as Bianchi et al. (2021), with higher degree of income inequality, the utility cost of government guarantees is higher for the poorer agents and for the society as a whole. Therefore, the optimal level of guarantees provided to the financial sector is lower, reducing insurance for the financial system. As a result, higher income inequality increases the borrowing cost of banks, and leads to a higher number of bank failures.
Table 2.1. Total and Per-capita Costs of Guarantees to Bank Liabilities 2008-2015 and Employment in the Financial Sector

<table>
<thead>
<tr>
<th>Country</th>
<th>Bailout Cost</th>
<th>Per-capita Cost</th>
<th>Financial sector employment to total employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max approved</td>
<td>Used</td>
<td>Max expected</td>
</tr>
<tr>
<td>Austria</td>
<td>75.0</td>
<td>19.3</td>
<td>9,014</td>
</tr>
<tr>
<td>Belgium</td>
<td>275.8</td>
<td>46.8</td>
<td>25,751</td>
</tr>
<tr>
<td>Cyprus</td>
<td>6.0</td>
<td>2.8</td>
<td>4,286</td>
</tr>
<tr>
<td>Denmark</td>
<td>580.0</td>
<td>145.0</td>
<td>105,646</td>
</tr>
<tr>
<td>France</td>
<td>319.8</td>
<td>92.7</td>
<td>4,918</td>
</tr>
<tr>
<td>Germany</td>
<td>447.8</td>
<td>135.0</td>
<td>5,454</td>
</tr>
<tr>
<td>Greece</td>
<td>93.0</td>
<td>62.3</td>
<td>8,363</td>
</tr>
<tr>
<td>Hungary</td>
<td>5.4</td>
<td>0.0</td>
<td>538</td>
</tr>
<tr>
<td>Ireland</td>
<td>376.0</td>
<td>284.3</td>
<td>83,741</td>
</tr>
<tr>
<td>Italy</td>
<td>80.0</td>
<td>85.7</td>
<td>1,344</td>
</tr>
<tr>
<td>Latvia</td>
<td>5.1</td>
<td>0.5</td>
<td>2,339</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>4.5</td>
<td>3.8</td>
<td>9,184</td>
</tr>
<tr>
<td>Netherlands</td>
<td>200.0</td>
<td>40.9</td>
<td>12,158</td>
</tr>
<tr>
<td>Portugal</td>
<td>28.2</td>
<td>16.6</td>
<td>2,712</td>
</tr>
<tr>
<td>Slovenia</td>
<td>12.0</td>
<td>2.2</td>
<td>5,825</td>
</tr>
<tr>
<td>Spain</td>
<td>200.0</td>
<td>72.0</td>
<td>4,276</td>
</tr>
<tr>
<td>Sweden</td>
<td>156.0</td>
<td>19.9</td>
<td>16,920</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>364.5</td>
<td>158.2</td>
<td>5,897</td>
</tr>
</tbody>
</table>

Note: Bailout costs are measured in billions of Euros. Per-capita costs of guarantees to banks’ liabilities and employment shares of financial institutions are computed taking in account population and employment figures of the year of the announcement. Announcement years: 2008 Austria, Belgium, Denmark, France, Germany, Hungary, Ireland, Latvia, Luxembourg, Netherlands, Sweden, United Kingdom; 2010 Greece; 2012 Italy and Spain; 2013 Cyprus and Slovenia; 2014 Portugal.
Source: Bailout Costs come from Millaruelo and del Río (2017); Population and Employment rate World Bank; Employees in domestic credit institutions EU structural financial indicators 2010 & 2015 releases.
### Table 2.2. Per-capita Costs of Guarantees to Banks Liabilities 2008-2015, as share of lowest income quartile top cut-off income

<table>
<thead>
<tr>
<th>Country</th>
<th>Nominal per-capita income</th>
<th>Disposable Income</th>
<th>First quartile top cut-off income level</th>
<th>Gini</th>
<th>Max expected</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>35,302</td>
<td>13,037</td>
<td>27.7</td>
<td>69.14%</td>
<td>17.79%</td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td>32,843</td>
<td>11,880</td>
<td>27.5</td>
<td>216.76%</td>
<td>36.78%</td>
<td></td>
</tr>
<tr>
<td>Cyprus</td>
<td>20,877</td>
<td>10,380</td>
<td>32.4</td>
<td>41.29%</td>
<td>23.66%</td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>44,012</td>
<td>16,973</td>
<td>25.1</td>
<td>622.43%</td>
<td>155.61%</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>30,950</td>
<td>12,926</td>
<td>29.8</td>
<td>38.05%</td>
<td>11.14%</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>31,013</td>
<td>12,120</td>
<td>30.2</td>
<td>45.00%</td>
<td>13.56%</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td>20,324</td>
<td>7,148</td>
<td>32.9</td>
<td>117.00%</td>
<td>78.39%</td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td>10,240</td>
<td>3,055</td>
<td>25.2</td>
<td>17.61%</td>
<td>0.00%</td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td>41,824</td>
<td>14,935</td>
<td>29.9</td>
<td>560.70%</td>
<td>423.96%</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>27,282</td>
<td>9,686</td>
<td>32.4</td>
<td>13.88%</td>
<td>14.86%</td>
<td></td>
</tr>
<tr>
<td>Latvia</td>
<td>11,234</td>
<td>2,443</td>
<td>37.5</td>
<td>95.74%</td>
<td>9.37%</td>
<td></td>
</tr>
<tr>
<td>Luxembourg</td>
<td>78,028</td>
<td>21,366</td>
<td>27.7</td>
<td>42.98%</td>
<td>36.30%</td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td>39,354</td>
<td>13,963</td>
<td>27.6</td>
<td>87.07%</td>
<td>17.80%</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>16,638</td>
<td>4,985</td>
<td>34.5</td>
<td>54.40%</td>
<td>32.02%</td>
<td></td>
</tr>
<tr>
<td>Slovenia</td>
<td>18,763</td>
<td>8,115</td>
<td>24.4</td>
<td>71.78%</td>
<td>13.16%</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>22,044</td>
<td>8,123</td>
<td>34.2</td>
<td>52.64%</td>
<td>18.95%</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>34,048</td>
<td>13,680</td>
<td>25.1</td>
<td>123.68%</td>
<td>15.77%</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>28,582</td>
<td>11,606</td>
<td>33.9</td>
<td>50.81%</td>
<td>22.05%</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Source: Nominal per-capita income data comes from the World Bank National Accounts Data. The first quartile top cut-off level and the Gini for the disposable income distribution come from Eurostat (EU-SICL survey). Figures are for the years of announcement in table 2.1.
In addition, in line with previous papers in the literature of sovereign default and inequality (for example Andreasen et al. (2019) and Ferriere (2015)), we find that a higher level of income inequality tends to increase the likelihood of sovereign default. This is because at higher levels of inequality, the welfare cost of debt repayment is especially high for the poorer households due to the concavity in household preferences, making the weighted average level of social welfare in the economy observed during downturns much lower than what would be observed for an economy with a representative agent, therefore increasing the incentives to default in comparative terms.

Furthermore, we extend our model in two directions. First, we consider the role of tax progressivity while keeping revenue to GDP constant to determine whether a more progressive tax policy can help achieve a better equilibrium outcome in economies that would otherwise be more unequal. Although progressive taxes might introduce some additional distortions into the productive process, their impact on the overall equilibrium is not obvious. We find that despite reducing the benefits from production to capital owners, more progressive taxation improves the distribution of costs of government guarantees, making its impact on poorer households less harmful, leading to an increase in the social welfare and a lower risk of financial crisis and sovereign default.

Second, we explore the robustness of our results to an alternative decision rule for fiscal policies. In this extension, instead of having a benevolent government maximizing social welfare, government policies on guarantees are determined by the preference of the median voter. We find that compared with the case of a benevolent government, the equilibrium level of government guarantees under the median voter regime is significantly lower when inequality is high. This is because when inequality is high, the benefit of government guarantees is distributed towards capital owners, while
the cost is shared equally among all voters. Consequently, workers gain little from guarantees and are increasingly against government guarantees to the financial sector when inequality becomes higher. Since the majority of the median voter regime are workers, government policy is determined by the preference of workers. As a result, the optimal level of government guarantee is lower.

The paper proceeds as follows: Section 2.2 reviews the related literature; Section 2.3 develops the model; Section 2.4 calibrates the model to the economy of Ireland. Section 2.5 presents the numerical results of the paper. Section 2.6 and 2.7 discuss two extensions of the model. Section 2.8 concludes.

2.2 Literature

This paper contributes to the theoretical framework of sovereign default which was introduced by Eaton and Gersovitz (1981), and later developed by Aguiar and Gopinath (2006) and Arellano (2008). Specifically, this paper relates to two strands of literature.

First, this paper relates to the literature on government bailouts to banks and the link between banking crises and sovereign defaults. Kaminsky and Reinhart (1999) present empirical evidence that banking crises often precede or accompany sovereign debt crises. Subsequent theoretical literature examine the feedback loop between banking crises and sovereign crises. Acharya et al. (2014) introduce a model where the government provides bailouts to banks to reduce the debt overhang of the financial sector. They show that a government bailout increases the government credit risk, which in turn reduces the credibility of the bailout and the value of the government bonds held by the financial sectors. Hur et al. (2021) study optimal bailout policies where shocks to bank capital can trigger banking crises and the government can provide asset guarantees to relax the financial constraint of the bank. They show that
even though bailouts mitigate the adverse effects of banking crises, it is optimal to ban bailout from an ex-ante perspective. Capponi et al. (2020) propose a model where banks are heterogeneous in their holdings of sovereign bonds. They study how the financial network structure and the distribution of ownership of sovereign debt within the banking sector affect the optimal bailout policy under the feedback loop between banking crises and sovereign debt crises. Our paper extends the model in Bianchi et al. (2021) which focuses on the risk transmission from the banking sector to the government through government bailouts. Our paper differs from previous papers by having heterogeneity among households and examining how income inequality affects the optimal level of the bailout.

Second, this paper contributes to the literature on the relationship between inequality and sovereign default. D’Erasmo and Mendoza (2016b) propose a model where the government issues domestic debt, and households differ from each other in their wealth and government bond holdings. They show that domestic sovereign defaults are driven by distributional incentives of the government. Ferriere (2015) shows that income inequality of households increases the risk of government default, and a progressive tax is an effective policy tool to reduce sovereign credit spreads. Andreasen et al. (2019) study how income distribution impacts the political economy of government borrowing and sovereign default. They show that income inequality and regressive taxes make defaults more likely for a given level of debt. Our paper considers households who derive income from different factors of production and studies how the income distribution of workers vs capital owners affect the government’s choice of bailout to banks and the risk of sovereign default.
2.3 Model

This section describes the model environment in detail. The model adopts the framework developed by Bianchi et al. (2021), where banks intermediate risky loans to firms in order to facilitate production. Government guarantees increase firms’ production by reducing the borrowing cost of firms. Our model extends the framework in one important dimension by having heterogeneity among households. The model features two types of households, who derive their income from different factors of production (labor vs capital). This setting allows us to study the effect of income inequality on government guarantees and the risk of sovereign default in the model.

2.3.1 Setup

Time is discrete and indexed by $t$. The model has a small open economy where both the government and the banking sector borrow from foreign investors. Firms employ labor and capital to undertake production subject to productivity shocks. Domestic banks intermediate loans to the firms in the economy.

2.3.2 Household

There are two types of households: workers and capital owners. Workers account for a fraction $\mu_L$ of all households. The share of capital owners in the population is $\mu_K = 1 - \mu_L$. The size of the total population is normalized to one.

2.3.2.1 Workers

Workers choose labor supply $n_t$, and consumption $c^L_t$ to maximize their utility:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t^L - g(n_t))\right]$$

(2.1)
Households (including both workers and capital owners) do not borrow directly from foreign lenders, instead, the government borrows and pays a lump sum transfer $T_t$ to smooth household consumption.

As a result, a worker’s problem reduces to the following static problem:

$$U^L = \max_{c_t, n_t} E_0 \left[ u(c_t - g(n_t)) \right]$$  \hspace{1cm} (2.2)

s. t.

$$c_t = (1 - \tau^N) w_t \theta n_t + T_t$$  \hspace{1cm} (2.3)

Here $\theta$ measures the individual productivity of the worker. A worker receives wage rate $w_t$ per unit of efficient labor, and is subject to income tax $\tau^N$.

The felicity function is in the form of constant relative risk aversion $u(\cdot) = \frac{(\cdot)^{1-\sigma}}{1-\sigma}$. The labor disutility function $g(\cdot)$ is in isoelastic form

$$g(n_t) = \frac{n_t^\omega}{\omega}$$  \hspace{1cm} (2.4)

where $\omega > 1$, so the Frisch elasticity of labor supply is $\frac{1}{\omega-1}$.

In order to isolate the effect of income inequality, I adjust $\theta$ to ensure that, for different $\mu_L$, aggregate labor supply is constant.\footnote{Without this adjustment, when $\mu_L$ varies, the population of workers changes as well, which leads to mechanical changes in aggregate labor supply.} I describe the details in the following paragraph.

The optimality condition for an individual worker is:

$$g'(n_t) = (1 - \tau^N) w_t$$  \hspace{1cm} (2.5)

Let $N_t$ be aggregate labor supply, $N_t = \mu_L n_t$

$$\left(\frac{N_t}{\mu_L}\right)^{\omega-1} = (1 - \tau^N) w_t$$  \hspace{1cm} (2.6)
Multiply both sides by \( \theta \), and define wage per unit of labor \( W_t = w_t \theta \):

\[
\left( \frac{\theta}{\mu_L} \right) N_t^{\omega-1} = (1 - \tau^N) \frac{w_t \theta}{w_t} = W_t
\]  

(2.7)

Here we adjust \( \theta = \mu_L^{\omega-1} \) to ensure that aggregate labor supply is determined by the after-tax wage rate and does not directly depend on \( \mu_L \):

\[
W_t = \frac{N_t^{\omega-1}}{1 - \tau^N}
\]  

(2.8)

### 2.3.2.2 Capital Owners

Capital owners receive the profit of firms and banks, both of which are subject to the capital tax \( \tau^K \).

\[
U^K = \mathbb{E} \left[ \sum_{t=0}^{\infty} u(c^K_t) \right]
\]  

(2.9)

s. t.

\[
c^K_t = \frac{(1 - \tau^K)(\Pi^F_t + \Pi^B_t)}{\mu_K} + T_t
\]  

(2.10)

Here \( \Pi^F \) and \( \Pi^B \) are profits of firms and banks.

### 2.3.3 Firms

Firms employ labor \( N_t \), time-invariant capital \( k \), and imported intermediate inputs \( M_t \) to produce final goods \( y_t \). The production function is Cobb-Douglass:

\[
y_t = z_t M_t^{\alpha_M} (\theta N_t)^{\alpha_N} k^{\alpha_k}
\]  

(2.11)

Here \( 0 < \alpha_M, \alpha_N, \alpha_k < 1 \), and \( \alpha_M + \alpha_N + \alpha_k = 1 \).

We regroup aggregate productivity \( z_t \) and individual productivity \( \theta \) together as total productivity \( Z_t \):

\[
y_t = \frac{Z_t}{z_t \theta^{\alpha_N}} M_t^{\alpha_M} (N_t)^{\alpha_N} k^{\alpha_k}
\]  

(2.12)
We assume that total productivity $Z_t$ follows an AR(1) process.

Intermediate inputs $M_t$ are purchased from the world market at a constant price $p^*$. At the beginning of each period, firms borrow intermediate loans $l_t$ from banks to pay for a fraction $\kappa$ of the cost of intermediate inputs in advance. These loans are repaid at the end of the period at an interest rate $r^F_t$.

Firms maximize their profits:

\[
\max_{N_t, M_t^*, l_t^F} \Pi^F_t = y_t - \mathcal{W}_t N_t - p^* M_t^* + l_t^F - (1 + r^F_t)l_t^F
\]

s. t.

\[
\kappa p^* M_t^* \leq l_t^F
\]

The first order conditions of the firms’ optimization problem determine the demand for labor and imported inputs:

\[
\frac{\partial \Pi^F_t}{\partial N_t} = \alpha^N Z_t M_t^* \alpha^M N_t^{(\alpha^M - 1)} k^{\alpha_k} - \mathcal{W}_t = 0 \quad (2.15)
\]

\[
\frac{\partial \Pi^F_t}{\partial M_t^*} = \alpha^M Z_t M_t^* (\alpha^M - 1) N_t^\alpha^N k^{\alpha_k} - (1 + \kappa r^F_t)p^* = 0 \quad (2.16)
\]

### 2.3.4 Banks

There is a measure one of ex-ante identical banks. At the beginning of each period, banks make intra-period working capital loans $l_t^F$ to firms at rate $r^F$. To finance these loans, banks issue bonds $l_t^B$ in the international financial market at rate $r^B$. At the end of each period, banks are hit with idiosyncratic shocks $\varrho$ and decide whether to go bankrupt.

