

DOES THE LAW OF DIMINISHING RETURNS APPLY TO INFANT  
MORTALITY DECLINE?

by

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## ABSTRACT

**Objective:** This paper examines time series data on infant mortality from 21 countries to demonstrate an appropriate test of the hypothesis that percentage reductions in infant mortality are larger when infant mortality is lower. Prior research expounding this hypothesis has dubbed it “the Matthew effect”.

**Method:** Time series for infant mortality can be modeled as  $X_t = \mu + \theta_1 X_{t-1} + \varepsilon_t$  where  $\varepsilon_t$  is identically and independently distributed. If  $\theta_1=1$  it is easily demonstrated that the time series has an asymptotic distribution with infinite variance. The correct test to apply in this situation is the Dickey-Fuller test which we use to test the statistical significance of  $\theta_1$  in a regression analysis of  $\Delta \log \text{IMR}_t = \mu + \theta_1 \log \text{IMR}_{t-1} + \varepsilon_t$ . Evidence that  $\theta_1$  is significant and negative would support the claim that there is a Matthew Effect in infant mortality. This paper uses time series data on IMR from 21 nations for 1870-1988. Several additional lagged values of  $\Delta \log \text{IMR}$  were appended using an Akaike Indicator Criterion to select the preferred specification. Transformations of IMR other than simple logarithms were explored.

**Results:** With the preferred specification, the Dickey-Fuller test rejected the presence of any Matthew Effect in all but three countries. The rejection of a Matthew Effect was robust to alternative specifications of the lag structure of IMR and to various transformations of IMR other than logarithmic.

**Conclusion:** Based on 20<sup>th</sup> century data there is scarce evidence that percentage reductions in infant mortality are generally smaller in higher mortality countries. Large percentage reductions in infant mortality are possible for countries at any stage in economic development and are likely to be reflective of durable advances in human knowledge, social institutions, and physical capital.

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## **Introduction**

The law of diminishing returns as applied to population health suggests that as populations become healthier it becomes ever more difficult to add percentage improvements to their health. The rationale is that societies will first undertake the most cost-effective interventions to reduce infant mortality, so that after substantial progress has been achieved, new gains require investments in less cost-effective measures. The so-called Matthew effect<sup>1</sup> in infant mortality violates the law of diminishing returns by asserting that percentage changes in infant mortality are larger where infant mortality rates are lower (4). If the Matthew effect is an appropriate characterization of the general pattern of infant mortality decline then there is a paradox. Matthew effects suggest that as infant mortality rates fall, it becomes ever less difficult for societies to improve infant health.

Some evidence for a Matthew effect is based on anecdotal consideration of the frustrating stall in sub-Saharan African infant mortality rate (IMR) decline over the past 20 years (5). Other evidence for a Matthew effect in infant mortality is based on work which ranked 133 countries in order of increasing infant mortality rates (IMR) in 1960 and in order of percentage decline in IMR from 1960 to 1995. A Spearman's rank correlation test found a positive and significant correlation in the rankings—best ranked countries for IMR were likely to also have good ranking for large percentage declines in IMR (4).

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<sup>1</sup> The term "Matthew effect" was coined by Merton 3. Merton RK. The Matthew effect in science. *Science* 1968;159(810):56-63. after the Biblical verse "Unto every one that hath shall be given..." Matthew 25:29.

If a Matthew effect is a general pattern in the improvement of the health of populations then we lack a theoretical account of why the law of diminishing returns has been suspended for health. There is conjecture that political instability (5), poor government decisions (4, 5), and epidemics such as AIDS (6, 7) may be responsible for some of the exceptional failures to bring down high rates of infant mortality. The possibility that exceptional failures are “exceptional” suggests that Matthew effects are not the rule.

The objective of this paper is to study the dynamics of infant mortality decline using data from 21 countries that span most of the 20<sup>th</sup> century. We will develop a more efficient test for the existence of a Matthew effect and apply it to the data. We hypothesize that Matthew effects are indeed exceptions to the rule. We also hope to share a historical insight that emerges during this exercise. Statistically, the random year to year fluctuations in infant mortality reflect durable shifts in social and human capital that affect the survival of subsequent cohorts of infants in the same way that fluctuations in the physical capital stock endure long after a year of intensive capital investment or a catastrophic natural disaster. Increments and decrements in capital last because capital does. The same durability that is associated with capital is a feature of the fluctuations in infant mortality. We will show that in a statistical sense increments and decrements to IMR last indefinitely in the same way as capital, perhaps because IMR is a measure of a stock of health resources.