The profit of banks can be expressed as:

\[
\Pi^B = \begin{cases} 
(1 + r^F)l_t^F - (1 + r^B)l_t^B - a(l_t^F) - \varrho & \text{if no-bankruptcy} \\
\phi(1 + r^F)l_t^F - e \left[ a(l_t^F) + \varrho \right] & \text{if bankruptcy}
\end{cases} \quad (2.17)
\]
Here $a(l^F)$ is the cost of producing loans, and $\varrho$ measures the fixed operation cost of each bank. The coefficient $e \in (0, 1)$ measures how the banking industry is being regulated. Higher $e$ indicates more regulations and larger monitoring costs. At the end of the period, if a bank does not declare bankruptcy, it receives loan payments from firms and repays its debt. If a bank declares bankruptcy, the bank retrieves a fraction $\phi$ of its loans, and pays the monitoring cost, which equals to $e$ times $(a(l^F) + \varrho)$.

A bank declares bankruptcy if the payoff of bankruptcy is larger than the payoff of repaying the debt. For each bank, the decision of bankruptcy depends on its idiosyncratic cost $\varrho$. Let $\varrho^*$ be the value of $\varrho$ that equalizes the payoff of debt repayment and the payoff of bankruptcy. The threshold value $\varrho^*$ is given by:

$$
(r^F - r^B)l^F - a(l^F) - \varrho^* = \phi(1 + r^F)l^F - e \left[ a(l^F) + \varrho^* \right]
$$

(2.18)

If $\varrho$ is greater than the threshold value $\varrho^*$, then the bank decides to go bankrupt. If $\varrho < \varrho^*$, then the bank will repay its debt. When $\varrho = \varrho^*$, the bank is indifferent between bankruptcy and repayment.

The shock $\varrho$ follows a uniform distribution $\varrho \sim U(0, \varrho_{max})$. Consequently, the probability of bank bankruptcy is:

$$
\pi = 1 - \frac{\varrho^*}{\varrho_{max}}
$$

(2.19)

It is worth noting that, because there is a continuum of banks, $\pi$ is also equal to the share of banks that declare bankruptcy.

Lastly, I assume that banks are perfectly competitive and the expected profit of each bank is zero.

$$
\mathbb{E} \Pi^B(l^F, l^B) = 0
$$

(2.20)

which gives the supply of firm loans:

$$
r^F = r^B \frac{1 - \pi}{1 - \pi(1 - \phi)} + a \frac{1 - \pi(1 - e) - \phi \pi}{1 - \pi(1 - \phi)} + \frac{\mathbb{E} \left[ \varrho^* + e(\varrho_{max} - \varrho^*) \right]}{2l^F}
$$

(2.21)
\((l^F, r^F, \varrho^*)\) will be computed simultaneously by combining equation (2.18), (2.21), and (2.36).

### 2.3.5 Foreign Lenders

There is a large number of risk neutral foreign lenders, who purchase the bonds of domestic banks and the bonds of government as long as the expected payoff of the bonds are larger than the cost of the funds.

Specifically, the foreign lenders purchase the bonds of banks when the following condition is satisfied:

\[
(1 - \bar{\pi})(1 + r_B) + \bar{\pi}\eta(1 + r_B) \geq 1 + r \tag{2.22}
\]

Here \(\eta\) is the fraction of the bank loans guaranteed by the government, \(\bar{\pi}\) is the expected probability of bankruptcy in next period, and \(r\) is the international risk free rate.

For government bonds, the foreign lenders purchase the bonds if the interest rate of government bonds satisfies

\[
(1 - \delta^*)(1 + r^*) \geq 1 + r \tag{2.23}
\]

Here \(\delta^*\) is the probability of default for the government and \(r^*\) is the interest rate of the government bond.

### 2.3.6 Government

The government issues default-able bonds that mature in one period to foreign investors, and rebates the proceedings of the credit operation as lump sum transfer to households.
2.3.6.1 Repayment

At the beginning of each period, the government chooses whether to default or repay its debt. In the case of repayment, the government chooses the fraction of bank loans that will be guaranteed in next period $\eta'$ and a new debt level $b'$ to maximize its value. The optimal policy $(b^*, \eta^*)$ solves:

$$V^R(b, \eta, z, \pi) = \max_{\eta', b'} \{U(c^L, c^K) + \beta \mathbb{E}_{z', \pi' | z, \pi} V(b', \eta', z', \pi')\} \quad (2.24)$$

subject to government budget constraint

$$\tau^N W_t N_t + \tau^K \Pi^F + b_t = q_t b_{t+1} + G_t + T_t \quad (2.25)$$

Here $q_t$ is the price of government bonds: $q_t = 1/(1 + r^s_{t+1})$, and $G_t$ is government expenditure on guarantees:

$$G_t = \kappa p M_t \pi \eta_t (1 + r^B) \quad (2.26)$$

The government value function is a weighted sum of the utility of workers and the utility of capital owners by their shares in the population.

$$U(c^L, c^K) = \mu_L u \left[ c^L - g(n) \right] + \mu_K u(c^K) \quad (2.27)$$

2.3.6.2 Default

In the case of government default, government debt reduces to zero and the government is excluded from the credit market in the period of default. The government guarantees to bank loans are not affected by the default of government debt. In each period after default, there is a probability $\psi$ for the government to re-enter the credit market.

Therefore, the optimal guarantee policy in default $\eta_t^*, d$ solves:

$$V^D(\eta, z, \pi') = \max_{\eta'} \{U(c^L, c^K) + \beta \mathbb{E}_{z', \pi' | z, \pi} \left[ (1 - \psi) V^D(\eta', z', \pi') + \psi V(0, \eta', z', \pi') \right] \}$$

68
subject to budget constraint

\[ \tau^N w_t N_t + \tau^K \Pi^F = G_t + T_t \]  \hspace{1cm} (2.28)

The default is also associated with a loss in the output.

\[ y^*_t = \begin{cases} y_t & \text{if } y_t \leq \bar{y} \\ \bar{y} & \text{if } y_t > \bar{y} \end{cases} \]  \hspace{1cm} (2.29)

The government decides on whether to default by comparing the value of repayment with the value of default. Let \( \delta \) be the indicator of government default, \( \delta = 1 \) when the government defaults, 0 otherwise.

\[ \delta = \begin{cases} 0 & \text{if } V^R(b, \eta, z, \pi) > V^D(\eta, z, \pi) \\ 1 & \text{if } V^R(b, \eta, z, \pi) \leq V^D(\eta, z, \pi) \end{cases} \]  \hspace{1cm} (2.30)

### 2.3.7 Market Clearing

#### Labor and Imported Inputs

Combining the supply of labor from equation (2.6) and the demand for labor and imported inputs from equation (2.15) and (2.16), we can solve

\[ N_t = \left\{ \frac{[\alpha^N N_t]^{(1-\alpha^M)} Z_k^{\alpha_k} \omega^{-1}}{[\alpha^M]^{\omega - \alpha^N - \alpha^M}} \right\}^{(1 - \tau^N \alpha^N)} \]  \hspace{1cm} (2.31)

\[ M^*_t = \left\{ \frac{[\alpha^N N_t]^{(1-\alpha^M)} Z_k^{\alpha_k} \omega^{-1}}{[\alpha^M]^{\omega - \alpha^N - \alpha^M}} \right\}^{(1 - \tau^N \alpha^N)} \]  \hspace{1cm} (2.32)

We can also solve \( y_t, \Pi^F_t, \) and \( W_t \) as follows:

\[ y_t = \left\{ \frac{[\alpha^N N_t]^{(1-\alpha^M)} Z_k^{\alpha_k} \omega^{-1}}{[\alpha^M]^{\omega - \alpha^N - \alpha^M}} \right\}^{(1 - \tau^N \alpha^N)} \]  \hspace{1cm} (2.33)

\[ \Pi^F_t = \alpha^k y_t \]  \hspace{1cm} (2.34)

\[ W_t = \frac{\alpha^N y_t}{N_t} \]  \hspace{1cm} (2.35)
Bank Loan

Given that the working capital constraint (2.14) always hold with equality, we can solve the demand for bank loans as:

\[ l_t^F = \frac{\kappa \Omega^M}{1 + \kappa r_t^F} y_t \]  

(2.36)

Combining the demand for bank loans (2.36) with the supply for bank loans (2.21), and two auxiliary equations (2.18) and (2.19), we solve for \((l^F, r^F, \varphi^*, \pi)\).

Resource Constraints

The government recognizes aggregate resource constraint by combining household budget constraint (2.3) (2.10) with government budget constraint (2.25)(2.28). Consequently, the consumption of workers and capital owners are as follows.

For workers,

\[ c^L = \left\{\frac{1 - (1 - \mu_L)\tau^N}{\mu_L} \alpha^N + \tau^K \alpha^k\right\} y - G + (b - q\beta')(1 - \delta) - L \]

For capital owners,

\[ c^K = \left\{\frac{\alpha^k [1 - (1 - \mu_K)]}{\mu_K} + \tau^N \alpha^N\right\} y - G + (b - q\beta')(1 - \delta) - L \]

Here \(G\) is the government expenditure on the guarantee:

\[ G = \kappa p^* M^* \pi \eta (1 + r^B) \]  

(2.37)

\(L\) is the cost of bank bankruptcy

\[ L = \phi \pi (1 + r^F) l^F \]  

(2.38)

2.3.8 Timing

The timing of decisions in the model is as follows.
0. The government guarantee $\eta_t$ is chosen in the previous period.

1. At the beginning of the period $t$, the shocks $Z_t$ and $e_t$ arrive.

2. Given the values of the shocks and the government guarantee $\eta_t$, banks make decisions on the supply of working capital loans to firms.

3. Firms employ labor, intermediate inputs, and undertake production.

4. Bank idiosyncratic shocks $\varrho$ are realized.

5. Banks make decisions on bankruptcy based on the realizations of $\varrho$.

6. The government decides on whether to default, and chooses the guarantee for the next period $\eta_{t+1}$ and the new borrowing $b_{t+1}$.

### 2.3.9 Recursive Equilibrium

**DEFINITION.** A Recursive Equilibrium is defined by: (i) a set of government value functions: $V, V^R, V^D$; (ii) policy rules for government default $\delta$, borrowing $b^*$, and guarantees $\eta^*$; (iii) labor supply $n$, bank loans $l^F$, and consumption $c^L$ and $c^K$. (iv) government bond price $q$, wage rate $w$, and interest rate on bank loans $r^F$ and on bank bonds $r^B$ such that

1. For a given bond price $q$, the value functions $V, V^R, V^D$, and policy functions $\delta, \eta^*, b^*$ solve the government optimization problem.

2. Firms maximize their profits.


4. Workers choose labor and consumption optimally.

5. Wage rate $w$ and labor $n$ satisfy the market clearing condition for labor.
6. Interest rate \( r^F \) and the quantity of loans \( l^F \) satisfy the market clearing condition for bank loans.

7. Bond price \( q \) is given by the expected zero profit condition

### 2.3.10 Inequality

In the model, the degree of inequality among households is determined by the relationship between the labor share of domestic income \( \lambda \) and the share of workers in the population \( \mu_L \). The labor share of domestic income is

\[
\lambda = \frac{\alpha^N}{\alpha^N + \alpha^k}
\]

(2.39)

If the labor share of domestic income equals to the share of workers in the population, \( \lambda = \mu_L \), workers and capital owners have the same level of individual income. There is no income inequality in the economy. If \( \lambda > \mu_L \), workers receive larger share of income than their share in the population. As a result, workers have higher income than capital owners.

Similarly, if \( \lambda < \mu_L \), capital owners receive higher income than workers. In this case, we can prove that, pre-tax Gini Index for household income equals to the difference between the share of workers in the population and the labor share of domestic income:

\[
\text{Gini Index} = \mu_L - \lambda
\]

(2.40)

### 2.3.11 Guarantees

A main benefit of government guarantees is to reduce the borrowing cost of banks. As shown in equation (2.22), higher \( \eta \) reduces the interest rate \( r^B \), which in turn reduces the interest rate of intermediate loans \( r^F \). As the cost of intermediate loans decreases, firms employ more intermediate inputs, which increase firms' production \( y \) and household welfare.
However, government guarantees can also reduce household welfare through two channels. First, given that the tax rate is fixed, the government faces a trade off between financing the guarantees and paying a transfer to households. A higher level of government guarantees reduces transfers to households and therefore limits the ability of households to smooth their consumption. Second, in the model, the government decides on the level of guarantees one period ahead. Due to the uncertainty of the monitoring cost $e$, the share of banks that actually go bankrupt $\pi$ might be significantly larger than the expected share of bankruptcy $\tilde{\pi}$. In this case, government guarantees to banks can lead to a large amount of unexpected fiscal expenditure which can cause the government to default. In the next section, I calibrate the model to study the effect of government guarantees quantitatively.

### 2.4 Calibration

We calibrate the model to the economy of Ireland. The Irish government provided guarantees for its banking sector during the Global Financial Crisis. The cost of these guarantees worsened the fiscal position of the Irish government and led to a sovereign debt crisis in 2011. The economy of Ireland thus provides a case study for the interaction between government guarantees to banks and the risk of sovereign default.

In the model, each period is one quarter. Table 2.3 reports all parameter values. There are 20 parameters in the model. The value of five parameters $(\sigma, \beta, \omega, \alpha^M, \kappa)$ are taken from Mendoza and Yue (2008).

The parameters for the TFP process $(\sigma_z, \rho_z)$ are calibrated to match the standard deviation and autocorrelation of Irish GDP. The labor share $\alpha^N$ and capital share $\alpha^k$ are chosen to be 55 percent and 45 percent in order to match the average labor share.
and capital share of GDP between 2004 and 2018 from Central Statistics Office of Ireland. The capital stock \( k = 1.4 \) is calibrated to match the average capital-to-GDP ratio for Ireland between 2012-2018. The loss of output in autarky is set to be 2% to match the probability of default for the government bond of Ireland. The default probability is calculated using the data of the Credit Default Swap for the government bond of Ireland. The probability of re-entering in the credit market \( \theta \) is chosen to be 0.08 which implies that the economy is expected to be denied market access for about 3 years.

The mean and the standard deviation of the monitoring cost \( e \) is calibrated to match the mean and the standard deviation of the probability of bank bankruptcy of Irish banks. The probability of bankruptcy is calculated using the credit default swaps (CDS) spread data for Irish bank bonds during the period between June 2004 and December 2012. We assume that banks’ operating cost is linear \( a(l^F) = al^F \). The parameter \( a \), retaining value of bank loans \( \phi \), and the maximum fixed cost of banks \( \varrho_{max} \) are taken from Bianchi et al. (2021). The risk free interest rate \( r = 1.8\% \) is set to match the interest rate of the Irish government bond.

For the baseline calibration, we choose the share of workers in the population \( \mu_L \) to be 0.85 to match the Gini index for the household income of Ireland. We set \( \tau^N = 0.25 \) and \( \tau^K = 0.35 \) to match the government revenue-to-GDP ratio (29.1 percent) and the share of wage income tax (45.2 percent) in total tax revenue for Ireland during the period between 2004 and 2012. The data on tax revenue of Ireland is from OECD (2020, 2022). In Section 2.5 and 2.6, we vary the value of \( \mu_L \) and the tax rate to study the impact of income inequality and progressive tax on government guarantees.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std.Dev of TFP</td>
<td>$\sigma_z$</td>
<td>0.24%</td>
<td>Match the std.dev of GDP</td>
</tr>
<tr>
<td>Auto-correlation of TFP</td>
<td>$\rho_z$</td>
<td>0.94</td>
<td>Match the auto-correlation of GDP</td>
</tr>
<tr>
<td>Share of Inputs</td>
<td>$\alpha^M$</td>
<td>0.58</td>
<td>From Mendoza and Yue (2008)</td>
</tr>
<tr>
<td>Share of Labor</td>
<td>$\alpha^N$</td>
<td>0.23</td>
<td>Match labor income in GDP</td>
</tr>
<tr>
<td>Share of Capital</td>
<td>$\alpha^K$</td>
<td>0.19</td>
<td>Match capital income in GDP</td>
</tr>
<tr>
<td>Working Capital Ratio</td>
<td>$\kappa$</td>
<td>0.7</td>
<td>From Mendoza and Yue (2008)</td>
</tr>
<tr>
<td>Capital stock</td>
<td>$k$</td>
<td>1.4</td>
<td>Match Capital-to-GDP ratio</td>
</tr>
<tr>
<td>Output in Autarky (% of $y_t$)</td>
<td>$\bar{y}$</td>
<td>98%</td>
<td>Match probability of default</td>
</tr>
<tr>
<td>Probability of reentering</td>
<td>$\theta$</td>
<td>0.08</td>
<td>Standard value</td>
</tr>
<tr>
<td>Relative Risk Aversion</td>
<td>$\sigma$</td>
<td>2</td>
<td>From Mendoza and Yue (2008)</td>
</tr>
<tr>
<td>Time Discount Factor</td>
<td>$\beta$</td>
<td>0.88</td>
<td>From Mendoza and Yue (2008)</td>
</tr>
<tr>
<td>Elasticity of Labor Supply</td>
<td>$\omega$</td>
<td>1.46</td>
<td>From Mendoza and Yue (2008)</td>
</tr>
<tr>
<td>Price of Imported Inputs</td>
<td>$p$</td>
<td>0.62</td>
<td>Match average level of guarantees</td>
</tr>
<tr>
<td>Mean of Monitoring Cost</td>
<td>$\bar{e}$</td>
<td>0.421</td>
<td>Match probability of bankruptcy</td>
</tr>
<tr>
<td>Std.Dev of Monitoring Cost</td>
<td>$\sigma_e$</td>
<td>2.6%</td>
<td>Match Std.Dev of bankruptcy</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r$</td>
<td>0.018</td>
<td>Risk-free interest rate from FRED</td>
</tr>
<tr>
<td>Retaining value in bankruptcy</td>
<td>$\phi$</td>
<td>0.02</td>
<td>From Bianchi et al. (2021)</td>
</tr>
<tr>
<td>Operating Cost of Banks</td>
<td>$a$</td>
<td>0.05</td>
<td>From Bianchi et al. (2021)</td>
</tr>
<tr>
<td>Fixed Cost</td>
<td>$\varrho_{max}$</td>
<td>0.004</td>
<td>From Bianchi et al. (2021)</td>
</tr>
<tr>
<td>Share of Workers in Population</td>
<td>$\mu_L$</td>
<td>0.85</td>
<td>Match Gini Index of Ireland</td>
</tr>
</tbody>
</table>
### Table 2.4. Moments

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Data</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Capital-to-Income</td>
<td>4.14</td>
<td>4.12</td>
</tr>
<tr>
<td>Mean $\pi$</td>
<td>3.72</td>
<td>3.51</td>
</tr>
<tr>
<td>Mean $\eta$</td>
<td>41.94</td>
<td>40.04</td>
</tr>
<tr>
<td>Mean $r_s$</td>
<td>2.28</td>
<td>2.46</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>3.50</td>
<td>3.14</td>
</tr>
<tr>
<td>$\rho(y)$</td>
<td>0.93</td>
<td>0.92</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>5.65</td>
<td>5.42</td>
</tr>
<tr>
<td>Default Prob</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>Gini</td>
<td>29.9</td>
<td>29.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional Moments</th>
<th>Data</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Debt-to-GDP</td>
<td>68.6</td>
<td>41.30</td>
</tr>
<tr>
<td>Mean $r^F$</td>
<td>4.68</td>
<td>7.32</td>
</tr>
<tr>
<td>$\sigma(r_s)$</td>
<td>2.61</td>
<td>3.63</td>
</tr>
<tr>
<td>$\rho(y, c)$</td>
<td>0.87</td>
<td>0.84</td>
</tr>
<tr>
<td>$\rho(r_s, y)$</td>
<td>-0.49</td>
<td>-0.20</td>
</tr>
<tr>
<td>$\rho(r^F, c)$</td>
<td>-0.13</td>
<td>-0.21</td>
</tr>
</tbody>
</table>

**Note:** Table 2.4 compares the moments from simulation and the data. Simulation results reported are average over 5000 simulations with each simulation lasts for 15000 periods. Interest rates, and probabilities are reported in percentage.
Table 2.4 compares the key business cycle moments from the data with the simulation results. We can see that the model captures the behavior of several key moments of the economic data of Ireland. First, the model generates empirically plausible average government debt-to-GDP ratio of 41.3% which is comparable with 68.6% in the data. Second, the standard deviation of the interest rate for government bond is 3.63% in the model, which is close to the standard deviation in the data: 2.61%. Third, the model also captures the correlation between household consumption and GDP. Last, the model captures the negative correlation between GDP and the interest rate of government bond as well as the negative correlation between household consumption and interest rate of firm loans.

2.5 Results

In this section, we use the calibrated model to study how inequality affects the optimal level of government guarantees, the stability of banks, and the risk of sovereign default. We find that a high degree of income inequality reduces the optimal level of government guarantees to banks, increases the probability of bank bankruptcy, and leads to higher risks of sovereign default.

2.5.1 Guarantees and Bankruptcy

To study the effect of income inequality, we vary the value of $\mu_L$, and examine how the optimal level of government guarantees to banks, the probability of bank bankruptcy, and the probability of government default change.

For any given $\mu_L$, we run 5000 simulations. Each simulation lasts for 12500 periods. Using the simulated data, we compute the average level of government guarantees to banks, the average probability of bank bankruptcy, and the frequency of government default.
Figure 2-1 plots the average level of government guarantees $\bar{\eta}$ as a function of $\mu_L$. We can see that the income inequality reduces government support for banks. When $\mu_L = 0.55$, the income is equally distributed among households. In this case, the optimal government guarantees cover for 100 percent of bank liabilities. As $\mu_L$ increases, government guarantees become smaller. When $\mu_L$ is above 0.99, government guarantees reduce to zero and the model converges to a model with no government guarantees.

The intuition is the following. When inequality is high, consumption level of the poorer households is low. Due to the concavity of utility function, the utility cost of guarantees (in the form of reduction of government transfers) for poorer households is high. At the same time, the poorer households account for a large fraction of population, and the utility of poorer households has a large weight in the social welfare. Therefore, high income inequality reduces the incentive for the government to provide guarantees to banks.

Higher income inequality also increases the risk of banking crises. Figure 2-2 shows the average fraction of banks that go bankrupt in equilibrium as $\mu_L$ varies. We can see that, as $\mu_L$ increases from 0.5 to 0.99, the probability of bank bankruptcy increases from 2.71% to 4.45%. This is because when the income inequality is high, the government provides less guarantees to banks as shown in Figure 2-1. When the government guarantee is smaller, the borrowing cost of the bank $r^B$ is higher and it is more likely for the bank to go bankrupt. When $\mu_L$ is above 0.95, the government guarantees reduce to 0, the model converges to a model with no government guarantees as shown by the red line in the figure.\textsuperscript{3}

\textsuperscript{3}The red dashed line shows the result from a model which is identical to the baseline model except the government guarantees $\eta$ is fixed at 0.
**Figure 2-1.** The average government guarantees to banks $\eta$ as a function of $\mu_L$.

**Figure 2-2.** The average probability of bank bankruptcy as a function of $\mu_L$. Blue solid line shows the result with government guarantees and the red dashed line shows the result with no guarantees.
2.5.2 Risk of Government Debt

Higher inequality also reduces the borrowing capacity of the government. Figure 2-3 shows the price of government bond for different $\mu_L$. We can see that, as $\mu_L$ increases, the default threshold of government debt-to-GDP ratio becomes smaller. Compared with the case where $\mu_L = 0.6$, a high degree of inequality where $\mu_L = 0.95$ reduces the default threshold by 25 percent of GDP. This result is consistent with findings of previous literature that inequality reduces the borrowing limit of the government (D’Erasmo and Mendoza, 2012; Andreasen et al., 2019).

In addition, inequality increases the cost of government guarantees by increasing the risk transmission from the banking sector to the government. Figure 2-4 shows the probability of government default in the model with different $\mu_L$ (blue line) and compares the result with the case where there is no government guarantees (red line). We can see that when inequality is low, compared with the case where there is no government guarantees, having government provide guarantees to banks reduce the risk of sovereign default. However, when inequality is high, government guarantees increase the probability of government default.

This is because the guarantee has opposing effects on government default. On one hand, government guarantees to banks improve the financing condition of the firm and increase the level of production which reduces the risk of government default. On the other hand, when inequality is high, the welfare of workers is low during economic downturn due to the concavity in the preference. The fiscal burden of guarantees further reduces the welfare of workers and the social welfare of all households in the case of debt repayment. Through this channel, government guarantees increase the risk of government default. When inequality is high, the second effect becomes larger, therefore government guarantees to banks increase the risk of sovereign default.
Figure 2-3. The price of government bonds for different $\mu_L$.

Figure 2-4. The probability of government default as a function of $\mu_L$. Blue solid line shows the result with government guarantees and the red dashed line shows the result with no guarantees.
2.6 Progressive Tax

In this section, we explore an extension to the model where we introduce progressive taxes as a way to restore the efficiency of the government guarantee. We show that a progressive tax can mitigate the effect of inequality, and lead to a higher level of government guarantees, a lower probability of bank bankruptcy, and a smaller risk of government default.

To generate progressive tax in the model, we vary the tax rate on the wage income and the capital income: $\tau^N$ and $\tau^K$. In order to isolate the effect of the tax progressiveness without changing the level of tax income, we keep the tax revenue-to-GDP ratio constant. As a result, $\tau^N$ and $\tau^K$ satisfies

\[
\text{Revenue to GDP} = \frac{\tau^N \alpha^N y_t + \tau^K \alpha^k y_t}{(\alpha^N + \alpha^k) y_t} = \frac{\tau^N \alpha^N + \tau^K \alpha^k}{\alpha^N + \alpha^k} = 0.291
\]

Here the degree of inequality is kept constant at $\mu_L = 0.85$. Since capital owners have a larger income than workers, when $\tau^K$ becomes larger, the tax is more progressive.

Figure 2-5 shows the relationship between the average level of government guarantees and the tax rate for capital income $\tau^K$. We can see that, as $\tau^K$ increases from 10 percent to 45 percent, the level of guarantees increase from 10 percent to 60 percent. The intuition is that, when inequality is high, the income of poorer household is low and the marginal utility cost of taxing poorer household is high. Therefore the government is reluctant to provide guarantees if a large part of the cost of guarantees are paid by the poorer households. The progressive tax reduces the social welfare cost of guarantees by making the richer agents who also benefit more from the guarantees pay for a larger share of the cost of guarantees. As a result, the government provides
higher level of guarantees to banks.

As the progressive tax increases government support for banks, banks become less likely to go bankrupt. Figure 2-6 shows how the probability of bank bankruptcy responds to the variation of tax progressiveness. We can see that the probability of bankruptcy decreases as tax becomes more progressive.

Figure 2-7 shows how the frequency of government default varies with $\tau_K$. We can see that, when taxes are regressive, the government that provides guarantees to banks is much more likely to default than the government that does not provide guarantees. A progressive tax mitigates the effect of inequality and the government that provides guarantees becomes less likely to default.

### 2.7 Median Voter Model

In the baseline model, we abstract from the discussion of political economy which can shape the government’s decisions on guarantees and default. Previous literature (Andreasen et al., 2019; Dovis et al., 2016) has shown evidence that inequality can create constraints on government policies through political processes. In this section, we consider a variation of the model where fiscal policies on government guarantees and debt borrowings are determined by the preference of the median voter instead of a benevolent social planner. We show that in the median voter model, income inequality has a larger impact on the optimal level of government guarantees than it does in the social planner model.

Figure 2-8 plots the optimal level of government guarantees to banks in the median voter model and compares it with the result from the social planner model. We can see that, for a given $\mu_L$, the optimal level of government guarantees is lower in the median voter model than it is in the social planner model. This is because when
inequality is high, the poorer household has larger impact on government policy in a median voter model than it is in a social planner model.

To study how government guarantees affect the decision of voters, we compute the benefit and the cost of government guarantees for both types of households. For a given level of government guarantee \( \eta \), let \( \Delta C^+_L \) and \( \Delta C^+_K \) be the increase of the consumption of workers and capital owners due to the guarantee:

\[
\Delta C^+_L = \frac{(1 - \tau^N)\alpha^N}{\mu_L} \Delta y - \Delta g(N) \tag{2.42}
\]

\[
\Delta C^+_K = \frac{(1 - \tau^K)\alpha^k}{\mu_K} \Delta y \tag{2.43}
\]

Here the increase of output:

\[
\Delta y = y(b, \eta, z, \pi) - y(b, 0, z, \pi) \tag{2.44}
\]

The change in the disutility from labor \( \Delta g(N) \) is:

\[
\Delta g(N) = \frac{N^\omega(b, \eta, z, \pi)}{\omega} - \frac{N^\omega(b, 0, z, \pi)}{\omega} \tag{2.45}
\]

The cost of guarantees is the same for all households. For a given level of guarantee \( \eta \), let \( C^- \) be the cost of guarantees:

\[
\Delta C^- = \kappa p^* M^* \pi \eta (1 + r^B) \tag{2.46}
\]

Figure 2-9 plots the benefit and the cost of government guarantees for capital owners and workers. We can see that, for a given level of guarantees, the cost of guarantees are the same for capital owners and workers, and remains constant for different \( \mu_L \). However, as \( \mu_L \) increases, capital owners receive a larger fraction of the benefit of guarantees while the benefit for workers decreases. As a result, workers are increasingly against government guarantees to banks. Since workers make up the majority of the population, the median voter is a worker. Consequently, the government provides less guarantees to banks in the median voter model.
Figure 2-5. The average government guarantees to banks $\eta$ as a function of $\tau^K$.

Figure 2-6. The average probability of bank bankruptcy as a function of $\tau^K$. 
Figure 2-7. The probability of government default as a function of $\tau^K$. Blue solid line shows the result with government guarantees and the red dashed line shows the result with no guarantees.
Figure 2-8. Average levels of government guarantees. Blue solid line shows the result where the government optimize social welfare. The red dashed line shows the result from the median voter model.

Figure 2-9. The benefits and costs of government guarantees for workers and capital owners.
2.8 Conclusion

In this paper, we extend a sovereign default model with endogenous financial crisis to include heterogeneity among households. In the model, households differ from each other by their source of income (capital vs labor). Our model has several implications regarding the impact of income inequality on government guarantees to banks, the stability of the financial sector, and the risk of sovereign default.

We find that high degree of inequality reduces the incentive for the government to provide guarantees to banks. As the government provides less support for banks, the borrowing cost of banks increases. As a result, banks are more likely to go bankrupt after experiencing negative shocks to productivity and monitoring costs. In addition, income inequality worsens the risk transmission from the banking sector to the government. When inequality is low, government guarantees reduce the risk of sovereign default. However, when inequality is high, letting the government provide guarantees to banks increases the risk of sovereign default.

Next, we explore the use of progressive tax to mitigate the effect of inequality. We find that, when there is a high degree of inequality among households, a moderate level of progressive tax can restore government guarantees and reduce the risk of financial crisis and the risk of sovereigns default.

Finally, we extend the model to include a voting process to discuss how inequality might affect government guarantees through political economy. We show that when government policy is determined by the preference of the median voter, due to unequal distribution of the benefit of government guarantees, inequality reduces the level of government guarantees more than it does in a social planner model.

There are several directions for future research. In particular, the model currently focuses on the risk transmission from the financial sector to the government while the
sovereign default risk has little impact on the stability of banks. In the future, the model can be extended to allow risk transmission from the government to banks to study the impact of income inequality on the feedback loop between banking crises and sovereign default.
Chapter 3
The Risk of Local Government Finance Vehicles in China

3.1 Introduction

The rapidly growing Chinese local government debt has long been viewed as a potential source of systemic risk for the Chinese economy. Until 2015, local governments in China were prohibited from issuing debt. Instead, they borrowed through local government finance vehicles (LGFV) which are non-government entities tied to local governments through various forms of contracts. Given the opaque structure of LGFVs, the relationship between local government finances and the risk of LGFV debt is not well understood. Since 2015, Chinese central government implemented a series of reforms aimed at “opening the front door” i.e. legalizing official local government debt and “shutting the back door” i.e. restricting LGFV debt (Ministry of Finance of the People’s Republic of China, 2017e). These policies weaken the ties between LGFVs and local governments and trigger concerns about the risk of LGFV debt.

This paper studies the risk of LGFV debt by investigating its relationship with local government finances and regulatory policies. In particular, I compute the yield spread between LGFV debt and official local government debt and examine how the
economic fundamentals of local governments such as government debt-to-GDP ratio, revenue-to-GDP ratio, and GDP growth rate affect the yield spread and the credit ratings of LGFV debt. In addition, I focus on regulatory policies that separate LGFVs and local governments, and employ a news study method to analyze the effect of the announcements of these regulatory policies on the yield spread of LGFV debt. Finally, I propose a model to explain the empirical results.