Equipped with an improved understanding of the dynamics of infant mortality decline one will be able to see a country with a high IMR, not as a lost cause, but as a tremendous opportunity for rapid and sustained improvements in human welfare.

## Methods

The data for this study come from the countries and years listed in the first two columns of Table 1. A test for the Matthew effect could be formulated as a hypothesis test of the null that  $\beta=0$  vs. alternative that  $\beta<0$  in the following equation:

$$[1] \ \% \Delta \text{IMR}_t = C + \beta \text{IMR}_{t-1} + \varepsilon_t$$

or in logs<sup>2</sup>

$$[2] \ \Delta \log(\text{IMR}_t) = C + \beta \log(\text{IMR}_{t-1}) + \varepsilon_t$$

where C is a constant trend e.g. a historically steady decrease (or increase) in IMR and  $\varepsilon_t$  is distributed normally

Dickey and Fuller have shown that the conventional t-test cannot be used to test the null that  $\beta=0$  (2) . The reason the conventional test is incorrect is because of the genuine possibility that the process under study has non-normal variance. One can demonstrate this possibility by adding  $\log(\text{IMR}_{t-1})$  to both sides of equation [2] to rewrite it as

$$[3] \ \log(\text{IMR}_t) = C + (1 + \beta) \log(\text{IMR}_{t-1}) + \varepsilon_t$$

Under the null that  $\beta=0$  this becomes

$$[4] \ \log(\text{IMR}_t) = C + \log(\text{IMR}_{t-1}) + \varepsilon_t$$

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<sup>2</sup>  $\Delta \log(\text{IMR}_t)$  is an approximation for  $d \log(\text{IMR}) = d \text{IMR} / \text{IMR} \cong \% \Delta \text{IMR}_t$

The problem with estimating equation [3] can be demonstrated if one considers estimating an IMR time series with 100 years of data from 1900-1999. One could repeatedly apply equation [3] to describe this process as

$$[4] \log(\text{IMR}_{1999})=100*C+\log(\text{IMR}_{1900})+100*\epsilon_t$$

or more generally as

$$[4'] \log(\text{IMR}_T)=T*C+\log(\text{IMR}_0)+T*\epsilon_t$$

As T approaches infinity the variance of  $\log(\text{IMR}_T)$  also approaches infinity. Under the null that  $\beta=0$  the distribution of the error term cannot be normal and the t-statistic ( $\beta/\text{sd}(\beta)$ ) will not follow the conventional t-distribution<sup>3</sup>.

Fortunately Dickey and Fuller have tabulated the appropriate distribution to use to test that  $\beta=0$  in equation [3].<sup>4</sup> The performance of the Dickey Fuller test is improved if equation [2] is augmented by adding additional lags of  $\Delta\log(\text{IMR})$  to “whiten” the remaining residual. A specification criterion such as the Akaike criterion can be used to select the appropriate number of lags to add in the augmented Dickey Fuller test. Since infant mortality has a regular downward drift, it is appropriate to include a constant downward term, modeled as a negative constant, C, that predicts a steady reduction in IMR.

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<sup>3</sup> Time series that can be characterized as conforming to equation [3] or [4'] are said to be non-stationary. Other terms for these processes include “random walks”, “Brownian Motion”, and “Integrated of Order 0”. See Hamilton, 1994. Hamilton JD. Times Series Analysis. Princeton, NJ: Princeton University Press; 1994.

<sup>4</sup> To conduct a Dickey-Fuller test one computes the conventional t-statistic that emerges after ordinary least squares regression of equation [3] and then compares this t-statistic to the  $\alpha$  cutoff values from the Dickey-Fuller distribution (2).



We applied Akaike selection tests to select the most appropriate lag structure and then ran augmented Dickey-Fuller tests in a model that included a deterministic trend in log IMR.<sup>5</sup> Support for the Matthew effect would require that the  $\beta$  coefficient is negative and statistically significant.

### **Results**

Figure 1 shows the pattern of decline in log (IMR) in the 22 countries. Inspection of these figures supports the inclusion of a linear deterministic downward trend in the statistical models. For comparison Figure 2 graphs a convex curve consistent with diminishing returns as well as a concave curve consistent with a Matthew Effect. From simple visual inspection of the IMR trends in figure 1 it is difficult to conclude whether IMR decline is generally concave or complex.