In the following part of the paper, I refer to the debt that a local government borrows directly from creditors as official local government debt and the debt that a local government borrows through LGFVs as LGFV debt. To study the risk of LGFV debt, I first compute the yield spread between LGFV debt and official local government debt for all city-level local governments. I use the ordinary least squares (OLS) regression to analyze the relationship between local government finances and the yield spread of LGFV debt. I find that higher local government debt-to-GDP ratio increases the yield spread of LGFV debt. Interestingly, regardless of whether the local government issues official debt or LGFV debt, it has the same amount of impact on the yield spread of LGFV debt. Moreover, I find that local government revenue and expenditure have significant impacts on the yield spread of LGFV debt. Higher revenue-to-GDP ratio reduces the yield spread and higher expenditure-to-GDP ratio increases the yield spread of LGFV debt.

Second, I examine the relationship between local government finances and the credit ratings of LGFV debt. I find that the debt structure of the local government has significant impact on the credit ratings of LGFV debt. Higher official local government debt-to-GDP ratio is associated with better credit ratings, but higher LGFV debt-to-GDP ratio weakens the credit ratings of LGFV debt. One possible explanation is that credit rating agencies might consider high official local government debt as a signal for the favorable attitude of the central government towards the local government.
given that the limit of official government debt of each city needs to be approved by
the central government. In addition, the revenue structure of the local government
also affects the credit ratings of LGFV debt. Higher tax revenue of local government
is associated with higher credit ratings of LGFV debt, but higher government fund
revenue (revenue from land sales) reduces the credit ratings of LGFV debt.

Next, to study the impact of regulatory policies on the risk of LGFV debt, I
construct a dataset of regulatory policies issued by the central government of China on
LGFV debt since 2017. Using this dataset, I examine how the yield spread of LGFV
debt changes on the day of policy announcements. I find that regulatory policies that
emphasize the separation between LGFVs and local governments increase the yield
spread of LGFV debt on the day of policy announcement. This effect is significant for
LGFV debt of all credit rating levels.

However, the policy effects vary with local government finances. In particular,
the regulatory policies have stronger effects on local governments with lower debt. It
implies that when a regulatory policy is announced, even though the yield spreads of all
LGFV debt increase on average, the yield spread increases more for local governments
with lower debt-to-GDP ratio. After I decompose local government debt into official
debt and LGFV debt, I find that the counterintuitive result above is almost entirely
caused by the negative correlation between the official local government debt-to-GDP
ratio and the change of yield spread when regulatory policies are announced.

To explain the empirical results, I propose a theoretical model for LGFV debt. In
the model, a local government and a finance vehicle issue debt separately, but combine
their income to pay for the debt. A key assumption of the model is that official local
government debt has the senior status over LGFV debt. To avoid the default of LGFV
debt, both official local government debt and LGFV debt need to be paid off. As a
result, increasing either official debt or LGFV debt increases the likelihood for the LGFV debt to default, hence increasing the yield spread of LGFV debt. This result of the model is consistent with the empirical observation that official local government debt and LGFV debt have the same amount of impact on the risk of LGFV debt.

The model also explains the counterintuitive result that, when regulatory policies that separate local government and LGFVs are announced, the change of yield spread is negatively correlated with local government debt ratio. In the model, when the local government and the LGFV are separated, the two entities do not combine their income any more. Instead each entity has to pay for its own debt with its own income. Consequently, the separation releases the LGFV from the obligation to pay for the more senior official local government debt, and reduces the yield spread of LGFV debt. This effect is stronger when the official local government debt is larger, which explains why the change of yield spread is negatively correlated with official local government debt-to-GDP ratio when regulatory policies are announced to separate local governments and LGFVs.

### 3.1.1 Related Literature

This paper is related to several strands of literature.

First, this paper contributes to the literature on the yield spread and credit risk of local government debt in China. Ambrose et al. (2015) explore the linkage between local government debt and local housing market. They show that higher expected house price growth is associated with lower default risk of local government debt. Similarly, Ang et al. (2015) find that higher local real estate GDP is associated with lower yield spread of local government debt. Luo and Liu (2016) show that the finance of local government instead of the finance of LGFV plays an important role in
determining the issuing price of the LGFV debt.\textsuperscript{1} Niu et al. (2016) study the joint dynamic of yield spreads of LGFV and the yield of treasury bond in China. They show that LGFV debt exhibits considerable local risks and can lead to systemic risk of the treasury bonds. Liu et al. (2021) investigate the relationship between the yield spread of LGFV and the implicit local government debt of city and province level. They find that higher local government implicit debt-to-revenue ratios are associated with higher yield spreads of LGFV debt. Gao et al. (2021) present evidence that political considerations affect the default decision of local governments in China. They show that Chinese local governments choose to default on banks with weaker political power when they are able to identify the lenders. This paper makes a contribution to this literature by studying how the economic fundamentals of local government and regulatory policies on LGFVs affect the risk of LGFV debt.

Second, this paper contributes to the literature which studies the development of government bond market and the rise of LGFV debt in China. Wu (2016) and Amstad and He (2018) provide overviews on the local government bond in China. Jin and Rial (2016) show that infrastructure projects carried out through LGFVs were largely unregulated PPPs (Public-Private Partnership). Lam and Wang (2018) study the official local government debt and discuss the impediments of the official local government bond market in China. In addition, Huang and Du (2018) show that land purchases play an important role in the finance of LGFVs. They show that the issuance of LGFV debt often involves the LGFV to purchase land from the government first, and then use the land as a collateral for borrowing. Chen et al. (2020) examine the connection between the 2009 stimulus package in China following the global financial crisis and the rise of shadow banking activities. They show that the rollover pressure of the stimulus loans leads to the rise of unofficial government debt.

\textsuperscript{1}The paper is written in Chinese.
through LGFVs. Liu and Wu (2017) provide evidence that the implicit support from the central government to local government is correlated with excessive borrowing of the local government. This paper focuses on the change of regulatory policies on local government debt in China since 2017 and estimates the impact of policies that separate local governments and LGFVs on the riskiness of LGFV debt.

The rest of this paper is organized as follows. In Section 3.2, I introduce the institutional background of local government debt in China. In Section 3.3, I describe the data and summary statistics. Section 3.4 and Section 3.5 discuss the results of the empirical study. In Section 3.6, I present a theoretical model to explain the empirical result. Section 3.7 concludes.

3.2 Local Government Debt in China

In China, there are four levels of local governments under the central government, including (from high to low): provincial, prefecture city, county, and township. Before 2015, the Budget Law of China prohibited local governments of all levels from issuing debt. Other than a few special cases where the Ministry of Finance borrowed on behalf of local governments, there was no official local government debt. In 2015, the new Budget Law established channels for local governments to borrow official debt. This reform is sometimes referred to by the Ministry of Finance as “opening the front door”. Blue solid line of Figure 3-1 shows the official local government debt-to-GDP. We can see that the amount of official local government debt increased significantly since 2015.

While the official channel for local governments to issue debt is restricted, local governments establish finance vehicles (LGFV) to borrow from investors. These vehicles are typically registered as corporations and tied to local governments through
guarantee contracts, buy-back agreements, and overlapping leadership teams. The red dashed line of Figure 3-1 shows the total LGFV debt as a percentage of GDP from 2015 to 2022. We can see that as the share of official local government debt rises, the LGFV debt-to-GDP ratio stays roughly constant.

As the new Budget Law legalizes official local government debt, the central government tightens the regulation for the LGFV debt (this process is sometimes referred as “shutting the back door”). Since 2017, the central government introduced a series of policies aimed at separating LGFVs and local governments. The regulation of LGFV debt has gone through three phases. The first phase is from 2017 to mid 2018. During this period, the central government announced a set of policies to restrict fund flows between LGFVs and local governments, limit local government guarantees to LGFVs, and prohibit state-owned banks from lending to local governments through LGFVs. The second phase is from mid 2018 to mid 2019. During this period, the central government relaxed regulations of LGFVs amid a slow down of domestic economy. The regulatory policies released during this period often include phrases such as 'support the legitimate financing needs of LGFVs'. The third phase covers the period after 2019. Several LGFV debt defaulted during this period. The central government introduced policies to further separate LGFV debt from local governments.

3.3 Data

3.3.1 Economic Fundamentals

I obtain data on official government debt and LGFV debt from Wind Financial Terminal. The data set covers all official local government debt and LGFV debt from January 2015 to March 2022. The list of LGFVs are from the database of China Banking Regulatory Commission.
For each debt, the data set contains information on characteristics of the debt, including issuer, volume, and maturity date etc. For debt issued by LGFVs, the data set also contains the geographical location of the issuing LGFV and the credit ratings of the debt at issuance. I use the location of the LGFV to match the LGFV with its corresponding local government on the city level.

The credit ratings of LGFV debt is assigned by China Securities Credit Investment (CSCI). The ratings include 9 levels (from safest to riskiest): AAA, AA, A, BBB, BB, B, CCC, CC, C. Each level between AA to B can also have a “+” sign which indicates a positive outlook or a “-” sign which indicates a negative outlook. For example, the detailed credit rating levels between AA and A are (from safe to risky): AA+, AA, AA-, A+, A, A-.

In addition, the data set also contains annual data for economic fundamentals of each city. These variables include: government revenue, government expenditure, local GDP, local population etc. In particular, local government revenue data include two parts: tax revenue and government fund revenue. The government fund revenue
is usually from the land sales. Compared with tax revenue, local governments have larger flexibility in the use of government fund revenue. Panel A of Table 3.1 reports the summary statistics of the data on economic fundamentals.

3.3.2 Yield on Local Government Debt

I also obtain the data on the yield of official local government debt and LGFV debt from Wind Financial Terminal. This is the yield to maturity computed by Shanghai Clearing House given the market price of the bond. The data set covers the daily closing price for each official local government debt and LGFV debt for all trading days from May 2017 to March 2022.

Using the data, I compute the yield spread between LGFV debt and official local government debt. This yield spread measures the excessive risk of the LGFV debt. Specifically, I match each LGFV debt with an official debt from the same local government with a similar maturity date. (The maturity dates of the two debt are within 100 days.)

The yield spread $s_{i,t}$ is calculated as

$$s_{i,t} = r_{i,t}^l - r_{i,t}^o$$

Here $r_{i,t}^l$ is the yield of LGFV debt $i$ at day $t$, and $r_{i,t}^o$ is the yield of official local government debt from the same local government with a similar maturity date.\(^2\) Panel B of Table 3.1 reports the summary statistics of the yield data. On average, the yield of LGFV is higher than the yield of official local government debt by about 2.3 percent. Figure 3-2 plots the median yield spread of all LGFV debt. Figure 3-3 shows the median yield spread for LGFV debt of different credit ratings. We can see that LGFV debt with higher credit ratings has lower yield spreads.

\(^2\)The superscript “$l$” denotes LGFVs, and the superscript “$o$” denotes official debt.
### Table 3.1. Summary Statistics

<table>
<thead>
<tr>
<th>Panel A: Fundamentals</th>
<th>Mean</th>
<th>Median</th>
<th>Std.Dev</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue-to-GDP</td>
<td>16.35</td>
<td>16.20</td>
<td>5.12</td>
<td>41.61</td>
<td>4.62</td>
</tr>
<tr>
<td>Expenditure-to-GDP</td>
<td>17.43</td>
<td>15.57</td>
<td>7.69</td>
<td>142.41</td>
<td>5.01</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>32.76</td>
<td>31.11</td>
<td>11.79</td>
<td>83.54</td>
<td>11.00</td>
</tr>
<tr>
<td>Share of LGFV Debt</td>
<td>48.82</td>
<td>47.96</td>
<td>13.41</td>
<td>80.01</td>
<td>6.11</td>
</tr>
<tr>
<td>GDP Growth Rate</td>
<td>6.56</td>
<td>6.68</td>
<td>7.63</td>
<td>61.46</td>
<td>-35.91</td>
</tr>
<tr>
<td>Loan Prime Rate</td>
<td>4.08</td>
<td>4.15</td>
<td>0.22</td>
<td>4.31</td>
<td>3.70</td>
</tr>
<tr>
<td>CIVIX-50</td>
<td>0.01</td>
<td>-0.07</td>
<td>1.24</td>
<td>11.98</td>
<td>-6.35</td>
</tr>
<tr>
<td>GDP-per-Capita</td>
<td>76.82</td>
<td>71.50</td>
<td>37.08</td>
<td>202.21</td>
<td>9.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Yield</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield of Official Debt</td>
<td>3.23</td>
<td>3.20</td>
<td>0.55</td>
<td>4.59</td>
<td>1.05</td>
</tr>
<tr>
<td>Yield of LGFV Debt</td>
<td>5.53</td>
<td>5.51</td>
<td>1.49</td>
<td>27.70</td>
<td>-7.72</td>
</tr>
<tr>
<td>Yield Spread</td>
<td>2.36</td>
<td>2.07</td>
<td>1.52</td>
<td>25.28</td>
<td>-0.43</td>
</tr>
<tr>
<td>Change of Yield Spread</td>
<td>0.00</td>
<td>-0.00</td>
<td>0.05</td>
<td>5.35</td>
<td>-2.84</td>
</tr>
</tbody>
</table>

**Note:** Table 3.1 reports the summary statistics of the data. All values except GDP per capita are reported in percentage. GDP per capita is measured in thousand yuan.
Figure 3-2. Median yield spread between LGFV debt and official local government debt

Figure 3-3. Yield spreads between LGFV debt of different credit ratings and official local government debt
3.3.3 Policy News

To study the impact of regulatory policy on the risk of LGFV debt, I obtain a list of policy announcements about local government debt from the State Council and its three regulatory agencies: Ministry of Finance, Banking Regulatory Commission, and National Development and Reform Commission. These three agencies are primarily responsible for regulating local government debt.

In order to match the time span of the yield data, I set the time window from May 2017 to March 2022 and obtain all the policies that were announced during this period and mentioned local government debt or LGFV in their text. The policy announcements in the year of 2017 are from the official website of the corresponding agency (Links are included in References). The policies announced after 2018 are obtained from the State Council Policy Document Database (State Council, 2022). For each policy document, there is a date at the end of the document which indicates when the document is distributed to local governments and financial entities. I use this date as the announcement date of the policy. Table 3.2 lists the policy announcements.

The list of policies include both the policies that separate LGFVs from local governments and policies that support LGFV financing needs. These two sets of policies can have opposite effects on the perceived riskiness of LGFVs. To distinguish between these two sets of policies, I categorize the policies into three groups based on the content of the policy. The first group contains 11 policies which emphasized the separation between local governments and LGFVs. These documents either mention the separation of the balance sheet/leadership of local governments and LGFVs or restrict lendings to local governments through LGFVs. The second group contains 3 policies which mention “supporting the financing needs of LGFVs”. The third group contains the rest of the list which did not give clear signal on the status of LGFVs.
3.4 Risk of LGFV Debt

In this section, I study how economic fundamentals affect the risk of LGFVs.

3.4.1 Yield Spread

First I run the regression between the yield spread of LGFV debt and selected economic fundamental variables using the panel data:

\[ \bar{s}_{it} = \alpha_0 + \eta_t + \mu_i + \alpha X_{it} \]  

Here \( \bar{s}_i \) is the average yield spread of the \( i \)th LGFV debt year \( t-1 \). The explanatory variables \( X_{it} \) include: local government debt-to-GDP ratio, government revenue-to-GDP ratio, government expenditure-to-GDP ratio, and the GDP growth rate of the city for \( i \)th LGFV in the year \( t \). Both time effect \( \eta_t \) and LGFV fixed effects \( \mu_i \) are included.