Table 1 shows the results of the augmented Dickey Fuller test for the null hypothesis that the percentage change in infant mortality is not related to the previous level of infant mortality. Even by the more stringent cutoff value of  $\alpha=0.10$  the test rejects the null hypothesis in only 3 of the 21 countries studied. In the three countries, in which  $\beta$  was significantly nonzero--Belgium, France, and Italy--  $\beta$  was negative. The test results thus rejected the existence of a Matthew effect in infant mortality for all but three countries. Table 1 shows the actual number of lags selected under the Akaike criterion, however the overall

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<sup>5</sup> The 4 year gap in German data (1942-1945) and the exclusion of the post war East German population led us to exclude Germany from the Dickey-Fuller analysis.

rejection of Matthew effects was robust when multiple different lag orders were applied to the data.

### **Discussion**

In general any time series can be represented as  $Y_t = C + \beta Y_{t-1} + \gamma \epsilon_t$ , where  $-1 \leq \gamma \leq 1$ . If  $|\gamma| < 1$  then any innovative random change in the time series are attenuated over time, but if  $|\gamma| = 1$  then random disturbances are remembered by the process indefinitely. Our findings suggest that  $\gamma = 1$  in  $\log(\text{IMR}_t) = C + \log(\text{IMR}_{t-1}) + \gamma \epsilon_t$ . In other words, transient random disturbances in infant mortality time series remain unattenuated over time. How might this occur? Societies produce the prevailing rate of infant mortality in the social and physical environment using their stock of physical, social, and human capital. Diffused innovations in parents' infant care practices, and in medical care endure rather than evaporate. These successful ways of caring for infants are remembered by mothers, fathers, nurses, and doctors. Hospitals, delivery rooms, infant car seats, vaccines, and neonatal ICUs are durable innovations. By the same token, epidemics of vertically transmitted diseases and parental neglect would have been similarly durable during the 20<sup>th</sup> century in the countries studied.

The results of this analysis are limited to only 21 countries and to only the dramatic period of change in infant mortality seen during the 20<sup>th</sup> century. For most of human history infant mortality rates have been unremittingly high. The notion of an improvement in infant mortality may have seemed inconceivable to our historical predecessors. But once improvements occur our results show that it is exceptional to find countries where higher IMR makes progress less likely.

A simple explanation for the exceptional existence of Matthew Effects is that the first one or two decades of the transition from the historical baseline of high infant mortality to the achievement of any sustained decline represent a concave transition (Figure 2). Populations that remain at the high historical baseline mortality are not actively engaged in producing lower mortality rates. The law of diminishing returns presumes intentional activity to maximize an objective (e.g. improved infant survival). Rational maximizers ordinarily undertake the most cost-effective measures first. Once the goal is near, the easiest things to do have already been done and progress slows. Comparing countries that have not even begun a mortality transition to countries that have started to lower infant mortality would falsely suggest that greater percentage mortality declines always occur in populations where mortality is lower. The existence of Matthew effects very early in the process of mortality decline could reflect a comparison between societies that have begun to engage in improving child health and societies that have not. Such a comparison says little about the future course of events in either group.

### **Conclusion**

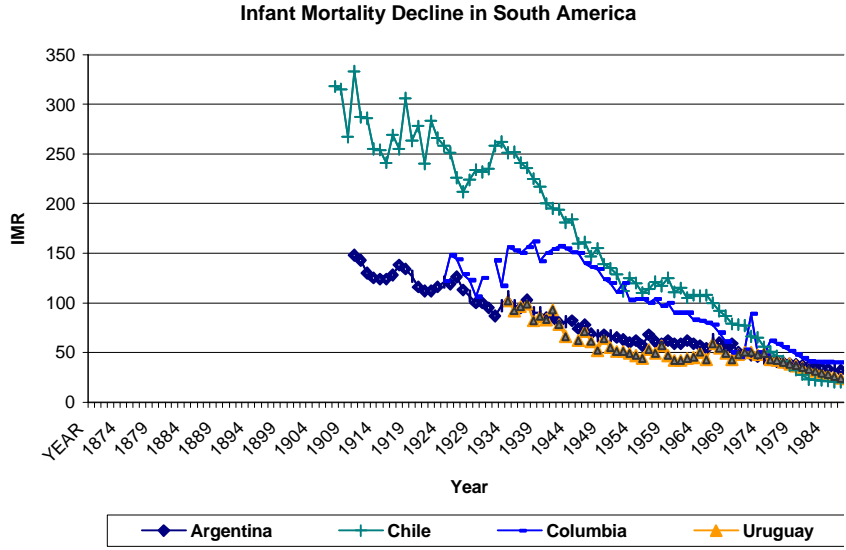
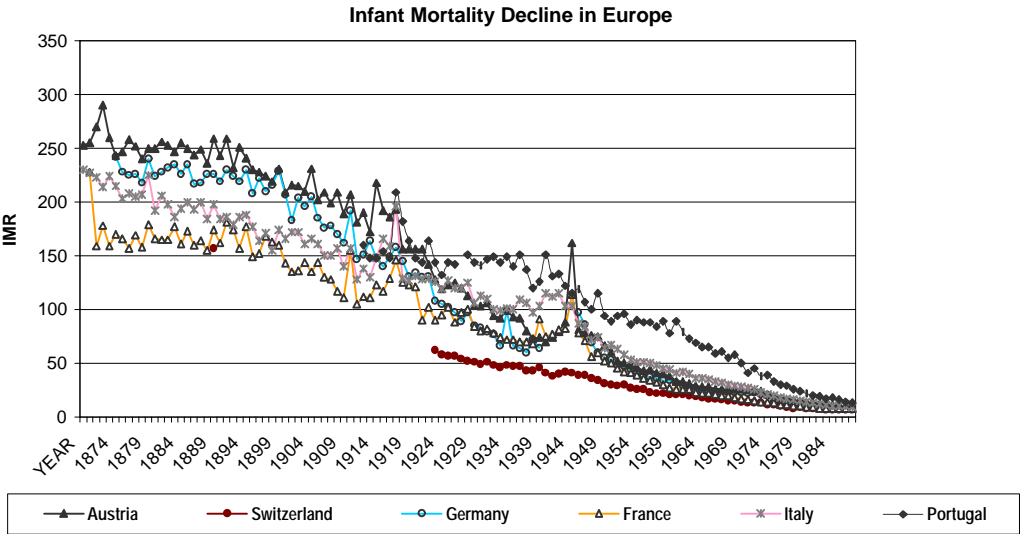
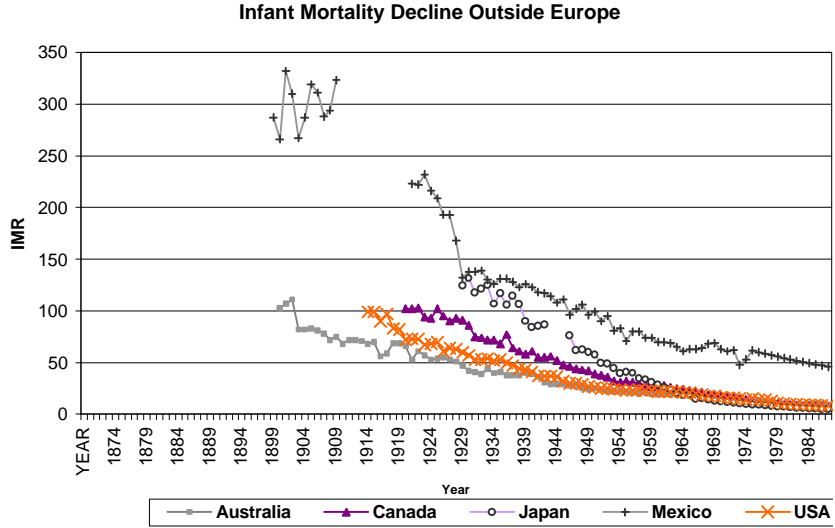
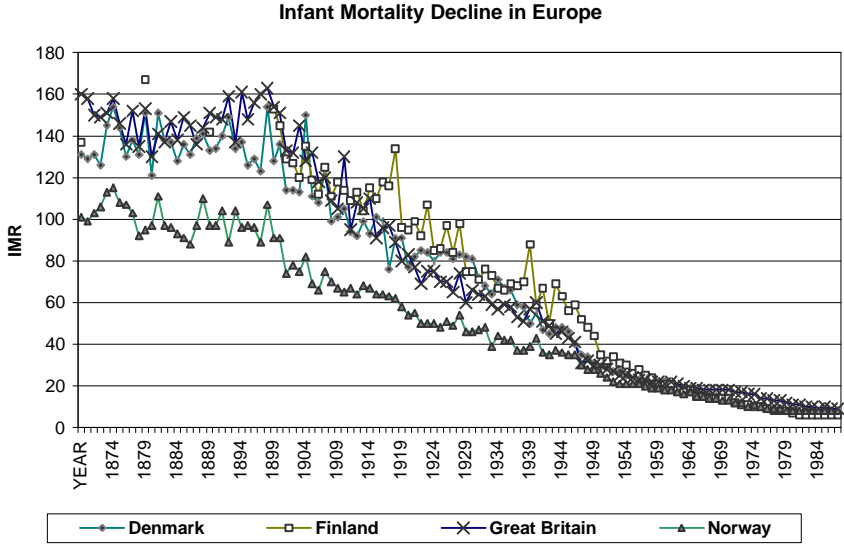
Analysis of data from 21 countries spanning a total of 1770 country-years, primarily during the 20<sup>th</sup> century shows that percentage reductions in infant mortality are seldom correlated with the pre-existing level of infant mortality. In only 3 countries ( Belgium, France, and Italy) there was a negative correlation between infant mortality level and its rate of decline.