Column 1 of Table 3.3 reports the result of the panel regression. We can see that the debt-to-GDP ratio of the local government significantly increases the yield spread of LGFV debt. On average, 1 percent increase of local government debt-to-GDP ratio increases the yield spread of LGFV debt by 4.6 basis points. In addition, local government revenue and expenditure to GDP ratio also have significant impact on the yield spread of LGFVs. Specifically, increasing government revenue-to-GDP ratio by 1 percent reduces the yield spread of LGFV by about 2.4 basis points. At the same time, 1 percent increase in government expenditure-to-GDP ratio increases the yield spread of LGFV debt by 6 basis points.
Table 3.2. List of Policy Announcements

<table>
<thead>
<tr>
<th>Date</th>
<th>Issuer</th>
<th>$I_{news}$</th>
<th>Document Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>2017-05-16</td>
<td>MOF</td>
<td>0</td>
<td>Measures for the Administration of Special Bonds for Local Government Land Reserves</td>
</tr>
<tr>
<td>2017-06-02</td>
<td>MOF</td>
<td>0</td>
<td>Notice on the types of local government special bonds that seek to balance the benefits and financing of pilot development projects</td>
</tr>
<tr>
<td>2017-06-03</td>
<td>MOF</td>
<td>0</td>
<td>Firmly Stop Illegal Local Government Borrowing in the Name of Government Purchasing Service</td>
</tr>
<tr>
<td>2017-11-16</td>
<td>MOF</td>
<td>1</td>
<td>Regulation of the Administrative Database of Public-Private Partnership Projects</td>
</tr>
<tr>
<td>2017-11-17</td>
<td>State Council</td>
<td>1</td>
<td>Strengthening Risk Management of PPP Projects of State Owned Enterprises</td>
</tr>
<tr>
<td>2018-02-08</td>
<td>NDRC</td>
<td>1</td>
<td>Further Strengthening the Ability of Enterprise Bonds to Serve the Real Economy and Strictly Preventing Local Debt Risks</td>
</tr>
<tr>
<td>2018-02-24</td>
<td>MOF</td>
<td>0</td>
<td>Guidelines for the Issuance of Government Bond in 2018</td>
</tr>
<tr>
<td>2018-03-28</td>
<td>MOF</td>
<td>0</td>
<td>Regulate Investment and Financing Behavior of the Financing Enterprises to Local Governments and State-Owned Enterprises</td>
</tr>
<tr>
<td>2018-04-02</td>
<td>MOF</td>
<td>0</td>
<td>Measures for the Administration of Pilot Issuance of Special Bonds for the Reconstruction of Local Governments in Shanty Areas</td>
</tr>
<tr>
<td>2018-05-12</td>
<td>NDRC</td>
<td>1</td>
<td>Improving Market Refrain Mechanism and Strictly Preventing Foreign Debt Risks and Local Debt Risks</td>
</tr>
<tr>
<td>2018-06-28</td>
<td>State Council</td>
<td>1</td>
<td>Further Strengthening Urban Rail Transit Planning and Construction Management</td>
</tr>
</tbody>
</table>

Note: Table 3.2 shows the list of policy announcements related to local government finance vehicles (LGFV). The first and the second column list the dates of the announcements and the issuance authorities of the policies. The issuance authorities include: State Council, Ministry of Finance (MOF), Banking and Insurance Regulatory Commission (BIRC), and National Development and Reform Commission (NDRC). The third columns list the indicators for the policy announcements. The indicator equals to 1 if the policy emphasized the separation between local government and LGFV, equals to -1 if the policy emphasized the support of LGFVs, and 0 otherwise. The last column shows the title of the policy documents.
### Table 3.2. List of Policy Announcements (Continue)

<table>
<thead>
<tr>
<th>Date</th>
<th>Issuer</th>
<th>$I_{news}$</th>
<th>Document Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018-07-18</td>
<td>MOF</td>
<td>1</td>
<td>A number of cases of local government accountability for illegal debt guarantees</td>
</tr>
<tr>
<td>2018-07-23</td>
<td>State Council</td>
<td>-1</td>
<td>Note of Executive Meeting of State Council</td>
</tr>
<tr>
<td>2018-08-17</td>
<td>BIRC</td>
<td>-1</td>
<td>Guidelines for Using Credit to Improve the Quality and the Efficiency of the Real Economy</td>
</tr>
<tr>
<td>2018-09-13</td>
<td>State Council</td>
<td>1</td>
<td>Strengthening the Asset and Liability Constraints of State-owned Enterprise</td>
</tr>
<tr>
<td>2018-10-11</td>
<td>State Council</td>
<td>-1</td>
<td>Maintaining the Strength in the Field of Improving the Basic Infrastructure</td>
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<tr>
<td>2018-12-20</td>
<td>MOF</td>
<td>1</td>
<td>Guidelines for the Disclosure of Local Government Debt Information</td>
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<tr>
<td>2018-12-05</td>
<td>State Council</td>
<td>1</td>
<td>Government Investment Regulations</td>
</tr>
<tr>
<td>2019-06-21</td>
<td>MOF</td>
<td>0</td>
<td>Measures for the Budget Management of Land Reserve Projects</td>
</tr>
<tr>
<td>2021-04-14</td>
<td>State Council</td>
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<td>Further Deepening the Reform of the Budget Management System</td>
</tr>
<tr>
<td>2021-05-22</td>
<td>MOF</td>
<td>1</td>
<td>Transfer Four Non-tax Government Income including: State-owned Land Use Rights, Special Income from Mineral Resources, Sea Area Use Fee, and Uninhabited Island Use Fee to Tax Authorities for Collection</td>
</tr>
<tr>
<td>2021-07-10</td>
<td>BIRC</td>
<td>1</td>
<td>Guidelines for Further Prevent and Remove Risks of Hidden Local Government Debt</td>
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<tr>
<td>2021-11-11</td>
<td>MOF</td>
<td>0</td>
<td>Operational Guidelines for Adjusting the Use of Local Government Special Bonds</td>
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</table>

**Note:** Table 3.2 shows the list of policy announcements related to local government finance vehicles (LGFV). The first and the second column list the dates of the announcements and the issuance authorities of the policies. The issuance authorities include: State Council, Ministry of Finance (MOF), Banking and Insurance Regulatory Commission (BIRC), and National Development and Reform Commission (NDRC). The third columns list the indicators for the policy announcements. The indicator equals to 1 if the policy emphasized the separation between local government and LGFV, equals to -1 if the policy emphasized the support of LGFVs, and 0 otherwise. The last column shows the title of the policy documents.
### Table 3.3. Regression: Fundamentals

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<thead>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<tbody>
<tr>
<td><strong>Yield Spread</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.124</td>
<td>0.974***</td>
<td>0.940***</td>
<td>0.988***</td>
</tr>
<tr>
<td></td>
<td>(0.174)</td>
<td>(0.212)</td>
<td>(0.209)</td>
<td>(0.206)</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>4.626***</td>
<td>4.502***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.546)</td>
<td>(0.527)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Official Debt</td>
<td></td>
<td>4.221***</td>
<td>4.141***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.589)</td>
<td>(0.581)</td>
<td></td>
</tr>
<tr>
<td>LGFV Debt</td>
<td></td>
<td>4.705***</td>
<td>4.866***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.742)</td>
<td>(0.775)</td>
<td></td>
</tr>
<tr>
<td>Revenue-to-GDP</td>
<td>-2.397***</td>
<td>-0.923</td>
<td>-0.952</td>
<td></td>
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<td></td>
<td>(0.648)</td>
<td>(0.673)</td>
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<td>Tax Revenue</td>
<td></td>
<td></td>
<td></td>
<td>-6.064***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.894)</td>
</tr>
<tr>
<td>Government Fund</td>
<td></td>
<td></td>
<td>0.572</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.765)</td>
<td></td>
</tr>
<tr>
<td>Expenditure-to-GDP</td>
<td>6.023***</td>
<td>2.590**</td>
<td>2.641***</td>
<td>3.516***</td>
</tr>
<tr>
<td></td>
<td>(0.811)</td>
<td>(1.012)</td>
<td>(1.008)</td>
<td>(1.100)</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>0.619</td>
<td>0.738**</td>
<td>0.772**</td>
<td>0.718*</td>
</tr>
<tr>
<td></td>
<td>(0.379)</td>
<td>(0.365)</td>
<td>(0.367)</td>
<td>(0.371)</td>
</tr>
<tr>
<td>GDP per Capita</td>
<td>-0.085***</td>
<td>-0.091***</td>
<td>-0.066***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.015)</td>
<td></td>
</tr>
<tr>
<td>Num.Obs</td>
<td>2102</td>
<td>2102</td>
<td>2102</td>
<td>2102</td>
</tr>
</tbody>
</table>

**Note:** Table 3.3 reports the coefficients of panel regressions. The dependent variable is the yield spread between LGFV debt and official local government debt. Explanatory variables include: local government debt-to-GDP ratio which is decomposed into official local government debt-to-GDP ratio and LGFV debt to GDP ratio in the last two columns; revenue-to-GDP ratio which is decomposed into tax revenue to GDP ratio and local government fund revenue to GDP ratio in the last columns; expenditure-to-GDP ratio, GDP growth rate, and GDP per capita of the local region. Both time effect and LGFV fixed effect are controlled. Heteroscedasticity-robust standard errors are shown in parenthesis.***, **, * denote significance level of 1%, 5%, and 10% respectively.
Second, I include GDP per capita of each city in the regression to control for the level of economic development of different regions. Column 2 of Table 3.3 shows the result. We can see that GDP per capita is negatively correlated with the yield spread of LGFV debt. Increasing GDP per capita by 10,000 yuan (about 1,500 US dollars) reduces the yield spread of LGFV by 8.5 basis points. After controlling for the level of development, the coefficient for government debt-to-GDP ratio remains significant. However, the coefficients for government revenue and expenditure become less significant. This result suggests that the impact of government revenue and expenditure on the yield spread can be partially explained by the development level of the local economy: local governments in regions with higher GDP per capita tend to have better fiscal conditions and are able to borrow LGFV debt at lower costs.

Third, in order to investigate which type of government debt has a larger impact on the perceived risk of LGFV debt, I decompose the total local government debt-to-GDP ratio into official local government debt-to-GDP ratio and LGFV debt-to-GDP ratio, and regress the yield spread on these two components. Column 3 of Table 3.3 reports the results after the debt decomposition. We can see that both official government debt and LGFV debt are positively correlated with the yield spread of LGFV debt and the coefficients of these two types of debt are roughly equal. The coefficients for official government debt and LGFV debt are close. Increasing either type of government debt by 1 percent of GDP increases the yield spread of LGFV debt by about 4.5 basis points. This result indicates that market investors consider both official debt and LGFV debt as government debt. It is the total amount of local government debt that affects the perceived risk of LGFV debt.

Last, previous literature (Ambrose et al., 2015; Chen et al., 2022) find that the revenue structure of the local government plays an important role in determining the risk of LGFV debt. In particular, government revenue from land sales are found to be
the primary source of fund for the government to repay LGFV debt. To study the impact of revenue structure on the risk of LGFV debt, I decompose the revenue of local government into tax revenue and revenue from government fund. The majority of the government fund revenue is from land sales. Column 4 of Table 3.3 shows the results of the regression after the decomposition of government revenue. Contrary to the previous findings, I find that government fund revenue does not have a significant impact on the yield spread of LGFV debt while the tax revenue-to-GDP ratio is negatively correlated with the yield spread of LGFV.

3.4.2 Credit Ratings

In addition to the yield spread, the credit ratings of LGFV debt are also important signals for the perceived risk of LGFVs. In this subsection, I conduct an ordinal logistic regression (OLR) to study how the economic fundamentals of local governments affect the credit ratings of LGFV debt.

Ordinal Logistic Regression has been used in literature to analyze the relationship between the cumulative probability of an ordinal dependent variable and a set of explanatory variables (Peterson and Harrell, 1990; Agresti, 1996; Fuks and Salazar, 2008). In our case, the dependent variable is the credit ratings of LGFV debt. The model assumes that the cumulative logit of the ordinal variable is a linear function of the explanatory variables:

\[
\ln \left( \frac{\text{Prob}(y \leq j)}{1 - \text{Prob}(y \leq j)} \right) = \mu_j - \beta X
\]

Here random variable \( y \) is the credit ratings of LGFV debt, \( j \) is an integer from 0 to 9, each integer represents a credit rating level. The higher the number is, the higher the rating is. The explanatory variables \( X \) include: official local government debt-to-GDP ratio, LGFV debt-to-GDP ratio, tax revenue-to-GDP ratio, government
fund revenue-to-GDP ratio, government expenditure-to-GDP ratio, GDP growth rate and the GDP per capita.

For any given rating level \( j \), the coefficients \( \beta_x \) can be interpreted as the impact of independent variables on the probability of LGFV debt to get higher/lower ratings. For example, if the coefficient for government revenue-to-GDP ratio is positive, it implies that higher government revenue-to-GDP ratio increases the probability of LGFV debt to have a higher rating than \( j \).

Note that \( \beta_x \) does not depend on \( j \), which is a fundamental assumption of the model, implying that the relationship between the logit of credit ratings and the explanatory variables are the same for all credit rating levels \( j \). While the coefficients are the same for all \( j \), there is a different intercept \( \mu_j \) for each credit rating level.

Table 3.4 presents the result of the ordinal regression. The first column reports the coefficient for each explanatory variable and the significance level. We can see that all of the coefficients are significant at 1 percent level except for the growth rate of local GDP which is significant at 10 percent level. To interpret the result quantitatively, the second column converts the coefficients into its impact on odds ratio by taking the exponential of the coefficients. An odds ratio greater than 1 indicates that increasing the corresponding explanatory variable increases the probability for the LGFV debt to get a higher credit rating. For example, the odds ratio of official debt is 1.032 implies that 1 percent increase of official local government debt-to-GDP ratio increases the odds of the LGFV getting a higher credit rating by 3.2 percent. Similarly, the odds ratio smaller than 1 indicates that increasing the corresponding variable decreases the probability of LGFV debt getting a higher credit rating.
**Table 3.4.** Regression: Fundamentals

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Odds Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Official Debt</strong></td>
<td>0.031***</td>
<td>1.032</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td><strong>LGFV Debt</strong></td>
<td>-0.037***</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td><strong>Tax Revenue</strong></td>
<td>0.251***</td>
<td>1.286</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
</tr>
<tr>
<td><strong>Government Fund</strong></td>
<td>-0.012***</td>
<td>0.988</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td><strong>Expenditure-to-GDP</strong></td>
<td>-0.087***</td>
<td>0.916</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td><strong>GDP Growth</strong></td>
<td>0.412*</td>
<td>1.510</td>
</tr>
<tr>
<td></td>
<td>(0.225)</td>
<td></td>
</tr>
<tr>
<td><strong>GDP per Capita</strong></td>
<td>0.049***</td>
<td>1.051</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td><strong>Num.Obs</strong></td>
<td></td>
<td>1944</td>
</tr>
</tbody>
</table>

**Note:** Table 3.4 reports the coefficients of cross-section ordinal logistic regressions. The ordinal class is the credit ratings of LGFV debt. Each credit rating is an integer number from 0 to 9 which represents the following credit ratings (From low to high): BB-, BB, BB+, A-, A, A+, AA-, AA, AA+, AAA. Explanatory variables include: official local government debt-to-GDP ratio, LGFV debt-to-GDP ratio, local government tax revenue to GDP ratio, local government fund revenue to GDP ratio, local government expenditure-to-GDP ratio, GDP growth rate, and GDP per capita of the local region. Standard errors are shown in parenthesis. The first column reports the coefficients for each explanatory variables. The second columns coverts the coefficients into odds ratios. If the corresponding odds ratio of an explanatory variable is greater than 1, it indicates that increasing this variable improves the rating of the LGFV debt. If the odds ratio is smaller 1, it indicates that increasing the explanatory variable reduces the credit rating of LGFV debt. ***, **, * denote significance level of 1%, 5%, and 10% respectively.
In Table 3.4, first we can see that the official local government debt and LGFV debt have opposite effects on the credit ratings of the LGFV debt. Higher official local government debt-to-GDP ratio is associated with better credit ratings while higher LGFV debt worsens the credit ratings of LGFV debt. One possible explanation is that the credit rating agencies perceive the official local government debt ratio as a signal for the support of central government. Because the limit of official local government debt is required to be approved by the central government, high official local government debt-to-GDP ratio indicates more favorable attitude of the central government towards this local government.

Second, local government revenue and expenditure have significant impacts on the credit ratings of the LGFV debt. On the revenue side, higher tax revenue is associated with better credit ratings. On average, increasing the tax revenue by 1 percent of GDP increases the odds of a higher credit rating by 28.6 percent. However, high government fund revenue (mainly from selling lands) reduces the credit ratings of the LGFV debt. This is probably due to the fact that revenue from selling land is perceived to be more volatile and not sustainable by the credit rating agencies. On the expenditure side, higher government expenditure-to-GDP ratio worsens the credit rating of the LGFV debt. Increasing the expenditure by 1 percent of GDP reduces the odds for a higher credit rating by 8.4 percent.

Last, high economic development also improves the credit rating of LGFV debt significantly. Increasing the GDP per capita by 10000 yuan increases the odds of LGFV getting a higher credit rating by 5.1 percent.
3.5 News Study

In this section, I study how regulatory policies on local government debt affect the risk of LGFV debt. First, I compute the daily change of yield spread for each LGFV debt as

$$\Delta s_{i,t} = s_{i,t} - s_{i,t-1} \quad (3.4)$$

Here $s_{i,t}$ is the yield spread of LGFV debt $i$ at day $t$. Using the data, I study two questions: (i) how do policy announcements impact the yield spread; (ii) how does this impact interact with economic fundamentals of the local government?

3.5.1 Time Series

To examine the impact of policy announcements, I run a dummy variable regression:

$$Y_t = \beta_0 + \beta_1 I_t + \beta_x X_t \quad (3.5)$$

Here $Y_t$ is the median daily change of the yield spread across all LGFV debt at day $t$: $Y_t = \text{median}_{1 \leq i \leq N} \{ \Delta s_{i,t} \}$. The indicator $I_t$ is equal to 1 if there is a policy announced to separate the local government and the LGFV at day $t$, equal to -1 if there is a policy announced to support LGFVs, and equal to 0 otherwise.

In the baseline regression, there is no control variable $X_t$. In this case, the coefficient $\beta_0$ measures the average change of yield when there is no policy announced. The coefficient $\beta_1$ measures the additional change of yield spread on the day of policy announcements. The first column of Table 3.5 reports the result of the baseline regression. We can see that when there is no policy announcements, the average change of yield spread is close to zero. The policy announcement is associated with a significant increase of the yield spread where $\beta_1 = 0.02$, which implies that announcing
regulatory policies to separate LGFV and local government increases the median yield spread of LGFV debt by 2 basis points on the days of policy announcements.

Next, I run the dummy variable regression with additional control variables X_t, including: monetary policy shocks (measured by China loan prime rate (LPR) and an indicator of cutting reserve ratio I_t), and the volatility of stock market (measured by the volatility index of Shanghai-50 ETF: CIVIX-50). I obtain the loan prime rate data from the website of People’s Bank of China and CIVIX-50 data from Wind Financial Terminal. The second and the third columns of Table 3.5 report the regression results with control variables. The coefficient for policy announcement remains significant after controlling for monetary policy shocks and market volatility.

Robustness To ensure that the result is robust to the definition of change of yield spread, I run the regression of equation (3.5) using the mean of daily change of yield spread instead of the median. First, for each day t, I compute the change of yield spread \( \Delta s_{i,t} \) for each LGFV debt \( i \). Then, I remove extreme values of \( \Delta s_{i,t} \) by removing the value above 99 percentile or below 1 percentile. Last, I compute the average \( \Delta s_{i,t} \) across all LGFV debts as \( Y_t \). Column (4) to column (6) of Table 3.5 shows the result of the regression using this alternative definition of \( Y_t \). We can see that the coefficient for policy announcement remains significant. The result is robust.

In addition, I run the regression model for LGFV debt with different credit ratings. Table 3.6 presents the result for LGFV debts with credit ratings ranges from AA- to AAA.\(^3\) We can see that, the coefficient of the policy dummy is significant and quantitatively equals to the results in Table 3.5. The policy announcement is associated with a 2.1 basis point increase of the yield spread on the day of announcement.

\(^3\)There are 6 LGFV debt has ratings below AA-. Because the median/mean of yield spread for these LGFV debt are driven by idiosyncratic movements of data outliers, I remove these LGFVs.
Table 3.5. Time Series Regression: Policy Announcements

<table>
<thead>
<tr>
<th>Change of Yield Spread</th>
<th>Median</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$I_{\text{news}}$</td>
<td>0.020***</td>
<td>0.021***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>$CIVIX_{50}$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>LPR</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_r$</td>
<td>-0.010</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Num.Obs</td>
<td>1076</td>
<td>1076</td>
</tr>
</tbody>
</table>

**Note:** Table 3.5 reports the coefficients of OLS regressions. The dependent variable is the daily change of the yield spread between LGFV debt and official local government debt. The first three columns show the result when the change of yield spread is taken as the median value across all LGFV debt. The last three columns show the result when the change of yield spread is taken as the mean value of all LGFV debt after removing extreme values. The explanatory variable ($I_{\text{news}}$) is an indicator for policy announcements to separate the local government and LGFV. The control variables include the change of Chinese stock market volatility index: CIVIX-50, and the monetary policy shock which is represented by the loan prime rate (LPR), and a dummy for cutting bank reserve ratio ($I_r$). Heteroscedasticity-robust standard errors are shown in parenthesis. ***, **, * denote significance level of 1%, 5%, and 10% respectively.
Table 3.6. Regression: Policy Announcements on LGFV of Different Ratings

<table>
<thead>
<tr>
<th></th>
<th>AA-</th>
<th>AA</th>
<th>AA+</th>
<th>AAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>$I_{news}$</td>
<td>0.021***</td>
<td>0.022***</td>
<td>0.019***</td>
<td>0.022**</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.006)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$CIVIX_{50}$</td>
<td>0.001</td>
<td>0.001</td>
<td>-0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>LPR</td>
<td>0.158</td>
<td>0.095*</td>
<td>0.206***</td>
<td>0.194</td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
<td>(0.051)</td>
<td>(0.073)</td>
<td>(0.237)</td>
</tr>
<tr>
<td>$I_r$</td>
<td>-0.003</td>
<td>0.003</td>
<td>-0.002</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.014)</td>
<td>(0.010)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Num.Obs</td>
<td>1076</td>
<td>1076</td>
<td>1076</td>
<td>1076</td>
</tr>
</tbody>
</table>

Note: Table 3.6 reports the results of OLS regressions for LGFV of different credit ratings. The dependent variable is the median daily change of the yield spread between LGFV debt and official local government debt. The explanatory variable is a dummy for policy announcements. Control variables include Chinese stock market volatility index: $CIVIX_{50}$, and the monetary policy shock which is represented by the loan prime rate (LPR), and a dummy for cutting reserve ratio. Heteroscedasticity-robust standard errors are shown in parenthesis. ***, **, * denote significance level of 1%, 5%, and 10% respectively.
3.5.2 Cross Section

In this section, I examine how the impact of policy announcements interacts with economic fundamentals of the local government. I conduct the pooled cross section regressions:

\[ \Delta s_{i,k} = \gamma_0 + \gamma_2 X_{i,k} \]  

Here \( \Delta s_{i,k} \) is the change of yield spread on the day of the \( k \)th policy announcement for each LGFV debt \( i \). First, I conduct the baseline regression where the explanatory variables \( X_{i,k} \) include: local government debt-to-GDP ratio, revenue-to-GDP ratio, expenditure-to-GDP ratio, GDP growth rate of the local area, as well as GDP per capita of the region in the year before the policy announcement.

Column 1 of Table 3.7 reports the results of the baseline regression. First, we can see that the change of yield spread is negatively correlated with the total debt-to-GDP ratio of the local government. When other things are equal, 1 percent increase of the local government debt-to-GDP ratio reduces yield spread by 6.3 basis points on the day of policy announcement. Second, government budget also has a significant impact on the yield spread of LGFV debt when regulatory policies are announced. Specifically, increasing revenue by 1 percent of GDP reduces the yield spread by about 5 basis points. Increasing government expenditure by 1 percent of GDP leads to 2.4 basis point increases the yield spread by 2.4 basis points when the regulation policies are announced. Last, GDP growth rate of the local area is positively correlated with the change of yield spread of LGFVs on the day of policy announcement. This is partially due to the fact that local governments tend to borrow more LGFV debt to finance the government investment projects to achieve high GDP growth target. Increasing GDP growth rate by 1 percent in the year before the policy announcement leads to a 6.5 basis point increases of yield spread when regulation policies are announced.
Table 3.7. Regression: Interaction of Policy Announcements and Fundamentals

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.052***</td>
<td>0.051***</td>
<td>0.046***</td>
<td>0.052***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Debt-to-GDP</td>
<td>-0.063***</td>
<td>-0.060***</td>
<td>-0.060***</td>
<td>-0.088***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Official Debt</td>
<td>-0.093***</td>
<td>-0.088***</td>
<td>-0.083***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>LGFV Debt</td>
<td>-0.018</td>
<td>0.003</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue-to-GDP</td>
<td>-0.049***</td>
<td>-0.050*</td>
<td>0.067**</td>
<td>0.147***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.028)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>Tax Revenue</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gov Fund</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditure-to-GDP</td>
<td>0.024***</td>
<td>0.027***</td>
<td>0.020**</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>0.065***</td>
<td>0.048***</td>
<td>0.050***</td>
<td>0.033**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.012)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>GDP per Capita</td>
<td></td>
<td></td>
<td></td>
<td>-0.001***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.000)</td>
</tr>
<tr>
<td>Num.Obs</td>
<td>3711</td>
<td>3711</td>
<td>3711</td>
<td>3711</td>
</tr>
</tbody>
</table>

Note: Table 3.7 reports the results of pooled cross-section OLS regressions. The dependent variable is the change of the yield spread between LGFV debt and official local government debt on days when the central government announces regulation policies on LGFVs. Explanatory variables include: local government debt-to-GDP ratio which includes official local government debt-to-GDP ratio and LGFV debt to GDP ratio; revenue-to-GDP ratio which includes tax revenue to GDP ratio and government fund revenue to GDP ratio in the last two columns; expenditure-to-GDP ratio, GDP growth rate, and GDP per capita of the local region. Heteroscedasticity-robust standard errors are shown in parenthesis. ***, **, * denote significance level of 1%, 5%, and 10% respectively.
The negative correlation between government debt-to-GDP ratio and the change of yield spread of LGFV debt appears to be counterintuitive. It suggests that policy announcements have larger effects on LGFV debt when the government has lower debt-to-GDP ratio. To understand which type of government debt causes this result, I decompose local government debt into official local government debt and LGFV debt. Column 2 of Table 3.7 shows the results when I replace local government debt-to-GDP ratio with its two components in the regression. We can see that the negative correlation is driven mostly by official local government debt-to-GDP ratio which is negatively correlated with the change of yield spread. Increasing official local government debt-to-GDP ratio by 1 percent of GDP reduces the change of yield spread by 9.3 basis points on the days of policy announcements. At the same time, the coefficient for LGFV debt-to-GDP ratio is not significant. In Section 3.6, I propose a model to show that this result can be explained by the seniority of official local government debt over LGFV debt.

Next, to investigate the relationship between the structure of government revenue and the impact of policy announcements, I decompose government revenue into tax revenue and government fund revenue. Column 3 of Table 3.7 reports the regression result after the decomposition. We can see that tax revenue and government fund revenue have opposite effects on the yield spread when the regulatory policies are announced. High tax revenue increases the yield spread, but high government fund revenue reduces the yield spread on the days of policy announcements. This result shows that investors anticipate regulatory policies to have different effects on local government tax revenue and government fund revenue. Conventionally, the tax revenue of local government is strictly regulated, but local governments typically have large flexibility regarding the use of local government fund revenue. Investors anticipate that the separation policies would have a strong effect in preventing local government
from using tax revenue to pay for LGFV debt, but the restrictions on the use of
government fund revenue would be weaker. As a result, the separation policy has
larger impact on the government that relies on tax revenue than on the government
that has high government fund revenue.

Finally, I check the robustness of the results by using GDP per capita of each city
as a control variable. Column 4 of Table 3.7 report the results. We can see that the
GDP per capita is negatively correlated with the change of yield spread on the day of
policy announcement. Increasing GDP per capita by 10000 yuan reduces yield spread
change by 0.1 basis point. The coefficients for government debt-to-GDP ratio, official
government debt-to-GDP ratio, government revenue-to-GDP ratio and GDP growth
rate remain statistically and economically significant after controlling for GDP per
capita. However, the coefficient for government expenditure becomes insignificant,
which suggests that the positive correlation between expenditure-to-GDP ratio and
the yield spread can be explained by different levels of development of different cities.

3.6 Model

In this section, I present a model for local government debt in China to explain the
empirical results. We consider a one-period model with a small open economy. There
are two agents in the economy: a local government and a local government finance
vehicle (LGFV).

At the beginning of the period, the local government borrows official debt \( B_G \) from
risk neutral international investors. Here \( B_G \) is the face value of the zero coupon local
government bond which is due at the end of the period. Similarly, the LGFV borrows
one-period LGFV debt \( B_v \) from risk-neutral international investors.\(^4\) At the end of

\(^4\)The subscript ‘G’ denote the government, and ‘v’ stands for vehicles.
the period, the local government and the LGFV receive stochastic income $Y_G$ and $Y_v$.

We consider two cases. In the first case, the LGFV is tied to the local government. In the second case, the local government and the LGFV are two separate entities.

### 3.6.1 Connecting Local Government with LGFV

When the local government and the LGFV are tied together, the local government and the LGFV combine their income and use the combined income to pay for the total amount of debt. I assume that, between the official local government debt and the LGFV debt, the official local government debt is the senior debt. The local government and the LGFV will first allocate their income to pay for the official local government debt, and then use the remaining income to pay for the LGFV debt. If the remaining income is not large enough to cover the LGFV debt, the LGFV will default.

Let binary $I_v$ denotes whether the LGFV defaults on its debt. $I_v$ is equal to 1 if the LGFV defaults, 0 otherwise.

$$I_v = \begin{cases} 
1 & \text{if } Y_G + Y_v < B_G + B_v \\
0 & \text{otherwise}
\end{cases} \quad (3.7)$$

When the LGFV defaults, international creditors receive 0 payoff from the LGFV debt. Consequently, the interest rate of the LGFV debt $r_v$ is:

$$r_v = \frac{1 + r^f}{1 - \text{Prob}(I_v = 1)} - 1 = \frac{1 + r^f}{1 - \text{Prob}(Y_G + Y_v < B_G + B_v)} - 1 \quad (3.8)$$

Here $r^f$ is the risk-free interest rate.

In addition, I assume that the government debt is guaranteed by an outside authority (for example, in the context of local government debt in China, the official local government debt is often perceived to be guaranteed by the central government of China). If the combined income is too small to cover the official local government debt, the local government and the LGFV will default.
debt $Y_G + Y_v < B_G$, the guarantor will pay for the remaining part of official local government debt $B_G - (Y_G + Y_v)$ to international creditors. As a result, the interest rate of official local government debt equals to the risk-free interest rate: $r_G = r^f$.

The yield spread between LGFV debt and official local government debt is:

$$s = (1 + r^f) \times \frac{\text{Prob}(Y_G + Y_v < B_G + B_v)}{1 - \text{Prob}(Y_G + Y_v < B_G + B_v)}$$

The equation (3.9) highlights two testable properties of the yield spread of LGFV debt. The following two propositions describe these two properties.

**Proposition 3.** The yield spread $s$ is increasing in the total amount of general government debt $B_G + B_v$.

This is because the probability function $\text{Prob}(Y_G + Y_v < B_G + B_v)$ is increasing in the amount of general government debt (the sum of official debt and LGFV debt) for any non-singular distributions of $Y_G$ and $Y_v$.

This result is consistent with empirical findings in Table 3.3. As we have discussed in Section 3.4.1, the first two columns of Table 3.3 show that the yield spread of LGFV debt increases as the general local government debt becomes higher.

**Proposition 4.** Because the probability function $\text{Prob}(Y_G + Y_v < B_G + B_v)$ is commutative in $B_G$ and $B_v$, both official local government debt and LGFV debt have the same effect on the yield spread $s$.

Here the intuition is that, from the perspective of the LGFV, both official debt and LGFV vehicle debt need to paid off in order to avoid default. Therefore, increasing either type of debt increases the probability of default, and thus increases the yield spread.

Proposition 4 is consistent with empirical observations. Column 3 and Column 4 of Table 3.3 present the regression results between the yield spread of LGFV debt
and economic fundamentals of local government in China. We can see that the yield spread is positively correlated with both official local government debt and LGFV debt and the correlation coefficients of these two types of debt are approximately equal.

3.6.2 Separating Local Government and LGFV

In this subsection, I discuss how the yield spread of LGFV changes when the local government and the LGFV are separated.

When the local government and the LGFV are separate entities, each entity uses its own income to pay for its debt. The LGFV defaults if its income is smaller than its debt. Let $I'_v$ be the default indicator for the LGFV after the separation (I use prime to denote the variables after the separation).

$$I'_v = \begin{cases} 1 & \text{if } Y_v < B_v \\ 0 & \text{otherwise} \end{cases}$$ (3.10)

As a result, the interest rate of LGFV debt is

$$r'_v = \frac{1 + r^f}{1 - \text{Prob}(Y_v < B_v)} - 1$$ (3.11)

Since the official local government debt is risk free, the yield spread between LGFV debt and official local government debt after the separation is

$$s' = (1 + r^f) \times \frac{\text{Prob}(Y_v < B_v)}{1 - \text{Prob}(Y_v < B_v)}$$ (3.12)

We can compute the change of yield spread due to the separation by combining equation (3.9) and (3.12):

$$\Delta s = s' - s = \frac{\text{Prob}(Y_v < B_v)}{1 - \text{Prob}(Y_v < B_v)} - \frac{\text{Prob}(Y_G + Y_v < B_G + B_v)}{1 - \text{Prob}(Y_G + Y_v < B_G + B_v)} (1 + r^f)$$ (3.13)
It is worth noting that $B_v$ and $B_G$ have different impact on $\Delta s$. For LGFV debt $B_v$, a higher $B_v$ increases both $s$ and $s'$. Therefore, the impact of $B_v$ on $\Delta s$ is ambiguous. Depending on the specific form of the probability function, and the amount of government debt, a higher $B_v$ can either increase or decrease $\Delta s$. However, the official local government debt $B_G$ only increases $s$ and does not affect $s'$. As a result, a higher official local government debt reduces $\Delta s$. The intuition here is that the separation of local government and LGFV releases the LGFV from the burden of paying for the official local government debt therefore reduces the risk of LGFV debt. This effect is stronger when the amount of official local government debt is larger.