These findings offer no support for a so-called Matthew effect as a general characterization of infant mortality decline.

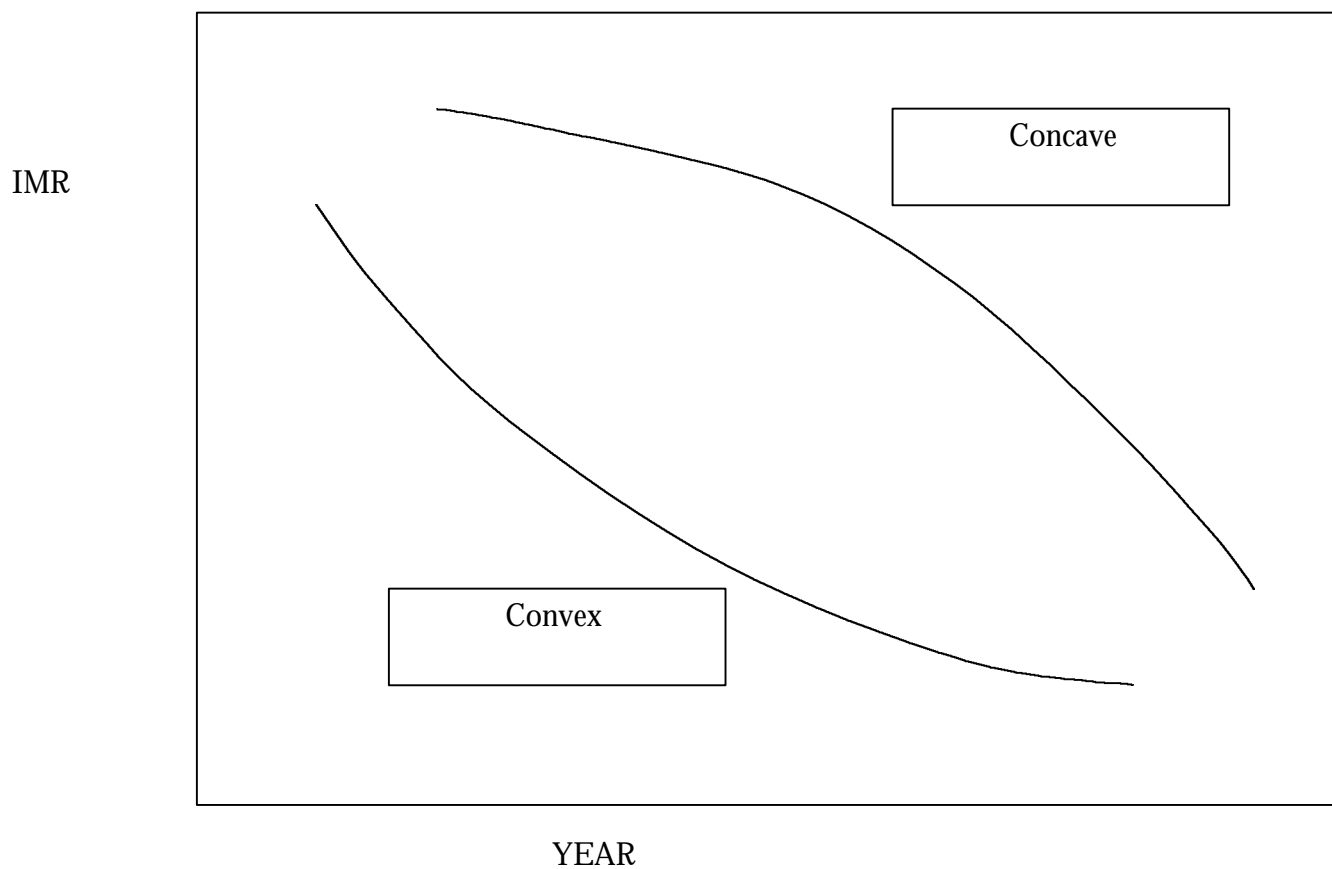
There is a real danger in a mistaken belief that Matthew effects are a general phenomenon in mortality decline. Donors and policymakers may wrongly conclude that greater health improvements can be achieved by investing a fixed pool of resources in countries that have already made progress. An erroneous belief in a Matthew effect could turn Matthew effects into a self-fulfilling prophecy.