Moreover, equation (3.13) suggests that, if the negative correlation between the yield spread $\Delta s$ and the amount of official local government debt $B_G$ is sufficiently large, $\Delta s$ can be negatively correlated with the total amount of debt ($B_v + B_G$). Proposition 3 summarizes the result.

**Proposition 5.** When there is an announcement to separate the LGFV and the local government, the change of yield spread $\Delta s$ is negatively correlated with the amount of official local government debt. At the same time, the impact of LGFV debt on $\Delta s$ is ambiguous. If the impact of official local government debt dominates, $\Delta s$ will be negatively correlated with the total amount of debt ($B_v + B_G$).

Proposition 5 explains the counterintuitive result from the news study in Section 3.5. As we discussed in Section 3.5, the first column of Table 3.7 shows that when there is a policy announced to separate the local government and the LGFV, the change of yield spread for LGFV debt is negatively correlated with the total amount of local government debt (including both official debt and LGFV debt). Column 2 of Table 3.7 shows the regression results after I decompose general local government debt into official local government debt and LGFV debt. We can see that the negative
correlation between government debt and the change of yield spread is mostly driven by official local government debt. These results are consistent with Proposition 5.

Finally, because the official local government debt in the model is risk free, the interest rate of official local government debt remains constant \( r'_G = r_G = r^f \). When the local government and the LGFV separate, the change of yield spread \( \Delta s \) is entirely caused by the change of the interest rate of LGFV debt:

\[
\Delta s = s' - s = (r'_v - r'_G) - (r_v - r_G)
\]

\[
= (r'_v - r_v) - (r'_G - r_G)
\]

\[
= (r'_v - r_v)
\]

Proposition 4 describes this property.

**Proposition 6.** When there is a policy announced to separate the local government and the LGFV, the change of yield spread \( \Delta s \) is caused by the change of the interest rate of LGFV debt, while the interest rate of official local government debt remains constant.

To test Proposition 6, I employ the news study method in Section 3.5.1. Specifically, I decompose the change of the yield spread of LGFV debt into two parts: (i) the change of the yield of LGFV debt \( \Delta r^v \) and the change of the yield of official local government debt \( \Delta r^G \). Then I replace the dependent variable in equation (3.5) with \( \Delta r^u \) and \( \Delta r^o \), and run the dummy variable regression.

Table 3.8 shows the regression result. The first three columns of Table 3.8 report the coefficients where the dependent variable is the median daily change of the yield of LGFV debt: \( y = \text{median}_{1 \leq i \leq N} \{ \Delta r^v_{i,t} \} \). Here \( \Delta r^v_{i,t} = r^v_{i,t} - r^v_{i,t-1} \). The last three
columns of Table 3.8 report the coefficients where the dependent variable is the median daily change of the yield of official local government debt.

We can see that when there is a regulatory policy announced to separate LGFVs and local governments, the interest rate of LGFV debt increases significantly. At the same time, the change of the interest rate of official local government is small. This result is consistent with Proposition 6.

3.7 Conclusion

The risk of debt issued by local government finance vehicles (LGFV) in China is not well understood despite the fact that LGFVs play an important role in financing local government investment projects which are the main sources of economic growth in many regions of China. This paper studies the risk of LGFV debt by examining how economic fundamentals and regulatory policies affect the yield spread of LGFV debt. Using a data set that covers information on local government debt between 2017 and 2022, I find that higher local government debt-to-GDP ratio significantly increases the yield spread of LGFV debt. This effect occurs regardless of whether the local government borrows official debt or LGFV debt. I also find that the yield spread is smaller in regions with higher GDP per capita. This result is due to the fact that the governments in higher income regions tend to have better fiscal conditions.

To study how the regulatory policies on LGFVs affect the risk of LGFV debt, I conduct a news study to investigate how the yield spread of LGFV debt changes on the days of policy announcements. The results suggest that the regulatory policies that separate LGFVs from local governments significantly increase the yield spread of LGFV debt on the days of policy announcements. The magnitude of the impact of policies varies with economic fundamentals of local governments. Specifically, I find
that the impact of the regulatory policy is smaller if the local government has higher debt-to-GDP ratio.

To explain the empirical results, I propose a model for local government debt in China. Using the model, I show that the negative correlation between the impact of regulatory policy on the yield spread of LGFV debt and local government debt-to-GDP ratio can be explained by the seniority of official local government debt over LGFV debt.

One factor that could affect the risk of LGFVs and has not been addressed in this paper is the political connection of the local government officials. There has been evidence that political network of local government officials can affect the borrowing and the default decisions of local governments (Ang et al., 2015; Gao et al., 2021). Further research can be done to study how the political network of government officials affects the risk of LGFV debt and the effect of regulatory policies.
### Table 3.8. Regression: Interest Rate Vs Policy Announcement

<table>
<thead>
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<th>Change of Interest Rate</th>
<th>LGFV</th>
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<th>Official Debt</th>
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<td></td>
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<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Constant</td>
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<td>-0.002**</td>
<td>0.023**</td>
<td>-0.001*</td>
<td>-0.001**</td>
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<td>(0.012)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>( I_{\text{news}} )</td>
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<td>0.010**</td>
<td>0.010**</td>
<td>-0.008***</td>
<td>-0.008***</td>
<td>-0.008**</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>( CIVIX_{50} )</td>
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<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
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<td>0.001</td>
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<tr>
<td></td>
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<td>LPR</td>
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<tr>
<td>( I_r )</td>
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</tr>
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<td>1076</td>
<td>1076</td>
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</tr>
</tbody>
</table>

**Note:** Table 3.8 shows the results of OLS regressions. The dependent variable in the first three columns is the median daily change of the yield for LGFV debt. The dependent variable for the last three columns is the median change of yield for formal government debt. The explanatory variable \( I_{\text{news}} \) is an indicator for policy announcements to separate the local government and LGFV. The control variables include the change of Chinese stock market volatility index: CIVIX-50, and the monetary policy shock which is represented by the loan prime rate (LPR), and a dummy for cutting bank reserve ratio \( I_r \). Heteroscedasticity-robust standard errors are shown in parenthesis. ***, **, * denote significance level of 1%, 5%, and 10% respectively.
Appendix A

Appendix to Chapter 1

A.1 Theory

A.1.1 Proof of Lemma 1

In state 'a', for any $(e, \tau, \pi)$, let $v^a(b, e, \tau, \pi)$ be the value of the government when it chooses fiscal policy $(e, \tau)$. The value is the same if the government does not adjust fiscal policy and inherits $(e, \tau)$ from the previous period

$$v^a(b, e, \tau, \pi) = v^n(b, e, \tau, \pi)$$  \hspace{1cm} (A.1)

In state 'a', the government optimizes fiscal policy

$$v^a(b, \pi) \geq v^a(b, e, \tau, \pi) \; \forall \; e, \tau, \pi$$  \hspace{1cm} (A.2)

Combining condition (A.1) and (A.2)

$$v^n(b, e, \tau, \pi) \leq v^a(b, \pi) \; \forall \; e, \tau, \pi$$  \hspace{1cm} (A.3)

Because in state 'a', defaults only happen strategically (See Section A.2 for proof), the default threshold $b^a(\pi)$ satisfies

$$v^a(b^a(\pi), \pi) = v^d(\pi)$$  \hspace{1cm} (A.4)
Combining (A.3) and (A.4),

\[ \nu^n(\bar{b}^a, e, \tau, \pi) \leq \nu^d(\pi) \quad \forall e, \tau, \pi \]

In addition, both \( \nu^n \) and \( \nu^a \) are decreasing in \( b \), so \( \forall b > \bar{b}^a \)

\[ \nu^n(b, e, \tau, \pi) < \nu^n(\bar{b}^a, e, \tau, \pi) \leq \nu^d(\pi) \quad \forall e, \tau, \pi \]

Hence the default threshold in state 'n' \( \bar{b}^n(e, \tau, \pi) \) satisfies

\[ \bar{b}^n(e, \tau, \pi) \leq \bar{b}^a(\pi) \quad \forall e, \tau, \pi \quad (A.5) \]

**A.1.2 Proof of Proposition 1**

Let \( b \) denote government debt-to-GDP ratio of the current period, and \( b' \) be the debt ratio of next period.

First, I show that in state 'n', if government debt-to-GDP ratio is decreasing \( b' < b \), then there is no rollover crises.

In state 'n', if the government does not default in current period

\[ \nu^n(b, e, \tau, \pi) > \nu^d(\pi) \quad (A.6) \]

Because \( \nu^n(b, e, \tau, \pi) \) is decreasing in \( b \), if \( b' < b \), the government will not default strategically in next period either

\[ \nu^n(b', e, \tau, \pi) \geq \nu^n(b, e, \tau, \pi) = \nu^d(\pi) \quad (A.7) \]

If the government can rollover its debt in current period, the government budget constraint implies that

\[ \frac{b}{G} + e - \tau - S \geq \max_{b'} \{ q(b', e, \tau, \pi) b' \} \quad (A.8) \]
Because $b' < b$, the government will be able to rollover its debt next period. In this case, the bond price is

$$q(b', e, \tau, \pi) = \beta$$  \hfill (A.9)

Combining condition (A.8) with equation (A.9),

$$\frac{b}{G} + e - \tau - S \geq \max_{b'} \{\beta b'\}$$  \hfill (A.10)

Given that $b' < b$, $\max_{b'} \{\beta b'\} = \beta b$.

$$\frac{b}{G} + e - \tau - S \geq \beta b$$  \hfill (A.11)

Here I assume the risk-free interest rate is larger than the income growth rate: $\frac{1}{\beta} > G$. We can solve $b$ as

$$b \leq \frac{G(\tau + s - e)}{1 - \beta G} \hfill (A.12)$$

As long as $b$ satisfies condition (A.12), and $b' < b$, the government will not default due to rollover crisis.

Note when $b > \hat{b}$, government debt-to-GDP ratio is increasing which contradicts the assumption that $b' < b$. Hence, there is no rollover crises when $b' < b$.

Second, I show that rollover crises cannot happen in state 'a'. This is because, in state 'a', the government can choose fiscal policy $(e, \tau)$ to put government debt-to-GDP ratio on a decreasing path. As I show in previous paragraph, the rollover crisis will not happen in this case.

Third, I show that in state 'n', if government debt-to-GDP ratio is increasing $b' > b$, a rollover crisis can happen, and the threshold for the rollover crisis $\bar{b} \approx \phi \bar{b}'$.

In state 'n', the rollover crisis happens when

$$\frac{b}{G} + e - \tau - S > \max_{b'} \{q(b', e, \tau, \pi) b'\}$$  \hfill (A.13)
Depends on whether the government defaults next period, the bond price

\[
q(b', e, \tau, \pi) = \begin{cases} 
\beta & \text{if } b' \leq \bar{b}^n(e, \tau, \pi) \\
\phi \beta & \text{if } \bar{b}^n(e, \tau, \pi) < b' \leq \bar{b}^a(\pi) \\
0 & \text{if } b' > \bar{b}^a(\pi) 
\end{cases}
\]  

(A.14)

As a result

\[
\max_y \{q(b', e, \tau, \pi) b' \} = \max \{\beta \bar{b}^n(e, \tau, \pi), \beta \phi \bar{b}^a \} 
\]  

(A.15)

When government debt-to-GDP ratio reaches the default threshold in current period \(b = \bar{b}^n(e, \tau, \pi)\). Because \(b' > b > \bar{b}^n(e, \tau, \pi)\), the government can roll over its debt only if it adjusts fiscal policy in next period:

\[
\frac{\bar{b}^a}{G} + e - \tau - S = \beta \phi \bar{b}^a 
\]  

(A.16)

Solve \(\bar{b}^n(e, \tau, \pi)\) as

\[
\bar{b}^n = \beta G \phi \bar{b}^a + (\tau + S - e) G 
\]  

(A.17)

Here \(\beta G\) is close to 1, and the primary balance-to GDP ratio \((\tau + S - e)\) is much smaller than \(\bar{b}^a\)

\[
\bar{b}^n \approx \phi \bar{b}^a 
\]  

(A.18)

A.1.3 Proof of Proposition 2

Given that the default threshold \(\bar{b}^n\) is determined by a rollover crisis,

\[
\frac{\bar{b}^n}{G} + e_t - \tau_t = \beta \phi \bar{b}^a 
\]  

(A.19)

we can solve \(\bar{b}^n(e_t, \tau_t, \pi^{Low})\)

\[
\bar{b}^n(e_t, \tau_t, \pi^{Low}) = G [\beta \phi \bar{b}^a(\pi^{Low}) + \tau_t - e_t] 
\]  

(A.20)
If the seigniorage \( s_t \) satisfies
\[
s_t \geq \frac{1 - \beta G}{G} + e_t - \tau_t \tag{A.21}
\]
We can rearrange (A.21)
\[
\beta b_{t-1} \geq \frac{b_{t-1}}{G} + e_t - \tau_t = \beta b_t
\implies b_{t-1} \geq b_t \tag{A.22}
\]
The government debt-to-GDP ratio is on a decreasing path. Proposition 1 ensures that the default threshold \( \bar{b}^a \) is determined by strategic default where
\[
v^n(\bar{b}^n, \cdot) = v^d(\cdot) \tag{A.23}
\]
In this case, \( \bar{b}^n \) is close to \( \bar{b}^a \). If the seigniorage does not satisfy condition (A.21), the government debt-to-GDP ratio is on an increasing path \( b_{t-1} < b_t \). The threshold for rollover crisis \( \bar{b}^n \) is
\[
\bar{b}^n(e_t, \tau_t, \pi^{High}) = G\left[\beta \phi \bar{b}^a(\pi^{High}) + \tau_t + s_t - e_t\right] \tag{A.24}
\]
Combining equation (A.20) and (A.24), we can solve the increase in the default threshold due to monetary backstop as
\[
\frac{\bar{b}^n(e_t, \tau_t, \pi^{High}) - \bar{b}^n(e_t, \tau_t, \pi^{Low})}{\Delta \bar{b}^n} = G\beta \phi \left[\frac{\bar{b}^a(\pi^{High}) - \bar{b}^a(\pi^{Low})}{\Delta \bar{b}^a}\right] + Gs_t \tag{A.25}
\implies \Delta \bar{b}^n = G\beta \phi \Delta \bar{b}^a + Gs_t \tag{A.26}
\]
The first part on the right hand side (RHS) of equation (A.25) indicates that the monetary backstop increases the threshold of rollover crisis by increasing the debt limit in state 'a'. The second part on the RHS indicates that the monetary backstop also directly relaxes the government budget constraint producing a seigniorage in current period. The seigniorage \( s_t \) tends to be much smaller than the existing debt
stock $\bar{b}$. When $\phi$ is small, the first part on the RHS is also small. As a result, when the seigniorage is not large enough to put the government debt on a downward path, the impact of the monetary backstop on the default threshold is small.

### A.1.4 Derivation for equation (1.33) and (1.35)

**Equation (1.33)**

We can show that equation (1.33) measures the benefit of introducing a monetary backstop as equivalent changes in trend income.

Because the value function can be normalized by the level of trend income,

$$V_{t} = v_{t} \tilde{Y}_{t} \quad (A.27)$$

I assume that, after introducing the monetary backstop, the value of the government is equal to the value in the case when there is no monetary backstop but the trend income increases to $\hat{Y}$

$$V_{t} = v_{t} \hat{Y} \quad (A.28)$$

The equation (1.33) can be written as

$$\Delta V = \frac{V_{t} - V_{t}}{V_{t}} = \frac{v_{t} \hat{Y}_{t} - v_{t} Y_{t}}{v_{t} Y_{t}} = \frac{\hat{Y}_{t} - Y_{t}}{Y_{t}} \quad (A.29)$$

Hence, equation (1.33) measures the welfare benefit of introducing the monetary backstop as an equivalent increase in the trend income.

**Equation (1.35)**

Similarly, we can show that equation (1.35) measures the welfare cost of removing the monetary backstop in fraction changes in trend income. Because the value function is
homothetic in $Y$

$$V^π_t = v^π_t Y_t$$  \hfill (A.30)

Here $v^π_t$ is the value of the household normalized by the level of trend income $Y_t$.