The characterization that developing countries offer the promise of “good health at low cost” was based on the premise that the law of diminishing returns applies to population health (9). The pattern in the 20<sup>th</sup> century decline of infant mortality is characterized by randomly timed innovative improvements with long duration. These innovations were principally responsible for the fall of infant mortality and should be as successful or more so in countries with high infant mortality.

Figure 1. Twentieth Century Infant Mortality Decline



**Figure 2.** Concave vs. convex patterns of IMR decline. The concave pattern is consistent with the existence of Matthew effects. The convex pattern is consistent with diminishing returns to scale.



## Augmented Dickey Fuller Tests on IMR from 22 Countries

Country	Years	Lags by AIC*	ADF	5% cutoff	10% cutoff
<b>Argentina</b>	1911-1988	6	<b>-1.1</b>	-3.5	-3.26949
<b>Australia</b>	1890-1988	4	<b>-2.0</b>	-3.5	-3.248924
<b>Austria</b>	1870-1988	4	<b>-1.5</b>	-3.4	-3.215902
Belgium	1870-1988	7	-4.2	-3.5	-3.218288
<b>Canada</b>	1921-1988	8	<b>-2.0</b>	-3.5	-3.298478
<b>Switzerland</b>	1924-1988	2	<b>-0.5</b>	-3.5	-3.290173
<b>Chile</b>	1908-1988	6	<b>-3.1</b>	-3.5	-3.26372
<b>Colombia</b>	1925-1988, excl. 1932	5	<b>-2.2</b>	-3.5	-3.325439
<b>Denmark</b>	1870-1988	5	<b>-1.3</b>	-3.4	-3.216683
<b>Finland</b>	1900-1988	4	<b>-0.7</b>	-3.5	-3.247476
France	1871-1988	1	-3.6	-3.4	-3.214379
<b>Great Britain</b>	1870-1988	8	<b>-2.3</b>	-3.5	-3.219113
Italy	1870-1988	1	-4.0	-3.4	-3.213638
<b>Japan</b>	1930-1988 exc.1944-1946	8	<b>-2.6</b>	-3.5	-3.393706
<b>Mexico</b>	1900-1910, 1922-1988	8	<b>2.1</b>	-3.5	-3.295617
<b>Netherlands</b>	1900-1988	2	<b>-1.9</b>	-3.5	-3.244683
<b>Norway</b>	1870-1988	1	<b>-3.1</b>	-3.4	-3.213638
<b>Portugal</b>	1913-1988, excl. 1928	8	<b>-0.7</b>	-3.5	-3.304504
<b>Sweden</b>	1870-1988	8	<b>-0.4</b>	-3.5	-3.219113
<b>Uruguay</b>	1935-1988, excl. 1945	7	<b>-3.2</b>	-3.5	-3.219113
<b>USA</b>	1915-1988	8	<b>-1.2</b>	-3.5	-3.282638

\*AIC=Akaike Information Criterion    \*\* Bold results are countries where IMR is a random walk

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