I assume that the value of the government after removing the monetary backstop is equal to the value in the case where the central bank continues providing the monetary backstop, but the trend income is reduced to $\tilde{Y}_t$

$$V^{πf}_t = v^π_t \tilde{Y}_t$$  \hfill (A.31)

We can rewrite equation (1.35) as

$$\Delta V = \frac{V^π_t - V^{πf}_t}{V^π_t} = \frac{v^π_t Y_t - v^π_t \tilde{Y}_t}{v^π_t Y_t} = \frac{Y_t - \tilde{Y}_t}{Y_t}$$  \hfill (A.32)

Hence, equation (1.35) measures the welfare cost of removing the monetary backstop as an equivalent reduction in the trend income.

\section*{A.2 Empirical}

\subsection*{A.2.1 Data}

I obtain data on the government budget for advanced economies from the IMF World Economic Outlook (WEO) database. The definition of advanced economies follows the WEO categorization. To capture the long-run dynamics of government debt, I select countries that have data since 1992. \footnote{These countries are: Australia, Austria, Belgium, Canada, Chile, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, UK, and USA. See Appendix A for the detail of data.} For each country, the data set includes
yearly data on general government budget, such as debt-to-GDP ratio, revenue, and primary expenditure, and data on macroeconomic variables such as GDP, inflation, and real growth rates.

A.2.2 Decomposition of The Change of Debt-to-GDP

This appendix shows how to decompose the change of debt-to-GDP ratio into the contribution of primary balance, interest-growth differential, and stock flow residuals.

The change of debt-to-GDP ratio:

\[
\Delta d_t = d_t - d_{t-1} = \frac{D_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}}
\]  

(A.33)

Here \(D_t\) is the nominal amount of government debt outstanding at the end of time \(t\). \(Y_t\) is the nominal GDP of period \(t\). Let \(TR_t\) be the amount of total revenue of the government, \(TE_t\) be the amount of total expenditure. The nominal amount of government debt evolves according to:

\[
D_t = D_{t-1} - (TR_t - TE_t) + SF_t
\]  

(A.34)

\(SF_t\) is the difference between the annual change in the amount of debt outstanding and the budget deficit, here we refer to it as the nominal amount of the stock flow. The stock flow is usually the result of net acquisition of financial assets, debt adjustment effects, and statistical discrepancies.

The total government expenditure \(TE_t\) is the sum of the primary expenditure \(PE_t\) and the interest payment on the existing debt. Let \(i_t\) be the average nominal interest rate on the government bond.: \(TE_t = PE_t + i_t \times D_{t-1}\). We can rewrite equation
\begin{align*}
D_t &= D_{t-1} - [TR_t - (PE_t + i_t \times D_{t-1})] + SF_t \\
&= (1 + i_t)D_{t-1} - (TR_t - PE_t) + SF_t \\
&= (1 + i_t)D_{t-1} - PB_t + SF_t \quad \text{(A.35)}
\end{align*}

Here $PB_t = TR_t - PE_t$ is the amount of the primary balance.

Let $g^n_t$ be the nominal growth rate: $Y_t = (1 + g^n_t)Y_{t-1}$. We can substitute $D_t$ with equation (A.35):

\begin{align*}
\Delta d_t &= \frac{(1 + i_t)D_{t-1} - PB_t + SF_t}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} \\
&= \frac{(1 + i_t)D_{t-1}}{(1 + g^n_t)Y_{t-1}} + \frac{(-PB_t + SF_t)}{Y_t} - \frac{D_{t-1}}{Y_{t-1}} \\
&= \left[\frac{(1 + i_t)}{(1 + g^n_t)} - 1\right] d_{t-1} - pb_t + sf_t \quad \text{(A.36)}
\end{align*}

The first term on the right hand side of equation (A.35) represents the contribution of interest-growth differential to the change of debt-to-GDP. The interest rate and the growth rate here are both measured in nominal terms. In the $r - g$ part that I present in the main text, both the interest rate and the growth rate are in real terms.

To convert nominal value into real value, let $Z_t$ be real GDP at time $t$, and $P_t$ be the price index of all output at time $t$, the growth rate of nominal GDP is:

\begin{align*}
1 + g^n_t &= \frac{Y_t}{Y_{t-1}} \\
&= \frac{P_t Z_t}{P_{t-1} Z_{t-1}} \\
&= (1 + \pi_t)(1 + g_t) \quad \text{(A.37)}
\end{align*}

Here $g_t$ is the growth rate of real GDP. $\pi_t$ is the inflation rate calculated with the price of all output. \footnote{$P_t$ is calculated by dividing the GDP deflator with 100. $1 + \pi_t = \frac{\text{GDP deflator of time } t}{\text{GDP deflator of time } t-1}$}
Define the real interest rate $r_t$ as:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_t}$$  \hspace{1cm} (A.38)

we prove the decomposition in the main text:

$$d_t = \left[\frac{(1 + r_t)}{(1 + g_t)} - 1\right] d_{t-1} - pb_t + sf_t$$

### A.2.3 Event Study for Debt Decrease Episodes

Figure A-1 and A-2 show the result for debt decrease episodes.

At the beginning of debt decrease episodes, the real growth rate rises substantially, reduces the $r - g$ component. In addition, when the growth rate increases, the primary expenditure continue growing with the original low growth rate while the output and the revenue grow faster. The inertia of the primary expenditure causes the primary balance-to-GDP to increase, eventually reduces the $\Delta$Debt-to-GDP.

**Figure A-1.** The level of Output, Revenue, and Primary Expenditure around the beginning (set as time 0) of debt decrease period. The window covers from three years before the debt increase period to eight years into the debt increase period. All levels at year $-2$ are normalized to be 1
A.2.4 Additional Results

Figure A-3 shows the dynamics of government debt-to-GDP ratio for additional countries in the data set.
Figure A-2. Mean of selected macroeconomic variables for simulations (dashed line) and empirical data (solid line) at the beginning of a debt decrease episodes. The debt decrease episodes starts at time 0 which is marked with a black line. All variables are measured in percentage of GDP.
Figure A-3. Debt-to-GDP ratios for selected advanced economies. Debt increase episodes are marked as red area, debt decrease episodes are marked as blue area.
A.3 Numerical Methods

A.3.1 Solution Methods

I employ a value function iteration method to solve the model. The solution procedures are as follows.


2. Partition government debt-to-GDP ratio $b \in (-0.5, 6.5)$ into 400 points, government revenue-to-GDP ratio $\tau$ into 40 points, and government expenditure-to-GDP ratio $e$ into 12 points.

3. For given inflation rate for current period $\pi_t$ and inflation target for next period $\pi_{t+1}$, compute money-to-GDP ratio $m$ and seigniorage revenue-to-GDP ratio $s$.

4. Given any $(G, m)$, compute the optimal government fiscal policy in the state of default $(e^d, \tau^d)$, and the utility of that period $u^d$. Here when the government is in the state of default, the government cannot borrow to smooth the tax rate. The government optimization reduces to a static problem which can be solved separately.

4. Initialize value function $v(b, m, e, g, \tau)$, $v^a(b, m)$, $v^a(b, m, e, g, \tau)$, $v^d(G, m)$, the policy function $e(b, G, m)$ and $\tau(b, G, m)$. Starting from this step, I use the value function iteration method to solve for the government optimization problem.

6. For any given $(G, m)$, compute the value in default using equation (1.19).

7. For any given $(b, m, e, g, \tau)$, compute the value of the government when there is no adjustment $v^n(b, m, e, g, \tau)$ following equation (1.17). There are three key
issues worth noticing in this calculation. First, the price of government bond depends on the amount of new borrowings. Second, because the government can not adjust fiscal policy, the amount of new borrowings of the government will be determined by government budget constraint (1.16). As a result, the amount of new borrowings may not be a value on the grid of government debt-to-GDP ratio. To solve for new borrowings $b$ and value of the government, I conduct three following steps:

a. I interpolate value function $v$ and bond price $q$ linearly over government debt-to-GDP ratio $b$.

b. I compute the new borrowing using government budget constraint (1.16) and household consumption $c$ using resource constraint (1.28).

c. If household consumption is negative, the government can not raise enough money to cover its financing needs. In this case, I set $v^n$ to be $-1 \times 10^{10}$. If the household consumption is positive, I compute $v^n$ following equation (1.17).

(7) For any $(b, G, m)$, compute the value of the government in the state of adjustment in following steps:

a. Initialize the array for the value of the government choosing each possible fiscal policy: $v^a(b, G, e, \tau, m)$.

b. For any given policy $(e, \tau)$ on the grid, compute household consumption $c$ using the resource constraint (1.28).

c. If household consumption is negative, set $v^a(b, G, e, \tau, m) = -1 \times 10^{10}$. If household consumption is positive, set $v^a(b, G, e, \tau, m) = v^n(b, G, e, \tau, \pi)$.

d. Search for the optimal fiscal policy $(e, \tau)$ that maximizes $v^a(b, G, e, \tau, m)$.
Update fiscal policy and update the largest value of \( v^a(b, G, e, \tau, m) \) as the value of the government in the state of adjustment \( v^a(b, G, m) \).

(8) Compare value of continuation and value of default to decide on the indicator of default and actual value of the government using condition (1.21) and (1.19).

(9) Determine the inflation target for the next period using condition (1.24).

(10) For any given state \( (b', g, e, \tau, m) \), compute the price of the government bond \( q(b', g, e, \tau, m) \) using equation (1.25).

(11) Compute the maximum absolute difference between the value \( v \) in current iteration and \( v \) from previous iteration. If the difference is larger than \( 1 \times 10^{-4} \), repeat procedure (6) to (11).

**A.3.2 Simulation Procedure**

After I calibrated the model, I conduct three types of simulations using the model. First, I simulate an economy using the baseline model as follows:

1. Initialize the beginning state of the economy.

2. For each period \( t \), generate the growth rate \( G_t \) from the Markov process and shocks \( \xi_t \) to determine whether the government adjust fiscal policy in that period. If the inflation rate for current period \( \pi_t \) is high and condition (1.24) is satisfied, generate shocks to determine whether the inflation rate will return to low.

3. Given the state of the economy and the realization of the shocks, determine whether the government defaults in current period and determine the inflation target for next period based on the solution of the model.
4. Given the state of the economy and the realization of the shocks, choose the fiscal policy $e, g, \tau$ of the current period based on the solution of the model.

5. Compute the price of the government bond.

I run the model for 10000 histories. Each history lasts for 400 period. I drop the beginning 150 periods as the transition period.

Second, I run simulations for an economy where the monetary backstop is removed unexpectedly as follows:

6. I solve the model with no monetary backstop by setting $\pi^{High} = \pi^{Low} = 0.02$.

7. I run 10000 simulations of the economy with monetary backstop for 300 period following steps 1 to 5.

8. I use the state of the economy at the end of the simulation in step 7 as the beginning state, and run 10000 simulations for the model with no monetary backstop, each simulation lasts for 400 period. These 400 periods are the simulations of an economy where monetary backstop is removed unexpectedly.

Third, I run simulations for an economy where the monetary backstop is introduced unexpectedly:

9. I run 10000 simulations of the economy with no monetary backstop for 300 period following steps 1 to 5.

10. I use the state of the economy at the end of the simulation in step 9 as the beginning state, and run 10000 simulations for the model with monetary backstop, each simulation lasts for 400 period. These 400 periods are the simulations of an economy where monetary backstop is introduced unexpectedly.
Appendix B

Appendix to Chapter 2

B.1 Numerical Methods

B.1.1 Solution Methods

The numerical method to solve the model includes two main steps. First, because the optimization problems for households, firms, and banks are static, we solve these static problems separately. Second, after we solve the static part of the model, we use the value function iteration method to solve the government optimization problem.

The procedures for solving the static part of the model is the following,

1. Construct discrete shocks for the AR(1) distribution $z$, and $e$.

2. Partition the government debt $b \in (-0.15, 0)$ into 150 points, and government guarantee $\eta \in (0, 1)$ into 101 points.

2. For any given state $(b, \eta, z, e)$, we compute $\bar{\pi}$ and $r^B$ use equation (21).

3. Compute the bankruptcy threshold $\varrho^*$, the quantity of bank loans $l^F$, and the interest $r^F$ by solving equation system (2.36), (2.18), and (2.21) using Newton Method.

4. Use equation (30), (31), (32) (34) to solve for $N_t, M_t, W_t, Y_t$. 

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5. Compute household consumption \( c^L \) and \( c^K \), and the household labor supply \( n_t \).

Next, we solve the government optimization problem using the value function iteration method. The procedure is the following:

6. Initialize the guess of the value function \( V, V^R, \) and \( V^D \).

7. For any given state \((b, \eta, z, e)\), compute the felicity \( U(c^L, c^K) \), and the value of continuation using equation (2.24).

8. Compute the value of default \( V^D \).

9. Compare \( V^R \) and \( V^D \) to determine whether the government defaults and update the value of the government \( V \), save the value from the previous iteration as \( V_p \).

10. Compute the interest rate of government debt \( r^s \).

11. Compute the absolute difference between the value of the government \( V \) and the value of the government from previous iteration \( V_p \). If the difference is larger than \( 1 \times 10^{-4} \), repeat step 7 to 10. Until the difference is smaller than the threshold.

**B.1.2 Simulation Procedures**

This section describes the procedures for generating a history of the simulated outcomes of the model.

1. Initialize the beginning state of the economy.

2. For each period \( t \), draw random shocks \( z_t \) and \( e_t \).

3. Based on the shocks \((z_t, e_t)\) and the state variables \((\eta_t, b_t)\) of the economy, determine whether the government defaults or repay its debt.
4. Depends on whether the government defaults, generate policy variable $\eta_{t+1}$, $b_{t+1}$.

5. Given the government policy variables, compute $r^F_t$, $l^F_t$, and $\varrho_t$.

6. Compute out $Y_t$, labor $N_t$, and household consumption of the period.

In each simulation, we let the economy run for 15000 periods. The beginning 2500 periods is the transition phase. We report the data after the transition phase to reduce the impact of the initial state of the economy.

We run the model for 5000 times and compute the moments in Table 2.4 across periods and simulations. The data presented in Figure 2-1 to Figure 2-7 are computed as the average value of across periods and simulations.
Appendix C

Appendix to Chapter 3

C.1 Additional Results

In this appendix, I discuss the relationship between credit ratings of LGFV and the yield spread of LGFV debt. Specifically, I regress the average yield spread of each LGFV debt over the period of maturity on the credit ratings of the LGFV debt at issuance.

\[
\bar{s}_i = \gamma_0 + \gamma_1 H_i + \gamma_2 X_i
\]  

(C.1)

Here \( H_i \) is an integer between 0 and 9 which indicates the credit rating of the LGFV debt \( i \) at issuance. The credit ratings are between C and AAA. The higher the number is, the better the rating is. The dependent variable \( \bar{s} \) is the average yield spread of LGFV debt \( i \) since its issuance. Control variables \( X \) include local government debt-to-GDP ratio, revenue-to-GDP ratio, expenditure-to-GDP ratio, GDP growth rate of local economy, and GDP per capita of the city where the LGFV is located.

Table C.1 shows the results. Column 1 reports the coefficient of credit ratings when there is no control variables. We can see that as the credit ratings becomes higher, the yield spread of LGFV deb becomes smaller. On average, increasing the credit ratings by 1 level is associated with 7 basis point decrease of the yield spread.
Column 2 to Column 4 report the regression result when I control for local government finances and growth rate of local economy. We can see that the coefficients for credit ratings are significant after including the control variables in the regression, indicating that credit ratings of LGFV debt contains information beyond the fundamentals of local government and local economy.
Table C.1. Regression: Yield Spread on Credit Ratings

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<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
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<tr>
<td>Constant</td>
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<td>7.341***</td>
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<td>(0.462)</td>
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<td>(0.450)</td>
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<td>Revenue-to-GDP</td>
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<td>(0.959)</td>
<td>(0.965)</td>
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<td>(0.794)</td>
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<td>GDP Growth</td>
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Note: Table C.1 reports coefficients of OLS regressions. The dependent variable is the yield spread between LGFV debt and official local government debt. Explanatory variables include: credit ratings of LGFV debt at issuance, local government debt-to-GDP ratio, revenue-to-GDP ratio, expenditure-to-GDP ratio, GDP growth rate, and GDP per capita of the local region. Credit rating is an integer from 0 to 9, each integer represents a credit rating level. The higher the number is, the higher the rating is. Heteroscedasticity-robust standard errors are shown in parenthesis. ***, **, * denote significance level of 1%, 5%, and 10% respectively.
Bibliography


Vita

Tongli Zhang graduated from Tsinghua University with a Bachelor degree in Engineering Mechanics and Aerospace Engineering in 2013. He received a Master of Finance degree from The Johns Hopkins Carey Business School in 2014. He enrolled in the PhD in Economics program at Johns Hopkins University in 2016. He will join the Economist Program of the International Monetary Fund in 2022.